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Simple Stochastic Model for Planning the Inventory of Spare Components Subject to Wear-out

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We treat an industrial system which comprises of a number of identical components subject to wear-out. To support the system maintenance an appropriate inventory of spare components is needed. In order to plan the sufficient inventory of spare components, two variants of a simple stochastic model are developed. In both variants, the aim is to determine how many spare components are needed at the beginning of a planning interval to meet demand for corrective replacements during this interval. Under the first variant the acceptable probability of spare shortage during the planning interval is chosen as a decision variable. While in the second variant the adequate spare inventory level is assessed by taking into account the expected number of component failures within the planning interval. A comparison of both variants of the model shows that calculations involved in the second variant are simpler. However, it can only be used when the inventory of spare components can be planned for a relatively long period of time.

The determination of an adequate number of spare components according to both variants of our model depends on the form of the probability density function of component failure times. Since the components are subject to wear-out, this function exhibits a peak-shaped form that can be described by different statistical density functions. Advantages and disadvantages of using the normal, lognormal, Weibull, and Gamma density function in our model are discussed. Among the probability density functions studied, the normal density function is found to be the most appropriate for calculations in our model. The applicability of both variants of the model is given through numerical examples using field data on electric locomotives of Slovenian Railways.

Key words: industrial system, wear-out, maintenance, corrective replacements, spare components, inventory planning, stochastic modelling

1 Introduction

During the operation, the components of an industrial system deteriorate with usage, and frequently fail causing the system downtime and consequently a loss of income. In order to reduce the system downtime, various maintenance activities are performed. Maintenance costs represent a major part of the total system operating costs. Significant savings can be achieved by introducing an efficient maintenance policy. In defining the maintenance policy, greater attention should be paid to the availability of spare components at the times of component replacements.

Many mathematical models for the spare provisioning policy for deteriorating systems have been proposed in the literature. Usually, the optimal spare provisioning policy is defined by the minimization of the total system maintenance costs that comprise of the component replacement costs (corrective and preventive), and the inventory costs including ordering, holding and shortage costs. Such mathematical models are usually quite complex, containing a great number of parameters (see e.g. Brezavšček and Hudoklin, 2003; Diallo et al., 2008; Hu et al., 2008; Huang et al., 2008; de Smidt-Destombes et al., 2009; Wang et al., 2009), and are often difficult to implement. From a practical point of view, simpler methods for defining an efficient spare provisioning policy would be most desirable.

In our opinion, in many industrial plants such a traditional approach in defining the inventory policy can be simplified. In large systems the cost of system downtime due to the shortage of spare components frequently exceeds all the other elements of the maintenance costs substantially (see e.g. Brezavšček and Hudoklin, 2003). We think that in such a situation there is no need to optimize the total maintenance costs. It is enough to ensure the sufficient quantity of spare components to prevent the inventory shortage in a given time. This approach enables the development of a spare provisioning model which is much easier to implement than the traditional optimization models.

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In the paper, we will develop a simple stochastic model for planning the inventory of spare components needed to support maintenance of an industrial system if the shortage of spare components leads to high costs. The model is useful for components subject to wear-out, when the preventive replacements are performed according to the block replacement policy (Pham, 2003). Two variants of the model are presented. Under the first variant, the adequate number of spare components is calculated taking into consideration the acceptable probability of spare shortage during the planning interval. In the second variant, the expected number of component failures during the planning interval is used as the decision variable in planning an adequate spare inventory level.

The determination of an adequate number of spare components according to our model depends on the form of the probability density function of component failure times. Since the components are subject to wear-out this function exhibits a peak-shaped form that can be described by different statistical density functions. Advantages and disadvantages of using the normal, lognormal, Weibull, and Gamma density function in our model are discussed. The applicability of both variants of the model is shown using field data on electric locomotives of Slovenian Railways.

2 Preliminaries

The inventory of spare system components includes the spares needed for component preventive replacements, and the spares needed for corrective replacements. The number of spare components needed for preventive replacements in a given planning interval is known in advance. Therefore, in defining an efficient spare provisioning policy, the essential task is to ensure the sufficient number of spare components needed for corrective replacements in the interval between two successive preventive replacements.

The process of corrective replacements of a particular component during the system operation can be described by an ordinary renewal process. The renewal process is ordinary when all inter-renewal times are independent identically distributed random variables, all with the probability density function $f(t)$ (e.g. Cox, 1970). In our model two characteristics of an ordinary renewal process will be used: the number $N(t)$ of renewals in the interval $(0,t)$, and the renewal function $H(t)$ defined as the expected number of renewals in the interval $(0,t)$: $H(t) = E[N(t)]$.

The number $N(t)$ of component corrective replacements (i.e. renewals) is a random variable with the probability distribution $p_r(t) = P[N(t) = r]$, $r = 0, 1, 2, ...$ which can be calculated according to the equation

$$p_r(t) = F_r(t) - F_{r+1}(t) \qquad r = 0, 1, 2, ... \tag{1}$$

with $F_0(t) = 1$. The symbol $F_r(t)$ in the equation (1) denotes the r-fold convolution with itself of the cumulative distribution function $F(t) = \int_0^t f(x)dx$ When there are n independent identical components under observation, the process of their corrective replacements represents a superposition of n independent renewal processes. The probability distribution of the number of renewals of all n components in $(0,t)$ is given by the discrete convolution formula

$$p_r^{(n)}(t) = \sum_{i=0}^{r} p_{r-i}^{(n-1)}(t) p_i(t) \qquad r = 0,1,2,\ldots,\ n > 1 \tag{2}$$

with $p_r^{(1)}(t) = p_r(t)$. For an arbitrary n an analytical solution of $p_r^{(n)}(t)$ exists only when the analytical solution of (1) is available. Even then, the calculation of $p_r^{(n)}(t)$ is rather tedious. However, for large values of n the function $p_r^{(n)}(t)$ can be approximated by the normal density function with the mean $nH(t)$ and the variance $nV(t)$[1] (Haehling von Lanzenauer and Lundberg, 1974; Bergstrom, 2006).

The renewal function $H(t)$ can be calculated according to the equation

$$H(t) = \sum_{r=1}^{\infty} F_r(t) \tag{3}$$

For an arbitrary time t a simple solution of (3) is obtainable for some specific types of $f(t)$ only. For large values of time t a simple asymptotic formula for $H(t)$ can be used (Cox, 1970). When there are n independent identical components under observation, the expected number of renewals of all n components during time t is equal to $nH(t)$.

It is evident that the calculation of $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ depends on the probability density function $f(t)$ of inter-renewal times. In our case $f(t)$ is equal to the probability density function of component failure times.

3 Model development

A simple stochastic model for planning the inventory of spare components needed for corrective replacements of system components is developed. The model addresses the situation when the costs of system downtime due to the shortage of spare components considerably exceed all the other elements of the total maintenance costs.

The model is based on the renewal theory. In developing the model the following assumptions are considered:

- The system includes n identical components operating independently in the similar conditions.
- The components are subject to wear-out. The preventive replacements are performed according to the block replacement policy every t units of time.
- To meet demand for corrective replacements between planned preventive replacements, the inventory of spare components is replenished periodically. The planning interval is $T = k\tau$ where k is an integer. In the variant 1 of the model $k = 1$, while in the variant 2 k is a large integer ($k \to \infty$).

1 The symbol $V(t)$ denotes the variance of the number of renewals in the interval $(0,t)$ defined by the expression $V(t) = E[(N(t) - H(t))^2]$.

- At the beginning of T, Q spare components should be available.
- A failed system component is replaced immediately by a new one if a spare component is available. The replacement time and consequently the unplanned system downtime is negligible.
- If the replacement of the failed component cannot be performed due to the shortage of spare components the unplanned system downtime occurs.

The aim of the model is to determine the minimal number Q of spare components in the inventory at the beginning of the planning interval T to meet demand for component corrective replacements during T. Two variants of the model are presented. In the first variant the number Q is determined considering an acceptable probability of spare shortage during T. While in the second variant the number Q is assessed taking into account the expected number of component failures within T.

Variant 1

The assumption $k = 1$ means that $T = \tau$. Let the acceptable probability of spare shortage during T to be $P_s(T)$. The value $P_s(T)$ is predetermined considering the specific requirements of the system operation. We want to determine the minimal number of spare components Q at the beginning of T which ensures that the probability of spare shortage during T does not exceed the value $P_s(T)$. The number Q is the minimal integer that satisfies the relation

$$\sum_{r=Q+1}^{\infty} p_r^{(n)}(T) \le P_s\left(T\right) \qquad (4)$$

where the symbol $p_r^{(n)}(t)$ denotes the probability distribution of the number of corrective replacements of n components, given by the equation (2).

Variant 2

The inventory of spare components needed for corrective replacements is planned for the interval $T = k\tau$, where k is a large integer. We want to determine the number of spare components Q at the beginning of T in such a way that Q is at least equal to the expected number of component failures within T. Considering the proposed maintenance policy, the expected number of failures of n components in T is equal to $knH(\tau)$. The number Q i*kn*s then the minimal integer satisfying the relation

$$Q \;\ge\; knH(\tau) \qquad (5)$$

where $H(\tau)$ is calculated according to the equation (3). If the condition ($k \to \infty$) is fulfilled, then the sum of deviations of the actual number of component failures during T from the expected number of failures in T approaches zero. Therefore, when there are Q spare components available for corrective replacements at the beginning of T, the probability of spare shortage during T approaches zero.

The calculations in variant 2 of the model are much simpler than in variant 1. Besides, the inventory of spare components needed for corrective replacements during $T = k\tau$ is replenished once at the beginning of the interval T, while in the variant 1 the inventory should be replenished k times at the beginning of every interval $T = \tau$. On the other hand, the condition ($k \to \infty$) means that the inventory of spare components should be planned for a very long period of time. In practice, very large values of k could lead to the planning interval of several years what could be unreasonable. However, this variant of the model is applicable also if k is not a very large number, but the number n of components under consideration is large. In such a situation the dispersion of the number of component failures during T around the expected number $knH(\tau)$ is relatively small. This implies that the expected number of component failures in T can be used as the decision variable in inventory planning.

4 Selection of appropriate probability density function of component failure times

The calculation of an adequate number Q of spare components according to our model depends on the form of the probability density function $f(t)$ of component failure times. The function $f(t)$ for components subject to wear-out follows a peak-shaped curve. Times to failure are distributed around a peak value specific for a given deterioration mechanism (e.g. corrosion, fatigue cracking, diffusion). A general form of the function $f(t)$ for components subject to wear-out is shown in Fig. 1.

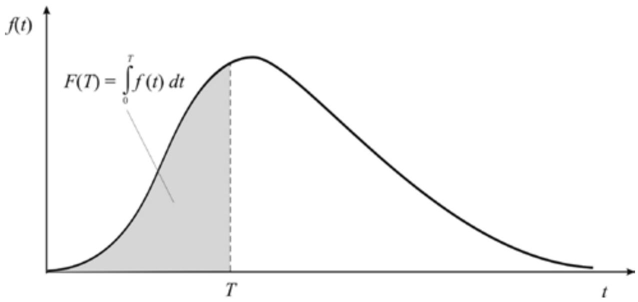

Figure 1: General form of the probability density function f (t) of failure times of components subject to wear-out

The normal, lognormal, Weibull, and Gamma probability density functions are frequently used to describe the function $f(t)$ for the components subject to wear-out (see e.g. Jardine and Tsang, 2006; Kececiouglu, 1995). Advantages and disadvantages of using a particular statistical density function in our model will be discussed. The following criterions will be taken into account:

- availability of the analytical expressions for the renewal characteristics needed in the model ($p_r(t)$, $p_r^{(n)}(t)$, and $H(t)$),
- difficulty of the numerical calculation of the renewal characteristics needed,
- simplicity of the assessment of the statistical density function parameters.

Normal (Gaussian) density function

$$f(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2} \qquad -\infty < t < \infty,\; -\infty < \mu < \infty,\; \sigma > 0$$

with parameters the mean μ and the standard deviation σ. The form of the normal density function for different values of parameters μ and σ is shown in Fig. 2.

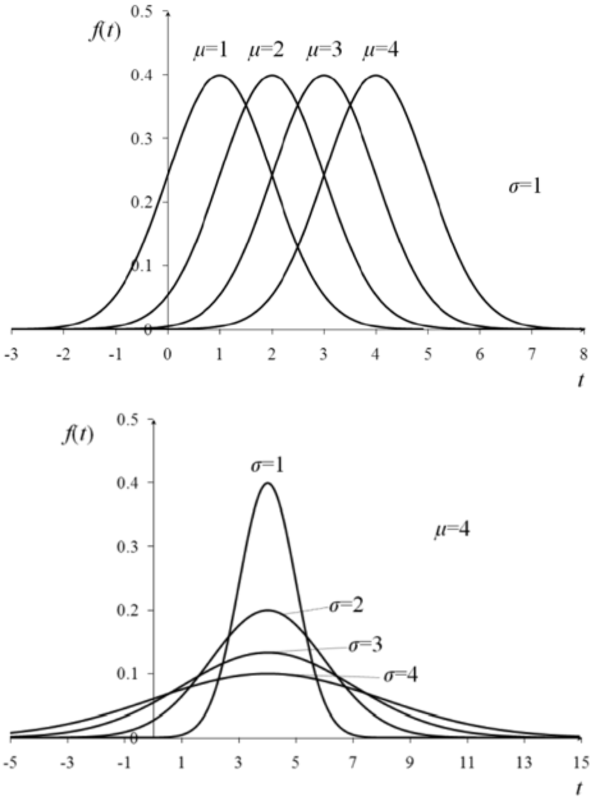

Figure 2: Form of the normal density function for different values of parameters μ and s

Advantages of using the normal density function

- The numerical calculation of $p_r(t)$, $p_r^{(n)}(t)$, and $H(t)$ is rather simple because the r-fold convolution of the normal distribution function $F(t)$ with parameters μ and σ is also a normal distribution function with parameters $r\mu$ and $\sigma\sqrt{r}$.
- The assessment of the values of parameters μ and σ is easy because the normal probability plotting paper is available (see e.g. http://www.weibull.com).

Disadvantages of using the normal density function

- Since time to component failure is a positive random variable the area under a normal curve for the negative values of time should be negligible. This is true when the ratio between μ and σ is significantly higher than 1. Otherwise a truncated normal distribution should be used (see e.g. Johnson et al., 1994; Kottegoda and Rosso, 1997).

Lognormal density function

The lognormal density function is in the relationship to the normal density function. If the random variable t is distributed according to a lognormal density function, the logarithm of t is distributed according to a normal density function. The lognormal density function is given by the expression

$$f(t)=\frac{1}{t\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln t-\mu}{\sigma}\right)^2} \qquad t>0,\ \mu>0,\ \sigma>0$$

where μ and σ are the mean and the standard deviation of ln t. The form of the lognormal density function for different values of parameters μ and σ is shown in Fig. 3.

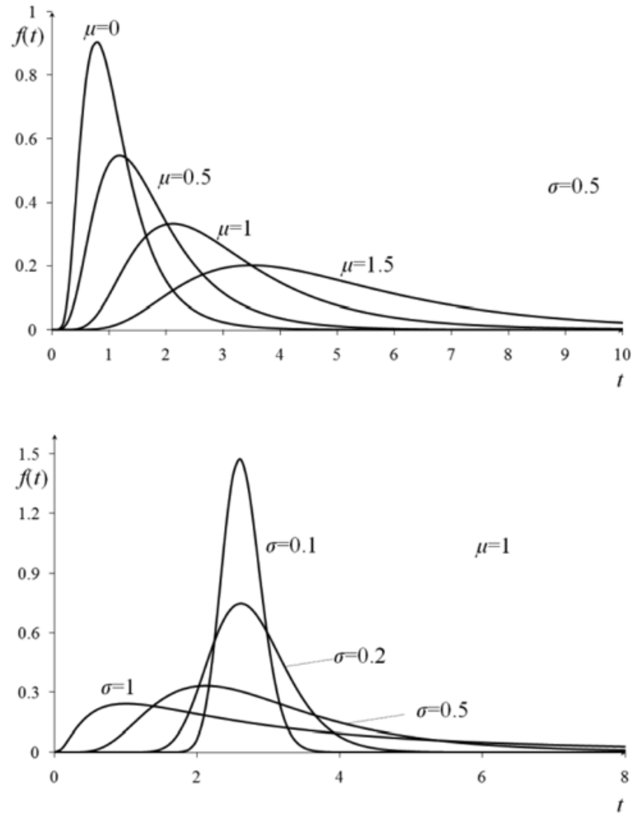

Figure 3: Form of the lognormal density function for different values of parameters μ and s

Advantages of using the lognormal density function

- The assessment of the values of parameters μ and σ is easy because the lognormal probability plotting paper is available (see e.g. http://www.weibull.com).

Disadvantages of using the lognormal density function

- The closed form of $F_r(t)$ is not available. Some approximate formulas are available in the literature but calculations are quite tedious (see e.g. Barouch and Kaufman, 1976; Romeo et al., 2003; Lam and Le-Ngoc, 2006).

Weibull density function

The two-parameter[2] Weibull density function is given by the formula

$$f(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}e^{-\left(\frac{t}{\eta}\right)^{\beta}} \qquad t\geq 0,\ \beta>0,\ \eta>0$$

2 In reliability theory, the three parameter Weibull density function is also used. The third parameter γ, $-\infty<\gamma<\infty$, is the location parameter. When $\gamma=0$ the density function starts at time t = 0.

where β denotes the shape parameter, and η denotes the scale parameter. The form of the Weibull density function for different values of β and η is shown in Fig. 4.

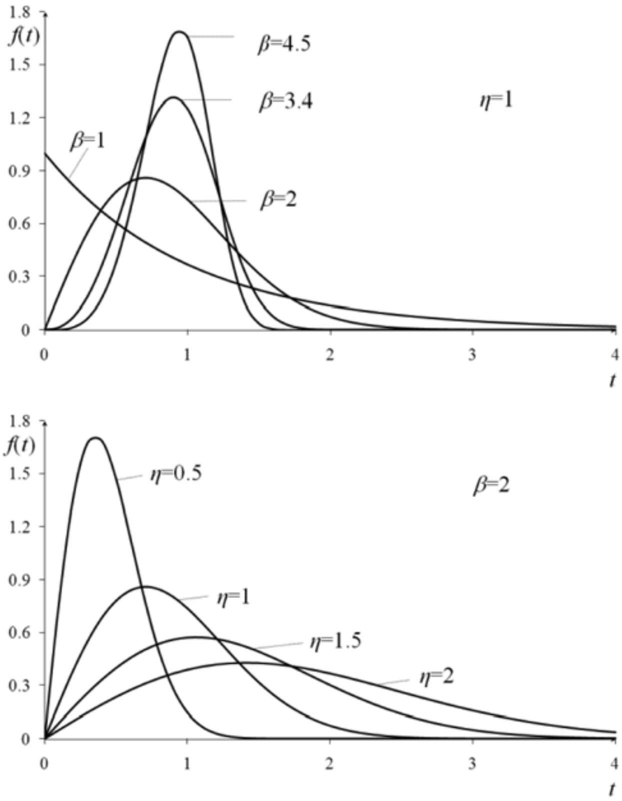

Figure 4: Form of the Weibull density function for different values of parameters β and η

It can be seen from Fig. 4 that the Weibull density function exhibits a peak-shaped form when $\beta > 1$. If $\beta = 1$, the Weibull density function becomes exponential with the parameter $\lambda = 1/\eta$.

Advantages of using the Weibull density function

- The assessment of the values of parameters β and η is easy because the Weibull probability plotting paper is available (see e.g. http://www.weibull.com).

Disadvantages of using the Weibull density function

- In the case of the Weibull density function with $\beta > 1$, the closed form of $F_r(t)$ is not available. The numerical calculations of $p_r(t)$, $p_r^{(n)}(t)$, and $H(t)$ are quite tedious (see e.g. Jiang, 2008). A comprehensive overview of different numerical calculations of the Weibull renewal function is given in the book by Rinne (2009).

Gamma density function

The two-parameter[3] Gamma density function is given by the formula

$$f(t) = \frac{1}{\eta\Gamma(\beta)}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\frac{t}{\eta}} \qquad t \geq 0,\ \beta > 0,\ \eta > 0$$

where β denotes the shape parameter, η denotes the scale parameter[4], and $\Gamma(.)$ denotes the Gamma function[5].

The form of the Gamma density function for different values of parameters β and η is shown in Fig. 5.

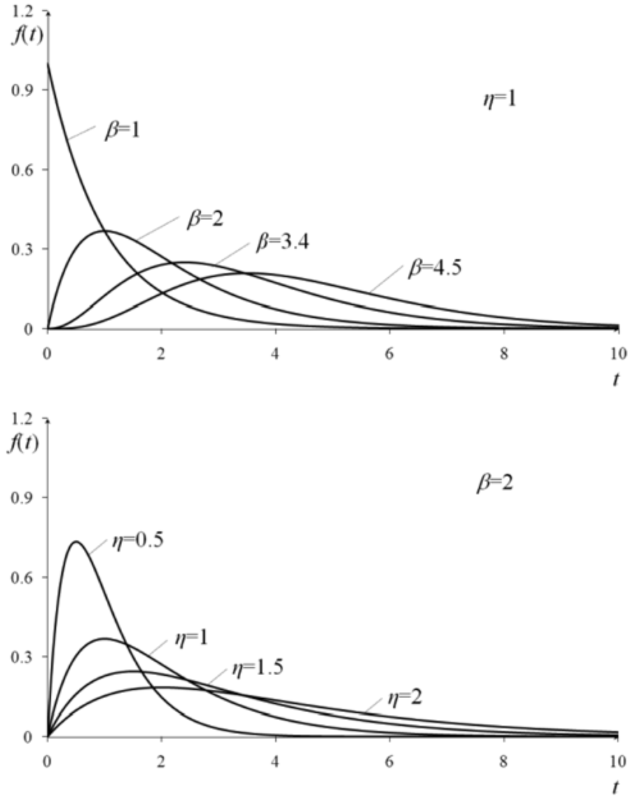

Figure 5: Form of the Gamma density function for different values of parameters β and η

It can be seen from Fig. 5 that the Gamma density function exhibits a peak-shaped form when $\beta > 1$. If β is equal to 1, the Gamma density function becomes exponential with parameter $\lambda = 1/\eta$.

Advantages of using the Gamma density function

- Analytical solutions for $p_r(t)$ and $H(t)$ exist if $\beta = 2$:

$$p_r(t) = \frac{2e^{-\frac{t}{\eta}}(1+r)\left(\frac{t}{\eta}\right)^{2r}\left[t+\eta(1+2r)\right]}{\eta\Gamma(3+2r)}$$

3 In reliability theory, the three parameter Gamma density function is also used. The third parameter γ, $-\infty < \gamma < \infty$, is the location parameter. When $\gamma = 0$ the density function starts at time $t = 0$.

4 When β is integer the Gamma density function becomes the Erlang density function. When the shape parameter is $\beta/2$ (β is any integer) and the scale parameter is equal to 2 the Gamma density function becomes the Chi-square density function.

5 $\Gamma(x) = \int_0^\infty z^{x-1} e^{-z} dz$

and $H(t)=\frac{1}{4}\left(e^{-\frac{2t}{\eta}}+\frac{2t}{\eta}-1\right)$

Using the expression for $p_r^{(n)}(t)$ given above, an analytical solution for $p_r^{(n)}(t)$ can be calculated according to the equation (2).

- The r-fold convolution of the Gamma distribution function with parameters b and η is also a Gamma distribution function with parameters $r\beta$ and η. Therefore, a numerical calculation of $p_r(t)$, $p_r^{(n)}(t)$ and $H(t)$ according to the equations (1), (2) and (3) is easy for any value of parameter b.

Disadvantages of using the Gamma density function

- The assessment of the values of parameters β and η is not trivial because the Gamma probability plotting paper is not commercially available. The general maximum likelihood method can be used to estimate the values of β and η (see Evans et al., 2000; Johnson et al., 1994). This is probably the reason why the Gamma density function is not widely used as a mathematical model for the probability density function of component failure times.

The results of suitability analysis of selected statistical density functions for using in our model are shown in Table 1.

We conclude from Table 1 that among the four statistical density functions studied, the normal density function is the most appropriate for use in our model.

5 Numerical example

We will illustrate the application of both variants of our model using field data on electric locomotives of Slovenian Railways. As in the paper Brezavšček and Hudoklin (2003), the arcing chamber which is an important locomotive component is studied. There are n = 120 components under observation. The components are subject to wear-out. Preventive replacements are performed according to the block replacement policy every t=23 weeks. It is assumed that times to component failure are distributed normally. The parameters of $f(t)$, μ and σ, are 44 and 12 weeks respectively.

Variant 1

The planning interval is $T = \tau = 23$ weeks. We want to determine the minimal number of spare components Q at the beginning of T which will ensure that the probability of spare shortage during T will not exceed 3%.

The probability distributions $p_r(t)$ and $p_r^{(120)}(T)$ of the number of component corrective replacements during T are calculated numerically according to the equations (1) and (2).

The results are shown in Tab. 1. Besides, the probabilities of spares shortage $\sum_{i=r+1}^{\infty} p_i^{(120)}$ for r = 0,1,2... are also added.

Considering that $P_s(T) = 0.03$, and the equation (4) we determine the minimal integer Q which satisfies the relation

$$\sum_{r=Q+1}^{\infty} p_r^{(120)}(T) \le 0.03$$

Using the results from Table 2 we obtain Q = 9. At the beginning of each interval T = 23 weeks between two successive preventive replacements, 9 spare components for corrective replacements during T are required. The shortage probability during T is equal to 2,32%, and the requirement above is fulfilled. Besides spares for corrective replacements, 120 spare components are needed for preventive replacements of all components at the beginning of T.

Variant 2

We have come to the conclusion that the period for planning the inventory of the arcing chambers is still acceptable up to 3.5 years which is approximately 184 weeks. Since τ = 23 weeks the value of k is 8. Because the number of components under consideration is relatively large we suppose that k = 8 is large enough to justify the usage this variant of the model.

We want to determine the number of spare components Q at the beginning of T=184 weeks which is at least equal to the expected number of component failures within T.

The value of the renewal function $H(\tau)$ is calculated according to the equation (3). From this we obtain $H(\tau)$ = 0.0401. The number of spare components needed for corrective replacements during T is determined according to the equation (5) as the minimal integer satisfying the relation

$$Q \ge 8 \cdot 120 \cdot 0.0401 = 38.50$$

We obtain Q = 39. Apart from 39 spares for corrective replacements, 960 spare components are needed to perform eight block preventive replacements of all 120 operating components.

Table 1: Suitability of different statistical density functions for using in the model

Criterion	Normal pdf	Lognormal pdf	Weibull pdf	Gamma pdf
The analytical expressions for the renewal characteristics are available	No	No	No	Only for some specific integer values of the parameter β (e.g. β = 2)
The numerical calculation of the renewal characteristics is easy	Yes	No	No	Yes
The assessment of the pdf parameters is easy	Yes	Yes	Yes	No

Table 2: Probability distribution of the number of component corrective replacements, and shortage probabilities in T = 23 weeks

r	0	1	2	3	4	5	6	7	8	9	10
$p_r(t)$	0.9599	0.04	6.40E-05	7.84E-08	…	…	…	…	…	…	…
$p_r^{(120)}(T)$	7.40E-03	3.70E-02	0.0918	0.1506	0.1839	0.1783	0.1428	0.0973	0.0576	0.03	0.014
$\sum_{i=r+1}^{\infty} p_i^{(120)}$	0.9926	0.9556	0.8638	0.7132	0.5292	0.3509	0.2081	0.1108	0.0532	0.0232	0.0092
r	11	12	13	14	15	16	17	18	19	20	21
$p_r^{(120)}(T)$	0.0059	0.0022	0.0008	0.0003	0.0001	2.07E-05	5.34E-06	1.29E-06	2.93E-07	6.25E-08	…
$\sum_{i=r+1}^{\infty} p_i^{(120)}$	0.0034	0.0011	0.0004	0.0001	2.77E-05	6.99E-06	1.66E-06	3.71E-07	7.81E-08	1.55E-08	…

6 Conclusion

In the paper, a simple stochastic model for planning the inventory of spare components needed to support maintenance of an industrial system is proposed. The aim of the model is to determine the minimal number of spare components in the inventory at the beginning of a given planning interval to meet demand for component corrective replacements during this interval. Two variants of the model are presented. In both variants it is assumed that components are subject to wear-out, and the preventive replacements are performed according to the block replacement policy. In the first variant of the model, the adequate number of spare components for corrective replacements is calculated considering the acceptable probability of spare shortage during the planning interval. In the second variant, the required number of spare components for corrective replacements is assessed taking into account the expected number of component failures within the planning interval.

In both variants of the model, the process of successive corrective replacements of a particular component is described by an ordinary renewal process. The determination of the characteristics of the renewal process depends on the form of the probability density function of component failure times. Since the components are subject to wear-out, this function can be described by a peak-shaped statistical density function. Advantages and disadvantages of using normal, lognormal, Weibull, and Gamma density function in the model are discussed. In our opinion, among the four statistical density functions studied, the normal density function is the most appropriate for calculating the probability distribution of the number of corrective renewals as well as the expected number of corrective renewals in a planning interval.

The applicability of the model is given through numerical examples using field data on electric locomotives of Slovenian Railways. Both variants of the model are useful for practical purposes. When the inventory of spare components can be planned for a relatively long period, we recommend the second variant because the calculations involved are much simpler than in the first variant.

The model proposed represents a simplification of rather complicated optimization models widely published in the literature. It is suitable for implementation in a variety of industrial systems where the costs of the system downtime due to the shortage of spare components considerably exceed all the other parameters of the system maintenance costs.

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Enostavni stohastični model za planiranje zaloge rezervnih komponent v obdobju izrabe

V prispevku obravnavamo proizvodni sistem, ki vsebuje določeno število identičnih komponent v obdobju izrabe. Komponente sistema lahko med delovanjem sistema odpovedo, kar povzroči zastoj sistema. Da lahko komponento, ki odpove, čim prej nadomestimo z novo, je potrebno imeti na zalogi zadostno količino rezervnih komponent. Predstavljeni sta dve varianti enostavnega stohastičnega modela, ki omogočata določitev minimalnega števila rezervnih komponent na zalogi na začetku intervala planiranja tako, da je zadoščeno izbranemu kriteriju. V prvi varianti modela je kot kriterij pri določanju ustrezne zaloge rezervnih komponent upoštevana verjetnost, da se zaloga tekom intervala planiranja izčrpa, medtem ko v drugi varianti planiramo zalogo rezervnih komponent glede na povprečno število odpovedi, ki jih v intervalu planiranja pričakujemo. Primerjava obeh variant modela je pokazala, da je druga varianta modela z matematičnega stališča enostavnejša, njena pomanjkljivost pa je v tem, da je uporabna le, kadar je možno zalogo rezervnih komponent planirati za daljše časovno obdobje.
Določitev ustreznega števila rezervnih komponent na podlagi obeh variant modela je odvisna od funkcije gostote verjetnosti za čas do odpovedi obravnavanih komponent. Kadar so komponente v obdobju izrabe, ima ta funkcija karakteristično kopasto obliko, ki jo lahko popišemo z ustrezno verjetnostno porazdelitvijo. Z vidika uporabnosti v modelu smo analizirali normalno, lognormalno, Weibullovo in gama verjetnostno porazdelitev. Ugotovili smo, da je za izračune, ki so v modelu zahtevani, najbolj prikladna normalna verjetnostna porazdelitev. Uporabnost obeh variant modela smo ponazorili z numeričnimi primeri in podatki o eksploataciji električnih lokomotiv iz Slovenskih železnic.

Ključne besede: proizvodni sistem, izraba, vzdrževanje, odpoved, zamenjava, rezervna komponenta, zaloga, planiranje, stohastično modeliranje