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# Extended NJL model with eight-quark interactions*

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**Abstract.** We present the results obtained in the three-flavour ($N_f = 3$) Nambu–Jona-Lasinio model which is extended by the $U_A(1)$ breaking six-quark 't Hooft interaction and eight-quark interactions. We address the problem of stability, and some phenomenological consequences of the models with multi-quark interactions.

A number of instructive models in low-energy QCD assume the existence of underlying multi-quark interactions and their importance for the physics of hadrons. They are efficient in the description of spontaneous chiral symmetry breaking ($\chi$SB), and in the study of the quark structure of light mesons. The Nambu–Jona-Lasinio (NJL) model [1] is a well-known example, where the local chiral symmetric four-fermion interactions under some conditions lead to the formation of fermion-antifermion bound states and as a result describe the $\chi$SB phenomenon. A modified form of these interactions has been widely considered to derive the QCD effective action at large distances [2]-[5].

One might ask if higher order multi-quark interactions are of importance. Indeed, along the lines suggested by an instanton-gas model, it can be argued [6] that there exists an infinite set of multi-quark terms in the effective quark Lagrangian starting from the NJL four-quark piece. In particular, the famous 't Hooft determinantal interaction [7] automatically appears if one keeps only the zero mode contribution in the mode expansion of the effective Lagrangian. This $2N_f$ multi-quark term manifestly violates the $U_A(1)$ axial symmetry of the QCD Lagrangian, offering a way out of the $U_A(1)$ problem.

The structure of QCD-motivated models at low energies with effective many-fermion interaction and a finite cutoff in the symmetry-breaking regime has been also considered in [8], where the authors, using the $1/N_c$ arguments and the fine-tuning condition in providing the scale invariance, classified the set of quasilocal vertices relevant for dynamical $\chi$SB. It has been found this way that in such effective models the vertices with four, six and eight fermions only should be retained in four-dimensional space-time.

Thus, it is tempting to consider the intuitive picture that describes the QCD vacuum based on a series of multi-quark interactions reflecting several tractable

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features of QCD, which include aspects of chiral symmetry and of the $1/N_c$ expansion. The bosonization of quark degrees of freedom leads then to the desirable effective Lagrangian with matter fields and a stable chiral asymmetric vacuum.

The NJL-type model with the $U_A(1)$ axial symmetry breaking by the 't Hooft determinant has been studied in the mean field approximation [9]-[14] for a long time. Numerous phenomenological applications show that the results of such an approach meet the expectations. Nevertheless, in this picture there is an apparent problem: the mean field potential is unbounded from below, and the 't Hooft term is the direct source of such an instability. A consistent approach requires obviously a stable hadronic vacuum.

To cure this disease of the model we consider the system of light quarks $u, d, s$ with multi-fermion interactions described by the Lagrangian

$$\mathcal{L}_{\text{eff}} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m)q + \mathcal{L}_{4q} + \mathcal{L}_{6q} + \mathcal{L}_{8q}. \tag{1}$$

Here, the quark fields $q$ have colour ($N_c = 3$) and flavour indices which are suppressed. We suppose that four-, six-, and eight-quark interactions $\mathcal{L}_{4q}$, $\mathcal{L}_{6q}$, $\mathcal{L}_{8q}$ are effectively local, for it is known that meson physics in the large $N_c$ limit is described by a local Lagrangian of this type [15]. The interaction Lagrangians $\mathcal{L}_{4q}$ and $\mathcal{L}_{6q}$ of the model in the scalar and pseudoscalar channels is given by two terms

$$\mathcal{L}_{4q} = \frac{G}{2}\left[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2\right], \tag{2}$$

$$\mathcal{L}_{6q} = \kappa(\det \bar{q}P_L q + \det \bar{q}P_R q). \tag{3}$$

The first one is the $U_L(3)\times U_R(3)$ chiral symmetric interaction specifying the local part of the effective four-quark Lagrangian in channels with quantum numbers $J^P = 0^+, 0^-$. The Gell-Mann flavour matrices $\lambda_a,\ a = 0, 1, \ldots, 8$, are normalized such that $\mathrm{tr}(\lambda_a\lambda_b) = 2\delta_{ab}$. The second term represents the 't Hooft determinantal interactions [7]. The matrices $P_{L,R} = (1 \mp \gamma_5)/2$ are projectors and the determinant is over flavour indices. The determinantal interaction breaks explicitly the axial $U_A(1)$ symmetry and Zweig's rule. The global chiral $SU(3)_L \times SU(3)_R$ symmetry of the Lagrangian (1) at $m = 0$ is spontaneously broken to the $SU(3)$ group, showing the dynamical instability of the fully symmetric solutions of the theory. In addition, the current quark mass $m$, being a diagonal matrix in flavour space with elements $\mathrm{diag}(m_u, m_d, m_s)$, explicitly breaks this symmetry down, retaining only the reduced $SU(2)_I \times U(1)_Y$ symmetries of isospin and hypercharge conservation, if $m_u = m_d \neq m_s$.

The eight-quark Lagrangian which describes the spin zero interactions contains two terms: $\mathcal{L}_{8q} = \mathcal{L}_{8q}^{(1)} + \mathcal{L}_{8q}^{(2)}$ [16], where

$$\mathcal{L}_{8q}^{(1)} = 8g_1\left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_i)\right]^2, \tag{4}$$

$$\mathcal{L}_{8q}^{(2)} = 16g_2\left[(\bar{q}_i P_R q_m)(\bar{q}_m P_L q_j)(\bar{q}_j P_R q_k)(\bar{q}_k P_L q_i)\right]. \tag{5}$$

Here the sum is taken over flavour indices $i, j, m = 1, 2, 3$; $\mathcal{L}_{8q}$ is a $U_L(3)\times U_R(3)$ symmetric interaction with OZI-violating effects in $\mathcal{L}_{8q}^{(1)}$. The first term $\mathcal{L}_{8q}^{(1)}$ coincides with the OZI-violating eight-quark interactions considered in [17]. The second term $\mathcal{L}^{(2)}_{8q}$ represents interactions without violation of Zweig's rule. $\mathcal{L}_{8q}$ is the most general Lagrangian which describes the spin zero eight-quark interactions without derivatives. It is the lowest order term in number of quark fields which is relevant to the case. We restrict our consideration to these forces, because in the long wavelength limit the higher dimensional operators are suppressed.

We view the main role of eight-quark forces considered as folows:
(i) They are of vital importance for the stability of the ground state built from four and six-quark interactions: the quark model considered follows the general trend of spontaneous breakdown of chiral symmetry and possesses a globally stable ground state, when relevant inequalities in terms of the coupling constants hold, $g_1 > 0, g_1 + 3g_2 > 0, Gg_1 > (\kappa/16)^2$ [16].
(ii) The low lying scalar and pseudoscalar mesonic spectra are almost insensitive to the eight-quark forces [18].
(iii) The 8q-interactions play an important role in determining the critical temperature, $T_c$, at which transitions occur from the dynamically broken chiral phase to the symmetric phase, lowering the value of $T_c$ with growing strength of the 8q couplings [19].
(iv) The multi-quark interactions introduce new additional features to the catalysis of dynamical symmetry breaking by a constant magnetic field H in $3 + 1$ dimensions: the first minimum catalyzed by a constant magnetic field (that is, a slowly varying field) is then smoothed out with increasing H at the characteristic scale $H \sim 10^{19}$G. The reason is that multi-quark forces generate independently another local minimum associated with a larger dynamical fermion mass. This state may exist even for multi-quark interactions with a subcritical set of couplings and is globally stable with respect to a further increase of the magnetic field [20].
(v) The OZI-violating terms with coupling strength $g_1$ affect the mechanism of $\chi$SB: starting from some critical value of the coupling $g_1 = (g_1)_{crit}$ the $\chi$SB is induced by the 6q 'tH̃ooft interactions, as opposed to the 4q NJL forces at $g_1 < (g_1)_{crit}$ [18].
(vi) It turns out that the mesonic spectra built on the spontaneously broken vacuum induced by the 't Hooft interaction strength, as opposed to the commonly considered case driven by the four-quark coupling, undergo a rapid crossover to the unbroken phase with a slope and at a temperature which is regulated by the strength of the OZI violating eight-quark interactions. This strength $g_1$ can be adjusted in consonance with the four-quark coupling G (keeping the remaining model parameters fixed) and leaves the spectra unchanged, except for the sigma meson mass which decreases. A first order transition behavior is also a possible solution within the present approach at large $g_1$ [21].
(vii) They may be also of importance in decays and scattering, not considered so far.

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