BLED WORKSHOPS
IN PHYSICS
VOL. 3, NO. 3

Proceedins of the Mini-Workshop
Quarks and hadrons *(p. 59)*
Bled, Slovenia, July 7-14, 2002

The Spin-Spin splitting of Bottomium – an Estimate based on leptonic decays of vector mesons *

D. Janc[a] and M. Rosina[a,b]

[b] J. Stefan Institute, 1000 Ljubljana, Slovenia
[a] Faculty of Mathematics and Physics, University of Ljubljana, 1000 Ljubljana, Slovenia

Abstract. The mass of η_b is estimated to be about 120 MeV below the mass of Υ, similarly as in the case of charmonium. The estimate is based on the experimental fact that the widths $\Gamma_{e^+e^-}$ for Υ and J/ψ are equal (apart from the factor 4 due to quark charges), and the hypothesis that both the spin-spin splitting and $\Gamma_{e^+e^-}$ of vector mesons are proportional to the density at the origin divided by quark mass squared.

1 Introduction

The comparison of the spin-spin splitting in charmonium and bottomium represents a valuable test of our understanding of the effective quark-quark interaction. Since the $b\bar{b}$ ground state, the η_b meson, has not yet been reliably observed, the interest in this state gives a strong motivation to experimentalists. Moreover, since theoretical predictions of the spin-spin splitting $\Delta m = m(\Upsilon) - m(\eta_b)$ vary strongly, this is also a challenge to theorists. (The estimates from perturbative QCD, from potential models and from lattice-inspired potential models lie in the range between 30 and 140 MeV.)

Recently, one candidate for η_b has been reported [1], with its mass $160 \pm 20 \pm 20\mathrm{GeV}/c^2$ below Υ. Though still inconclusive, such a large difference encourages further studies whether quark models or lattice calculations allow a high value for Δm.

We present a theoretical estimate which is based on general properties of the constituent quark models and depends only weakly on the details of the models.

2 Zero order approximation

2.1 Leptonic decay

The estimate for the mass of η_b is based on the remarkable fact that the partial width $\Gamma_{e^+e^-}(\Upsilon) = 1.32$ keV and $\Gamma_{e^+e^-}(J/\psi) = 5.26$ keV are equal (apart from the factor 4 due to quark charges). Assuming point-like quarks the leptonic decay of vector mesons can be represented by the graph in Fig. 1. The $Q\bar{Q}\gamma$ vertex can be

---

* Talk delivered by D. Janc.

expressed as $z_Q e\sqrt{\rho(0)}$ where $z_Q e$ is the quark charge. Then the partial width is described by van Royen - Weisskopf formula:

$$\Gamma^0_{e^+e^-} = z_Q^2\,\rho(0)\,\frac{16\pi\alpha^2}{m^2}. \tag{1}$$

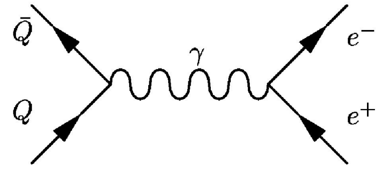

Fig. 1. The leptonic decay of vector mesons

Since the experimental values of $\Gamma_{e^+e^-}/z_Q^2$ are equal for Υ and J/ψ this fixes the ratio of the densities at the origin: $\rho(0)$ are proportional to m^2 where m is the vector meson mass. We conclude that (up to the assumed order of approximation) $\rho_\Upsilon(0)/m^2_\Upsilon = \rho_{J/\psi}(0)/m^2_{J/\psi}$.

2.2 Spin-spin splitting

In the nonrelativistic constituent quark model the spin-spin potential between heavy quarks is assumed to be the result of one gluon exchange between quarks which gives

$$\Delta H_{oge} = \frac{4}{3}\frac{2\pi\alpha_s}{3m_Q^2}\delta(\mathbf{r})\sigma_1\cdot\sigma_2$$

For very heavy quarks the spin dependent part of this interaction can be treated perturbatively and it yields the spin splitting between vector and pseudoscalar meson Δm proportional to $\rho(0)/m_Q^2$. If the quark mass is $m_Q = \frac{1}{2}\,m$, Δm is proportional to $\Gamma_{e^+e^-}/z_Q^2$. Since the latter is equal for bottomium and charmonium, it follows $\Delta m(\Upsilon) = \Delta m(J)/\psi = 117$ MeV. This prediction is within the error of the experimental candidate [1], but we have to wait for new experiments.

3 Corrections

It is well known, that there are large corrections to the van Royen - Weisskopf formula. Apart from first order correction in α_s, there are two additional corrections due to approximations which are implied in Eq (1). First approximation is, that we consider quarks to be point-like and the second is, that we neglect momentum of quarks inside the meson. We can write the partial width as

$$\Gamma_{e^+e^-} = R\Gamma^0_{e^+e^-}$$

where the factor R is 1, if we ignore this correction, or it is $R = (1 - 16\alpha_s/3\pi)$ if we consider just first order corrections. The current values for α_s at charmonium and bottomium relevant energies are α_s(3.1GeV)= 0.249 ± 0.010 and α_s(9.46GeV)= 0.178 ± 0.005, so neglecting other corrections, we have $R = 0.57$ and $R = 0.70$ respectively. It was shown [2] that the refinement due to momentum of quarks inside the meson is also of the same order, and is larger in charmonium as in bottomium. This correction depends on the potential model in which one calculate the meson wave function. If one considers only the first order corrections in α_s and refinements due to quarks momentum, one obtains for both charmonium and bottomium an overall correction to the original van Royen - Weisskopf formula $R = 0.85 \pm 0.05$. Since the factor R is almost the same in bottomium as in charmonium, we can again assume that the densities at the origin are still proportional to m^2.

4 Test – the η_c(2S) meson

We now test the assumptions of our estimation by looking into the charmonium sector, where we estimate the spin splitting between the 2S states η_c (2S) and ψ (2S). There are two very different experimental results about the mass of η_c (2S) state. The old results from 1982 is 3594 ± 5 MeV [4] while the Belle Collaboration reported the observation of $\eta_c(2S)$ in exclusive $B \to KK_sK^-\pi^+$ decay [3] with the mass 3654± 6 MeV. We can estimate the spin splitting from the leptonic decay width of ψ (2S) which is known to a large accuracy $\Gamma_{e^+e^-}(\psi(2S)) = 2.19 \pm 0.15$ keV:

$$m_{\psi(2s)} - m_{\eta_c(2S)} = \frac{\Gamma_{e^+e^-}(\psi(2S))}{\Gamma_{e^+e^-}(J/\psi)} \cdot \frac{m^2_{\psi(2S)}}{m^2_{J/\psi}} \cdot \left(m_{J/\psi} - m_{\eta_c}\right) =$$
$$(0.42 \pm 0.06) \cdot 1.41 \cdot 117\text{MeV} = 69\text{MeV} \pm 10\text{MeV}.$$

meson	m[MeV]	$\frac{\Gamma^{exp}_{e^+e^-}}{(3z_q)^2}$[keV]	$\Delta m_{exp.}$[MeV]	$\Delta m_{predict.}$[Mev]
$\eta_c(1S)$	2979.7		117	117 (input)
J/ψ	3096.9	1.32 ± 0.09		
$\eta_c(2S)$	3654 ± 6 [3] 3594 ± 5 [4]		32 ± 6 92 ± 5	69 ± 10
$\psi(2S)$	3686	0.55 ± 0.04		
$\eta_b(1S)$	9300 ± 40		160 ± 40	117
Υ	9460	1.32 ± 0.07		

Table 1. Second column: masses of the heavy mesons from [3] and [4]. Third column: leptonic decay width of vector meson. Fourth column: experimental data for spin-spin splitting. Last column: our prediction on spin-spin splitting.

Since the evaluation of the η_c(2S) mass in [3] is still in progress, we have to wait with our conclusions about our scheme.

References

1. The ALEPH Collaboration: CERN-EP/2002-009 (2002)
2. F. Bissey, J.-J. Dugne and J.-F. Mathiof, Eur. Phys. J **C24** (2002) 101
3. S.-K. Choi et al., (Belle Collaboration), Phys. Rev. Lett. **89** (2002) 102001; Erratum, Phys. Rev. Lett. **89** (2002) 12901
4. C. Edwards et al., Phys. Rev. Lett. **48** (1982) 70