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Mixed symmetric baryon multiplets in large N_c QCD: two and three flavours*

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Abstract. We propose a new method to study mixed symmetric multiplets of baryons in the context of the $1/N_c$ expansion approach. The simplicity of the method allows to better understand the role of various operators acting on spin and flavour degrees of freedom. The method is tested on two and three flavours. It is shown that the spin and flavour operators proportional to the quadratic invariants of $SU_S(2)$ and $SU_F(3)$ respectively are dominant in the mass formula.

1 Introduction

The $1/N_c$ expansion method proposed by 't Hooft [1] is a valuable tool to study nonperturbative dynamics in a perturbative approach, in terms of the parameter $1/N_c$ where N_c is the number of colors. The double line diagrammatic method of 't Hooft implemented by Witten [2] to describe baryons gives convenient power counting rules for Feynman diagrams. According to Witten's intuitive picture, a baryon containing N_c quarks is seen as a bound state in an average self-consistent potential of a Hartree type and the corrections to the Hartree approximation are of order $1/N_c$. These corrections capture the key phenomenological features of the baryon structure.

Ten years after 't Hooft's work, Gervais and Sakita [3] and independently Dashen and Manohar in 1993 [4] discovered that QCD has an exact contracted $SU(2N_f)_c$ symmetry when $N_c \to \infty$, N_f being the number of flavors. For ground state baryons the $SU(2N_f)$ symmetry is broken by corrections proportional to $1/N_c$. Since 1993-1994 the $1/N_c$ expansion provided a systematic method to analyze baryon properties such as ground state masses, magnetic moments, axial currents, etc [5–8].

A few years later the $1/N_c$ expansion method has been extended to excited states also in the spirit of the Hartree approximation [9]. It was shown that for mixed symmetric states the $SU(2N_f)$ breaking occurs at order N_c^0 instead of $1/N_c$ as for the ground and symmetric excited states.

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Presently a lattice test of $1/N_c$ baryon masses relations has been performed [10]. The lattice data clearly display both the $1/N_c$ and the SU(3) flavour symmetry breaking hierarchies.

Also, it was shown that the NN potential has an $1/N_c^2$ expansion and the strengths of the leading order central, spin-orbit, tensor and quadratic spin-orbit forces gave a qualitative understanding of the phenomenological meson exchange models [11].

2 The mass formula

Here we are concerned with baryon spectra. The general form of the baryon mass operator is [12]

$$M = \sum_i c_i O_i + \sum_i d_i B_i \tag{1}$$

with the operators O_i having the general form

$$O_i = \frac{1}{N_c^{n-1}} O_\ell^{(k)} \cdot O_{SF}^{(k)}, \tag{2}$$

where $O_\ell^{(k)}$ is a k-rank tensor in O(3) and $O_{SF}^{(k)}$ a k-rank tensor in SU(2), but invariant in SU(N_f). The latter is expressed in terms of SU(N_f) generators S^i, T^a and G^{ia} acting on spin, flavour and spin-flavour respectively. For the ground state one has $k = 0$. Excited states require $k = 1$ terms, which correspond to the angular momentum component and the $k = 2$ tensor term

$$L_q^{(2)ij} = \frac{1}{2}\left\{L_q^i, L_q^j\right\} - \frac{1}{3}\delta_{i,-j} \mathbf{L}_q \cdot \mathbf{L}_q \,. \tag{3}$$

The first factor in (2) gives the order $\mathcal{O}(1/N_c)$ of the operator in the series expansion and reflects Witten's power counting rules. The lower index i represents a specific combination of generators, see examples below. The B_i are SU(3) breaking operators. In the linear combination, Eq. (1), c_i and d_i encode the QCD dynamics and are obtained from a fit to the existing data. It is important to find regularities in their behaviour [13] and search for a possible compatibility with quark models [14].

A considerable amount of work has been devoted to ground state baryons summarized in several review papers as, for example, [5–7]. The ground state is described by the symmetric representation $[N_c]$. For $N_c = 3$ this becomes [3] or [**56**] in an SU(6) dimensional notation.

In the following we shall concentrate on the description of excited states only and the motivation will be obvious.

3 Excited states

Excited baryons can be divided into SU(6) multiplets, as in the constituent quark model. If an excited baryon belongs to the [**56**]-plet the mass problem can be treated similarly to the ground state in the flavour-spin degrees of freedom, but one has to take into account the presence of an orbital excitation in the space part of the wave function [15,16]. If the baryon belongs to the mixed symmetric representation [21], or [**70**] in SU(6) notation, the treatment becomes much more complicated.

There is a standard way to study the [**70**]-plets which is related to the Hartree approximation [9]. An excited baryon is described by symmetric core plus an excited quark coupled to this core, see *e.g.* [17–20]. In that case the core can be treated in a way similar to that of the ground state. In this method each SU($2N_f$) $\times$ O(3) generator is splitted into two terms

$$S^i = s^i + S^i_c; \quad T^a = t^a + T^a_c; \quad G^{ia} = g^{ia} + G^{ia}_c, \quad \ell^i = \ell^i_q + \ell^i_c, \tag{4}$$

where s^i, t^a, g^{ia} and ℓ^i_q are the excited quark operators and S^i_c, T^a_c, G^{ia}_c and ℓ^i_c the corresponding core operators.

In this procedure the wave function is approximated by the term which corresponds to the normal Young tableau, where the decoupling of the excited quark is straightforward. The other terms needed to construct a symmetric orbital-flavour-spin state are neglected, *i.e.* antisymmetry is ignored. An a posteriori justification is given in Ref. [21].

But the number of linearly independent operators constructed from the generators given in the right-hand side of Eqs. (4) increases tremendously the number of terms in the mass formula so that the number of coefficients to be determined usually becomes much larger than the experimental data available. Consequently, in selecting the most dominant operators one has to make an arbitrary choice, as for example in Ref. [17]. In particular the isospin operator as $t \cdot T^c/N_c$, although important, has been entirely ignored without any reason.

A solution to this problem has been found in Ref. [22], where the separation into a symmetric core and an excited quark is not necessary. The key issue is the knowledge of the matrix elements of the SU($2N_f$) generators for mixed symmetric states described by the partition $[N_c-1, 1]$ for arbitrary N_c. These can be obtained by using a generalized Wigner-Eckart theorem [23]. Using SU($2N_f$) generators acting on the whole system, the number of operators up to $1/N_c$ order in the mass formula is considerably reduced so that the physics becomes more transparent, as we shall see below.

3.1 The SU(4) case

The SU(4) case has been presented in Ref. [22]. Its algebra is

$$\begin{aligned} &[S^i, S^j] = i\varepsilon^{ijk}S^k, \quad [T^a, T^b] = i\varepsilon^{abc}T^c, \\ &[G^{ia}, G^{jb}] = \tfrac{i}{4}\delta^{ij}\varepsilon^{abc}T^c + \tfrac{i}{2}\delta^{ab}\varepsilon^{ijk}S^k, \end{aligned} \tag{5}$$

with $i, a = 1, 2, 3$. The matrix elements of the SU(4) generators were extracted from Ref. [23], initially proposed for nuclear physics where SU(4) symmetry is nearly exact. The transcription to a system of N_c quarks was straightforward. Instead of 12 operators up to order $\mathcal{O}(1/N_c)$ presented in Ref. [17] we needed only 6 operators for 7 experimentally known three- and four-star nonstrange resonances (no mixing angles). We have introduced the spin and isospin operators on equal footing, as seen from Table 1, and obtained the new result that the isospin term O_4 becomes as dominant in Δ resonances as the spin term O_3 does in N^* resonances, as indicated by the comparable size of the coefficients c_3 and c_4 in Table 1. Column 5 proves that by the removal of O_4 the fit deteriorates considerably.

Table 1. List of operators O_i and coefficients c_i in the $N = 1$ band revisited, 7 resonances of 3 and 4 stars status, no mixing angles.

Operator	Fit 1 (MeV)	Fit 2 (MeV)	Fit 3 (Mev)	Fit 4 (MeV)	Fit 5 (MeV)
$O_1 = N_c \mathbb{1}$	481 ± 5	482 ± 5	484 ± 4	484 ± 4	498 ± 3
$O_2 = \ell^i s^i$	-31 ± 26	-20 ± 23	-12 ± 20	3 ± 15	38 ± 34
$O_3 = \frac{1}{N_c} S^i S^i$	161 ± 16	149 ± 11	163 ± 16	150 ± 11	156 ± 16
$O_4 = \frac{1}{N_c} T^a T^a$	169 ± 36	170 ± 36	141 ± 27	139 ± 27	
$O_5 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	-29 ± 31		-34 ± 30		-34 ± 31
$O_6 = \frac{3}{N_c} L^i T^a G^{ia}$	32 ± 26	35 ± 26			-67 ± 30
$\chi^2_{\rm dof}$	0.43	0.68	0.94	1.04	11.5

3.2 The SU(6) case

Below we present preliminary results for SU(6). The group algebra is

$$[S^i, S^j] = i\varepsilon^{ijk} S^k, \quad [T^a, T^b] = i f^{abc} T^c,$$
$$[S^i, G^{ja}] = i\varepsilon^{ijk} G^{ka}, \quad [T^a, G^{jb}] = i f^{abc} G^{ic},$$
$$[G^{ia}, G^{jb}] = \frac{i}{4}\delta^{ij} f^{abc} T^c + \frac{i}{2}\varepsilon^{ijk}\left(\frac{1}{3}\delta^{ab} S^k + d^{abc} G^{kc}\right), \tag{6}$$

with i = 1,2,3 and a = 1,2,...,8. The analytic work was based on the extension of Ref. [23] from SU(4) to SU(6) in order to obtain matrix elements of all SU(6) generators between symmetric $[N_c]$ states first [24], followed later by matrix elements of all SU(6) generators between mixed symmetric states $[N_c - 1, 1]$ states [25]. The latter work has been recently completed by some new isoscalar factors required by the physical problem [26].

Theoretically the $[\mathbf{70}, 1^-]$ multiplet has 5 octets $(N, \Lambda, \Sigma, \Xi)$, 2 decuplets $(\Delta, \Sigma, \Xi, \Omega)$ and two flavour singlets $\Lambda_{1/2}$ and $\Lambda_{3/2}$. In the fit we take into account the 17 experimentally known resonances having a 3 or 4 star status and the two known mixing angles between the ${}^2N_J$ and ${}^4N_J$ ($J = 1/2, 3/2$) states. Table 2 exhibits the

9 operators used in the mass formula, from which the three B_i's break explicitly the SU(3) symmetry. The corresponding fitted coefficients c_i and d_i are indicated under a preliminary fit named Fit 1. We remind that in the symmetric core + excited quark procedure fifteen O_i (flavour invariants) and four B_i operators were included in the fit [27]. However the flavour operator $1/N_c\ t \cdot T^c$ was omitted, without any justification.

Like for nonstrange baryons, one can see that the dominant operators are the spin O_3 and flavour O_4. The latter has the form explained in Ref. [25]. It recovers the matrix elements of $O_4 = 1/N_c\ T^a T^a$ of nonstrange baryons (see Table 1). The operators O_3 and O_4 have similar values for the corresponding coefficients, which proves the importance of the flavour operators in the fit, like for the SU(4) case.

Table 2. Operators and their coefficients in the mass formula obtained from a numerical fit, mixing angles included, $\mathcal{S}$ denotes the strangeness.

Operator	Fit 1 (MeV)
$O_1 = N_c \mathbb{1}$	476.11 ± 4.09
$O_2 = l^i s^i$	63.6 ± 22.6
$O_3 = \frac{1}{N_c} S^i S^i$	165 ± 15
$O_4 = \frac{1}{N_c}(T^a T^a - \frac{1}{12} N_c(N_c+6))$	181.95 ± 11.6
$O_5 = \frac{3}{N_c} L^i T^a G^{ia}$	-19.4 ± 6
$O_6 = \frac{15}{N_c} L^{(2)ij} G^{ia} G^{ja}$	8.5 ± 0.3
$B_1 = -\mathcal{S}$	163.90 ± 12.04
$B_2 = \frac{1}{N_c} L^i G^{i8} - \frac{1}{2}\sqrt{\frac{3}{2}} O_2$	33.96 ± 31.55
$B_3 = \frac{1}{N_c} S^i G^{i8} - \frac{1}{2\sqrt{3}} O_3$	112.46 ± 62.14
χ^2_{dof}	2.85

In Tables 1 and 2 the operator O_2 contains the one-body part of the spin-orbit term, defined in Ref. [17], while O_5, O_6 and B_2 contain the total orbital angular momentum components L^i, as in Eq. (3). Using the total spin-orbit term it would hardly affect the fit. The contribution of terms containing the angular momentum is generally small, like for nonstrange baryons [22], see Table 1. The SU(3) breaking operator B_1 turns out to be important, as expected.

The $\chi^2_{\rm dof} = 2.85$ is larger than desired. We found that the basic reason is that it is hard to fit the mass of $\Lambda(1405)$ to be so low. The difficulty is entirely similar to that of quark models, where $\Lambda(1405)$ appears too high. An artificially larger mass of the order of 1500 MeV considerably improves the fit, leading to $\chi^2_{\rm dof} < 1$. More fits will be presented elsewhere [26].

The difference between our results and those of Ref. [18] can partly be explained as due to the difference in the wave function. In Ref. [18] only the component with $S_c = 0$ is taken into account and this component brings no contribution to the spin term in flavour singlets, so that the mass of $\Lambda(1405)$ remains low. In our case, where we use the exact wave function, both $S_c = 0$ and $S_c = 1$ parts of the wave function contribute to the spin term. This makes the spin term contribution identical for all states of given J irrespective of the flavour, which seems to us natural. Then, in our case, with a non vanishing spin term in flavour singlets as well, the mass formula accomodates a heavier $\Lambda(1405)$ than the experiment, like in quark models (for a review on the controversial nature of $\Lambda(1405)$ see, for example, Ref. [30] where one of the authors S.F. Tuan has predicted together with D.H. Dalitz this resonance in 1959, discovered experimentally two years later.)

4 Conclusion

The $1/N_c$ expansion method provides a powerful theoretical tool to analyze the spin-flavour symmetry of baryons and explains the success of models based on this symmetry. We have shown that the dominant contributions come from the spin and flavour terms in the mass formula both in SU(4) and SU(6). The terms containing angular momentum bring small contributions, which however slightly improve the fit. It is hard to fit the mass of $\Lambda(1405)$, a notorious problem in realistic quark models [28,29]. This suggests again a more complex nature of this resonance, as, for example, a coupling to a $\bar{K}N$ system , which might survive in the large N_c limit [31,32].

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