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Searching for tetraquarks on the lattice

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Abstract. The observed mass pattern of scalar resonances below 1 GeV gives preference to the tetraquark assignment over the conventional $\bar{q}q$ assignment for these states. We present a search for tetraquarks with isospins $0, 1/2, 1$ in lattice QCD using diquark anti-diquark interpolators [1]. We determine three energy levels for each isospin using the variational method. The ground state is consistent with the scattering state, while the two excited states have energy above 2 GeV. Therefore we find no indication for light tetraquarks at our range of pion masses $344 - 576$ MeV.

1 Introduction

The observed mass pattern of scalar mesons below 1 GeV, illustrated in Fig. 1, does not agree with the expectations for the conventional $\bar{q}q$ nonet. The observed ordering $m_\kappa < m_{a0(980)}$ can not be reconciled with the conventional $\bar{u}s$ and $\bar{u}d$ states since $m_{\bar{u}s} > m_{\bar{u}d}$ is expected due to $m_s > m_d$. This is the key observation which points to the tetraquark interpretation, where light scalar tetraquark resonances may be formed by combining a "good" scalar diquark

$$[qQ]_a \equiv \epsilon_{abc}[q_b^T C\gamma_5 Q_c - Q_b^T C\gamma_5 q_c] \quad (\text{color and flavor anti} - \text{triplet}) \tag{1}$$

with a "good" scalar anti-diquark $[\bar{q}\bar{Q}]_a$ [2]. The states $[qq]_{\bar{3}_f,\bar{3}_c}\ [\bar{q}\bar{q}]_{3_f,3_c}$ form a flavor nonet of color-singlet scalar states, which are expected to be light. In this case, the $I = 1$ state $[us][\bar{d}\bar{s}]$ with additional valence pair $\bar{s}s$ is naturally heavier than the $I = 1/2$ state $[ud][\bar{d}\bar{s}]$ and the resemblance with the observed spectrum speaks for itself.

Light scalar tetraquarks have been extensively studied in phenomenological models [2], but there have been only few lattice simulations [3–6]. The main obstacle for identifying possible tetraquarks on the lattice is the presence of the scattering contributions in the correlators. All previous simulations considered only $I = 0$ and a single correlator, which makes it difficult to disentangle tetraquarks from the scattering. The strongest claim for σ as tetraquark was obtained for $m_\pi \simeq 180 - 300$ MeV by analyzing a single correlator using the sequential empirical Bayes method [4]. This result needs confirmation using a different method (for example the variational method used here) before one can claim the existence of light tetraquarks on the lattice with confidence.

We study the whole flavor pattern with $I = 0, 1/2, 1$ and our goal is to find out whether there are any tetraquark states on the lattice, which could be identified with observed resonances $\sigma(600)$, $\kappa(800)$ and $a_0(980)$. Our methodology and results are explained in more detail in [1].

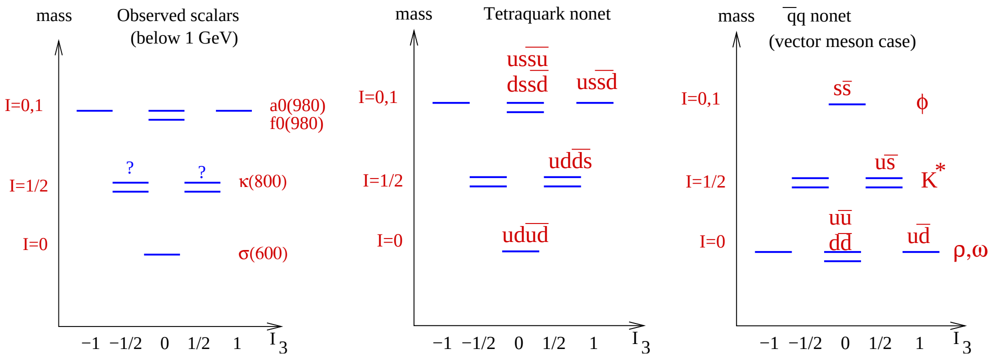

Fig.1. Schematic illustration of the observed spectrum for scalar mesons below 1 GeV (left), together with the expected mass spectrum for the nonet of scalar tetraquarks (middle), compared with a typical $\bar{q}q$ spectrum (right).

2 Lattice simulation

In our simulation, tetraquarks are created and annihilated by diquark anti-diquark interpolators

$$\mathcal{O}^{I=0} = [ud][\bar{u}\bar{d}] \;, \quad \mathcal{O}^{I=1/2} = [ud][\bar{d}\bar{s}] \;, \quad \mathcal{O}^{I=1} = [us][\bar{d}\bar{s}] \;. \tag{2}$$

In each flavor channel we use three different shapes of interpolators at the source and the sink

$$\mathcal{O}_1^I = [q_n Q_n][\bar{q}'_n \bar{Q}'_n] \;, \quad \mathcal{O}_2^I = [q_w Q_w][\bar{q}'_w \bar{Q}'_w] \;, \quad \mathcal{O}_3^I = [q_n Q_w][\bar{q}'_w \bar{Q}'_n] \;. \tag{3}$$

Here q_n and q_w denote Jacobi-smeared quarks with approximately Gaussian shape and two different widths: "narrow" (n) and "wide" (w) [8].

In order to extract energies E_n of the tetraquark system, we compute the 3×3 correlation matrix for each isospin

$$\begin{aligned} C_{ij}^I(t) &= \sum_{x} e^{ipx} \langle 0|\mathcal{O}_i^I(x,t)\mathcal{O}_j^{I\dagger}(0,0)|0\rangle_{\mathbf{p}=0} \\ &= \sum_n \langle 0|\mathcal{O}_i^I|n\rangle\langle n|\mathcal{O}_j^{I\dagger}|0\rangle\, e^{-E_n t} = \sum_n w_n^{ij} e^{-E_n t} \;. \end{aligned}$$

Like all previous tetraquark simulations, we use the quenched approximation and discard the disconnected quark contractions. These two approximations allow a definite quark assignment to the states and discard $[\bar{q}\bar{q}][qq] \leftrightarrow \bar{q}q \leftrightarrow vac$ mixing, so there is even a good excuse to use them in these pioneering studies. We work on two volumes $V = L^3 \times T = 16^3 \times 32$ and $12^3 \times 24$ at the same lattice spacing $a = 0.148$ fm [8]. The quark propagators are computed from the Chirally Improved Dirac operator [7]. We use $m_l a = m_{u,d} a = 0.02,\ 0.04$ and 0.06 corresponding to $m_\pi = 344,\ 475$ and 576 MeV, respectively. The strange quark mass is $m_s a = 0.08$. The analysis requires the knowledge of the kaon masses, which are $528,\ 576,\ 620$ MeV for $m_l a = 0.02,\ 0.04,\ 0.06$.

The extraction of the energies from the correlation functions using a multi-exponential fit $C_{ij} = \sum_n w_n^{ij} e^{-E_n t}$ is unstable. A powerful method to extract excited state energies is the variational method, so we determine the eigenvalues and eigenvectors from the hermitian 3×3 matrix $C(t)$

$$C(t)\boldsymbol{v}_n(t) = \lambda_n(t)\boldsymbol{v}_n(t)\ . \tag{4}$$

The resulting large-time dependence of the eigenvalues

$$\lambda_n(t) = w_n e^{-E_n t}\,[1 + \mathcal{O}(e^{-\Delta_n t})] \tag{5}$$

allows a determination of energies $E_{0,1,2}$ and spectral weights $w_{0,1,2}$. The eigenvectors $\boldsymbol{v}_n(t)$ are orthogonal and represent the components of physical states in terms of variational basis (3).

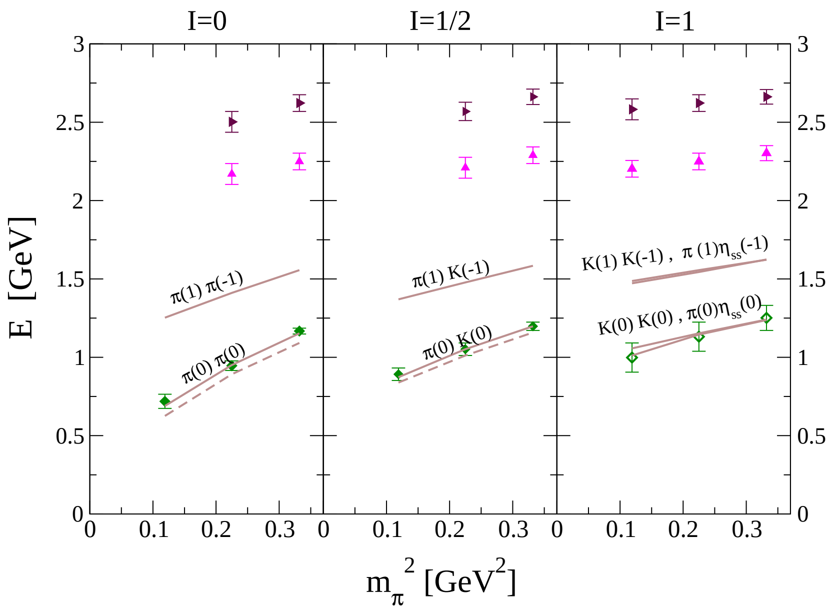

Fig.2. The symbols present the three lowest energy levels from tetraquark correlators in $I = 0, 1/2, 1$ channels at lattice volume $16^3 \times 32$. The lines give analytic energy levels for scattering states: full lines present non-interacting energies (6), while dashed lines take into account tree-level energy shifts.

3 Results

Our interpolators couple to the tetraquarks, if these exist, but they also unavoidably couple to the scattering states $\pi\pi$ ($I=0$), $K\pi$ ($I=1/2$) and $K\bar{K}$, $\pi\eta_{ss}$ ($I=1$) as well as to the heavier states with the same quantum numbers. The lowest few energy levels of the scattering states $P_1(\mathbf{k})P_2(-\mathbf{k})$

$$E^{P_1(\mathbf{j})P_2(-\mathbf{j})} \simeq m_{P1} + m_{P2},\ \ldots,\ \sqrt{m_{P1}^2 + \left(\frac{2\pi\mathbf{j}}{L}\right)^2} + \sqrt{m_{P2}^2 + \left(\frac{2\pi\mathbf{j}}{L}\right)^2},\ldots \quad (6)$$

are well separated for our L and we have to identify them before attributing any energy levels $E \simeq m_{\sigma,\kappa,a_0}$ to the tetraquarks.

Our main result is presented in Fig. 2, where the energy levels of the tetraquark system for all isospin channels are shown. These energy levels $E_{0,1,2}$ are extracted from $\lambda_{0,1,2}(t)$ with fitting details[1] given in [1].

The *ground state* energies in $I = 0, 1/2$ and 1 channels are close to $2m_\pi$, $m_\pi + m_K$ and $2m_K$, $m_\pi + m_{\eta_{ss}}$, respectively, which indicates that all ground states correspond to the scattering states $P_1(\mathbf{0})P_2(\mathbf{0})$. Another indication in favor of this interpretation comes from the study of the volume dependence of the spectral weights w, defined in (5). For the ground state we get $w_0(L=12)/w_0(L=16) \simeq 16^3/12^3$, as shown in Fig. 3. This agrees with the expected dependence $w_0 \propto 1/L^3$ for scattering states [4], which follows from the integral over the loop momenta $\int \frac{d\mathbf{k}}{(2\pi)^3} f(\mathbf{k},t) \to \frac{1}{L^3}\sum_{\mathbf{k}} f(\mathbf{k},t)$ with $dk_i = 2\pi/L$.

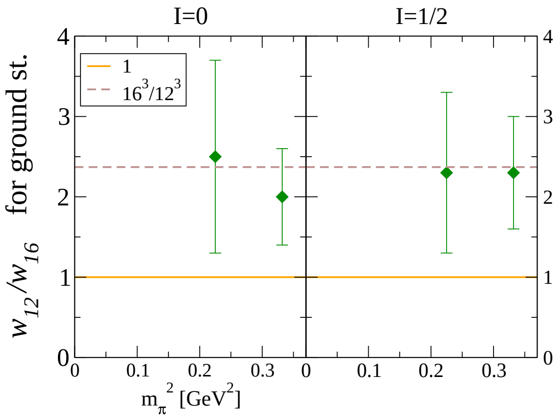

Fig.3. The ratio of spectral weights $w_0(L=12)/w_0(L=16)$ for $I = 0, 1/2$ as computed from the ground state eigenvalues for two volumes L^3.

The most important feature of the spectrum in Fig. 2 is a large gap above the ground state: *the first and the second excited states* appear only at energies above

[1] We observed a non-conventional time dependence of $\lambda_0(t)$ near $t \simeq T/2$, which is discussed in detail in [1].

2 GeV. Whatever the nature of these two excited states are, they are much too heavy to correspond to $\sigma(600)$, $\kappa(800)$ or $a_0(980)$, which are the light tetraquark candidates we are after. The two excited states may correspond to $P_1(\mathbf{k})P_2(-\mathbf{k})$ with higher $\mathbf{k}$ or to some other energetic state. We refrain from identifying the excited states with certain physical objects as such massive states are not a focus of our present study.

At first sight it is surprising that there are no states close to the energies of $P_1(1)P_2(-1)$ with $|\mathbf{k}| = 2\pi/L$ in the spectrum of Fig. 2. In [1] we argue that this is due to the fact that our basis (3) does not decouple the few lowest scattering states to separate eigenvalues. Our data supports the hypothesis that the few lowest scattering states contribute to the ground state eigenvalue.

4 Conclusions and outlook

We find no indication for light tetraquarks at our range of pion masses 344 − 576 MeV. However, one should not give up hopes for finding these interesting objects on the lattice. Indeed, our simulation does not exclude the possibility of finding tetraquarks for lighter $m_{u,d}$ or for a larger (different) interpolator basis. A stimulating lattice indication for σ as a tetraquark state at $m_\pi = 182 - 300$ MeV has already been presented in [4].

The present and past pioneering quenched tetraquark simulations, which discard disconnected diagrams, provide valuable information on the states with a definite quark assignment. The final conclusions will have to await dynamical simulations incorporating both disconnected quark diagrams and the $\bar{q}\bar{q}qq \leftrightarrow \bar{q}q \leftrightarrow vac$ mixing.

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