ANALYSIS OF GEOSTATISTICAL SURFACE MODEL FOR GPS HEIGHT TRANSFORMATION: A CASE STUDY IN IZMIR TERRITORY OF TURKEY

ANALIZA GEOSTATISTIČNEGA MODELA POVRŠJA ZA VIŠINSKO TRANSFORMACIJO GPS: ŠTUDIJA PRIMERA NA OBMOČJU IZMIRJA V TURČIJI

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ABSTRACT

The purpose of this study is evaluation of geostatistical surface model for transformation of GPS derived ellipsoidal heights to orthometric heights. The model was handled as its accuracy for surveying applications. 1148 reference points were used covering an area of 115×112 km with GPS and leveling data from the "Izmir geodetic infrastructure for the production of 1/5000 scaled digital photogrammetric maps and orthophotos" project. As a basic data, the differences between ellipsoidal and orthometric heights for each benchmarks were modeled by geostatistical interpolation method namely kriging. ArcGIS 10.0 Geostatistical Analyst was used with optimized parameters for modeling. The quality of the model was analysed by Cross Validation, splitted data and external data validation. The model provide about ± 5 cm absolute, 1 ppm relative accuracy. Also the consistency of the model with several geoid models namely TG03 (Turkish Geoid 2003) and EGM08 (Earth Gravitational Model 2008) geoids was approximately ± 7-10 cm.

IZVLEČEK

V članku je obravnavano vrednotenje geostatističnega modela ploskve za pretvorbo elipsoidnih višin, pridobljenih z meritvami GPS, v ortometrične višine. V raziskavi je bilo uporabljenih 1148 referenčnih točk, katerih višine so bile določene z geometričnim nivelmanom in meritvami GPS. Zajeto je območje velikosti 115 x 112 kilometrov v okviru projekta »Geodetska infrastruktura območja Izmir za izdelavo digitalnih fotogrametričnih zemljevidov in ortofotov v merilu 1 : 5000«. Osnovne podatke pomenijo razlike med elipsoidnimi in ortometričnimi višinami, ki so modelirane z metodo geostatistične interpolacije, in sicer krigingom. Uporabili smo programsko orodje ArcGIS 10.0 'Geostatistical Analyst' z optimiziranimi parametri modeliranja. Kakovost modela je bila analizirana z navzkrižnim preverjanjem, delitvijo podatkov in njihovim zunanjim preverjanjem. Ocenjena natančnost modela je približno 5 cm absolutno in približno 1 ppm relativno. Tudi skladnost modela z drugimi modeli geoida, in sicer z modeloma TG03 (turški geoid 2003) in EGM08 (Earth Gravitational Model 2008), znaša približno 7–10 cm.

KEY WORDS

GPS-leveling, interpolation, kriging, geoid

KLJUČNE BESEDE

GPS-nivelman, interpolacija, kriging, geoid

1 INTRODUCTION

The improvement in GPS technology makes it essential to determine precise geoid or similar surface model referring to a global geocentric datum. It is used for transformation of the GPS ellipsoidal heights to orthometric heights. The recent geodetic research has focused on obtaining "cm" level geoid both in theoretical and practical studies, e.g. Vanicek and Kleusberg, 1987;

Li and Sideris, 1994; Tóth et al., 2000; Kühtreiber, 2002; Chen et al., 2003; Soycan, 2006a; Abbak et al., 2012. Geoid is simply defined as the equipotential surface of the Earth's gravity field which best fits, in a least-squares sense, global mean sea level. It is the level surface of the Earth's gravitational field based on the assumption that the geoid heights vary from the sea surface in the equilibrium state to below the sea level. It is regarded as the reference surface for the vertical control surveys in geodesy. (Moritz, 1980; Torge, 1980; Soycan and Soycan 2003; Soycan, 2006b).

Geoid surface can be classified according to varying wavelengths. The long wavelength reflects the underground mass anomalies, while the medium and the short wavelength indicate significant components of the gravity anomalies depending on the topography. Finally, the ultra-short wavelength reflects both local topographic details and the disturbing effects of the underground mass.

The long wavelength can be characterized by global geoid model from geopotential coefficients easily, however, the medium and short wavelengths need regional geoid model based on gravity and DEM (Digital Elevation Model) data. Recent research applies combined techniques for determination of hybrid geoid model using, geopotential coefficients, gravity, DEM and GPS&Leveling data (Stopar et al.,2006, Erol et al., 2008; Corchete, 2010).

The term of geoid does not refer "height reference surface" which is theoretically computed based on orthometric and GNSS (ellipsoidal) heights. GPS&Leveling method (Marko and Kuhar, 2008) gained prominence with the improvements in GNSS technology for the determination of short and ultra-short wavelength.

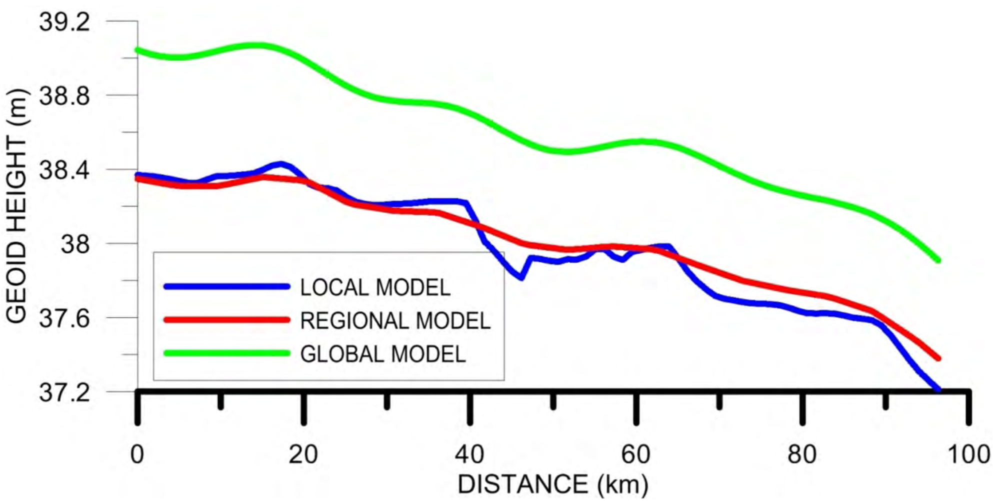

Figure 1: Representation of classified geoid surface

In particular, it is difficult to predict the local effects of the geoid by the global or regional geoid models. A more detailed local surface model is needed for the determination of short and ultra-short wavelength (IAG, Soycan and Soycan 2003; Soycan, 2006a; Soycan, 2006b; Stopar et al., 2006; Erol et al., 2008). The height differences derived from GPS and levelling data are the highest quality and most reliable data to predict the local effects. GPS&Leveling method is very effective solution for transformation GPS ellipsoidal heights to ortometric heights in practice.

These surfaces are smooth, yet their geometries are complex. It is difficult to express such surfaces mathematically; however we can approximate it with some analytical model. To identify these surfaces, the differences between orthometric and ellipsoidal heights can be modelled by geostatistical or deterministic methods with various interpolation techniques. For this purpose, users need reference points with orthometric heights from geometric levelling network and ellipsoidal heights from geodetic GPS/GNSS network.

With this method one actually compute "height reference surface" (Solheim, 2000; Koler et al., 2007; Kuhar et al., 2011). Several studies were carried out by researchers on geoid models and this kind of surface (Rapp 1992; Featherstone 2001; Kiamehr and Sjoberg 2005; Featherstone 2006; Ustun 2006; Benahmed and Fairheadb 2007; Kotsakis and Katsambalos 2010). This paper focuses on evaluation of geostatistical method known as kriging interpolation for this purpose.

2 SURFACE MODELING WITH GEOSTATISTICAL METHOD

The scientific literature presents various methods for scattered data interpolation. The well-known simpler interpolation algorithms include inverse distance weighting, bilinear interpolation, polynomial regression, triangulation, radial basis functions and nearest-neighbor interpolation. The diversity of methods leads to the conclusion that no method is better or worse than another. However they may differ from each other when considering the application area, surface features, data, accuracy and ease of calculation.

The method is an effective tool with their widespread applications in many areas for scattered data interpolation problems. Many scientific researches show that it can be applied routinely in most cases. It is intimately related to interpolation methods, but extends far beyond simple interpolation problems.

It goes beyond the interpolation problem by considering the studied phenomenon at unknown locations as a set of correlated random variables. The geostatistical techniques rely on statistical model that is based on random function (or random variable) theory to model the uncertainty associated with spatial estimation and simulation. It is a class of statistics used to analyze and predict the values associated with spatial or spatiotemporal phenomena. It incorporates the spatial (and in some cases temporal) coordinates of the data within the analyses. Many geostatistical tools have been developed as a practical means to describe spatial patterns and interpolate values for locations where samples were not taken. Those tools and methods have evolved not only to provide interpolated values, but also measures of uncertainty for those values (http://www.esri.com/software/arcgis/extensions/geostatistical).

One of the most useful geostatistical method providing accurate approximations is kriging. The kriging refers to a group of geostatistical techniques to interpolate the value of a random field (e.g., the elevation, z, of the landscape as a function of the geographic location) at an unobserved location from observations of its value at nearby locations. The next subsection gives a brief for kriging interpolation.

2.1 Kriging Interpolation

Kriging is an interpolator that can be exact or smoothed depending on the measurement error model. It is very flexible and allows investigating graphs of spatial auto- and cross-correlation. Kriging uses statistical models that allow a variety of output surfaces including predictions, prediction standard errors, probability and quantile. Ordinary Kriging method, which is mostly favoured among the kriging methods with its simplicity and ease to solve, was employed as the interpolation method in this study and it is summarized in following section.

It is an ideal method for a surface modelling with stable data (no significant change in gradient) and uniform distance between points. Ellipsoidal-Orthometric height difference of a point k ($N(\varphi,\lambda)_k$) is calculated using the N_i=h_i-H_i variables and the P_i weights by Equation 1. N_i values are derived from sampling reference points with known geographical coordinates, ellipsoidal and orthometric heights through n points, which used for modelling.

$$N(\varphi,\lambda)_k = \sum_{i=1}^{n} P_i N_i = P_1 N_1 + P_2 N_2 + \ldots\ldots\ldots\ldots + P_n N_n \tag{1}$$

Where, φ and λ geographical ellipsoidal coordinates, hi and Hi are ellipsoidal and orthometric heights respectively.

$N(\varphi,\lambda)_k$ can be re-written by means of a trend surface and using reduced values, so, equations (3) will be valid by

$$t\{N(\varphi,\lambda)_k\} = \sum_{i=1}^{n} P_i t\{N_i\}. \tag{2}$$

$$N(\varphi,\lambda)_k = N_{trend} + t\{N(\varphi,\lambda)_k\} \tag{3}$$

Where, $t\{N(\varphi,\lambda)_k\}$ and $t\{N_i\}$ denote reduced values. N_{trend} is considered as a trend surface that may be fitted by first or second order polynomial functions. The statements established using the error variance $\Sigma P_i \gamma = 1$, when Lagrange Multiplier (λ) taken into equation and then differentiated we can write,

$$\sum_{i=1}^{n} P_i \gamma(N_{ij}) + \lambda = \gamma(N_{ik}) \tag{4}$$

Where $\gamma(N_{ij})$ is semi-variances between two reference points (*i* and *j*) and $\gamma(N_{ik})$ semi-variance value with respect to the distance between the reference points (*i*) and a new point (*k*) subjected to prediction. To calculate the weights using Equation (4), another equation stating the sum of the weights equal to 1 is incorporated and then the equation set is solved. For instance, when these equations are considered for three reference points for the prediction of a point *k*, the following equations may be defined;

$$\begin{aligned}
&\lambda + P_1\gamma(N_{11}) + P_2\gamma(N_{12}) + P_3\gamma(N_{13}) = \gamma(N_{1k})\\
&\lambda + P_1\gamma(N_{21}) + P_2\gamma(N_{22}) + P_3\gamma(N_{23}) = \gamma(N_{2k})\\
&\lambda + P_1\gamma(N_{31}) + P_2\gamma(N_{32}) + P_3\gamma(N_{33}) = \gamma(N_{3k})\\
&0 \;+\; P_1 \;+\; P_2 \;+\; P_3 \;=\; 1
\end{aligned} \qquad (5)$$

Hence, the unknown weights can be obtained.

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \end{bmatrix} = \begin{bmatrix} 1 & \gamma(N_{11}) & \gamma(N_{12}) & \gamma(N_{13}) \\ 1 & \gamma(N_{21}) & \gamma(N_{22}) & \gamma(N_{23}) \\ 1 & \gamma(N_{31}) & \gamma(N_{32}) & \gamma(N_{33}) \\ 0 & 1 & 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} \gamma(N_{1k}) \\ \gamma(N_{2k}) \\ \gamma(N_{3k}) \\ 1 \end{bmatrix} \qquad (6)$$

We can predict *N* values for the location with the unknown value by using weights. The equations for kriging are contained in matrix and vectors that depend on the spatial autocorrelation among the measured sample locations and prediction location. The autocorrelation values can be available from the semivariogram model. Finally, the predicted values, variance of the prediction and the standard error (S_ε) can be calculated using the equations 1, 3 and 7, respectively.

$$S_\varepsilon^2 = \lambda + P_1\gamma(N_{1k}) + P_2\gamma(N_{2k}) + P_3\gamma(N_{3k})\,;\; S_\varepsilon = \sqrt{S_\varepsilon^2} \qquad (7)$$

3 THE CASE STUDY OF IZMIR TERRITORY, TURKEY

3.1 The project area and data

This study was performed within the borders of İzmir Municipality area, covering approximately 115 km x 110 km between 37.87° and 38.91° north latitudes and 26.47° and 27.76° east longitudes (Figure 2). It was a part of a national research project, namely "Izmir geodetic infrastructure for the production of 1 / 5000 scaled digital photogrammetric maps and orthophotos" carried out by co-operation of Izmir Metropolitan Municipality and Yildiz Technical University. The purpose of the project was the determination of precise surface model for transformation of GPS derived ellipsoidal heights to orthometric heights by using the common points of GPS and leveling networks.

The topography of the project area is rather uneven and displays significant variations in the local scale and it is involving wavy, flat and highland components with heights spanning from 0 to 1497 m. The roughness of the topography leads to local variations in surface model. The homogeneity, density and distribution of reference points in three-dimensional space are the parameters which directly affect the resulting accuracy. Therefore, the reference points were selected considering the topography as soon as possible.

First of all, a GPS network is created based on Turkey's National Fundamental GPS Network (TUTGA) and it was calculated in ITRF96 datum, 2005 epoch (TUTGA-99A). 1017 GPS benchmarks were positioned via the principle of hierarchical network densification in GPS net. The average of position errors are 14.8 mm, 13.1 mm, and 18.8 mm for latitude, longitude, and ellipsoidal height components, respectively.

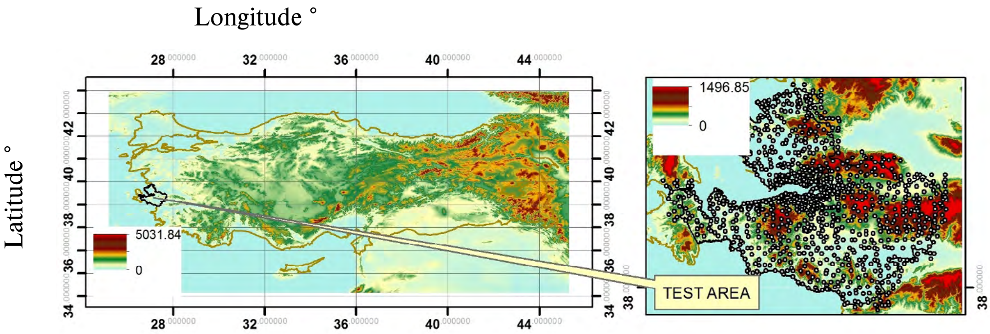

Figure 2: The project area boundaries, topography and reference points (units of the elevations are meters)

On the other hand, the leveling network based on Turkey's National Vertical Control Network (TUDKA) with 2525 points was established for similar purposes to determine orthometric heights. As a result of the adjustments to the leveling network, the reference standard deviation was 13.5 mm for 1 km of the leveling path. The average of mean square errors of leveling points was 32 mm with minimum and maximum values of 3 mm and 69 mm, respectively.

Input datasets	**Searching neighborhood:** Smooth	**Variogram Semivariogram**	**Model type:** Stable	**Outputs**
Data Records: 1148 **Method:** Kriging **Type:** Ordinary **Output type:** Prediction **Trend type:** None	**Smoothing factor:** 0.2 **Angle:** 0 **Major semiaxis:** 0.0491 **Minor semiaxis:** 0.0491	**Number of lags:** 12 **Lag size:** 0.006136598534437977 **Nugget:** 0.001342469167948521 **Measurement error:** %100 **ShiftON:** No	**Parameter:** 0.2 **Range:** 0.04909278827550382 **Anisotropy:** No **Partial sill:** 0.00788331909228199	**Output Format:** Grid **Grid Spacing:** 30'' **Grid Size:** 151 rows×124 columns **Grid Coverage:** 26.4747°-27.7280° N 37.8746° -38.8998° E Nmin: 36.5548 m Nmax: 38.8259 m **Area Covered by grid:** 12648 km^2

Table 1: Input and output parameters for kriging model

The impact of accuracy of orthometric and ellipsoidal heights to the accuracy of "surface model" is taken into consideration during the pre-processing of the data for outlier detection. 857 favorable points were obtained from GPS and leveling network. Besides, 301 previously established points from the Izjrs-2001 project were also used (Ayan et al., 2001). Finally, 1148 reference points (data density is approximately 4.75km^2/ point) with geographical coordinates were used for surface modeling by ArcGIS 10.0 Geostatistical Analyst. We applied geostatistical methods (kriging) to our data set. Since the flexibility of kriging can require a lot of decision-making during the process, optimized parameters were selected based on minimizing the RMS error (Table 1).

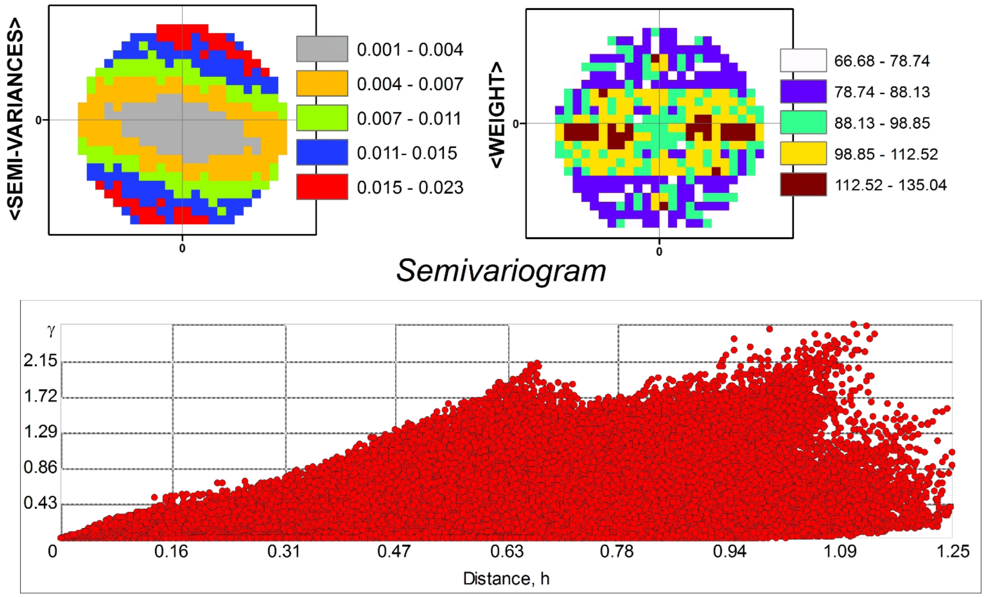

Figure 3: Representation of predicted surface and variography

Figure 3 shows the semi variances (top left), weights (top right), and variography (bottom) for used geostatistical model.

4 ANALYSIS OF MODEL

Several statistical information such as; Mean error (ME), Root Mean Square (RMS) error, Standard error (SE), standard deviation (STD), median absolute deviation (MAD), mean standardized error (MSE), root mean square standardized (RMSS) error and etc. can be used for understanding of goodness of the model.

In the following sections, the statistical analysis results of surface model are shown by applying various validation procedures.

4.1 Internal Validation

The most important indicator about quality of the model is RMS errors and its standardized form. RMS indicates how closely model predicts the measured values. The smaller this error, the better predictions could be performed. RMS error can be available from a Cross Validation (CV) analysis for kriging. The primary use of this procedure is to compare the predicted value to the observed value in order to obtain useful information about model parameters (Featherstone and Sproule, 2006; Soycan and Soycan 2009).

As with cross-validation, it is expected the following results:

- The average of errors close to zero
- A small RMS error for prediction
- A standardized mean prediction error near zero

- An average standard error similar to the RMS (If the average of the standard errors are close to the RMS, we are correctly assessing the variability in prediction; if the average of the standard errors are greater than the RMS, we are overestimating the variability of our predictions; if the average of the standard errors are less than the RMS, we are underestimating the variability in our predictions)
- Another option about this issue is to divide each prediction error by its estimated prediction standard error for calculating RMSS Error. It should be close to one if the prediction standard errors are valid. If the RMSS error is greater than one, model is underestimating the variability in predictions. If the RMSS error is less than one, model is overestimating the variability in predictions.

The right side of Figure 4 shows a color map for standard errors of the prediction. It represents the quality of the model depending on the geographical location. As expected, the standard errors seem to be less in the center and in the areas having high data density. It is varied between 0.013 m to 0.129 m with 0.0606 m mean.

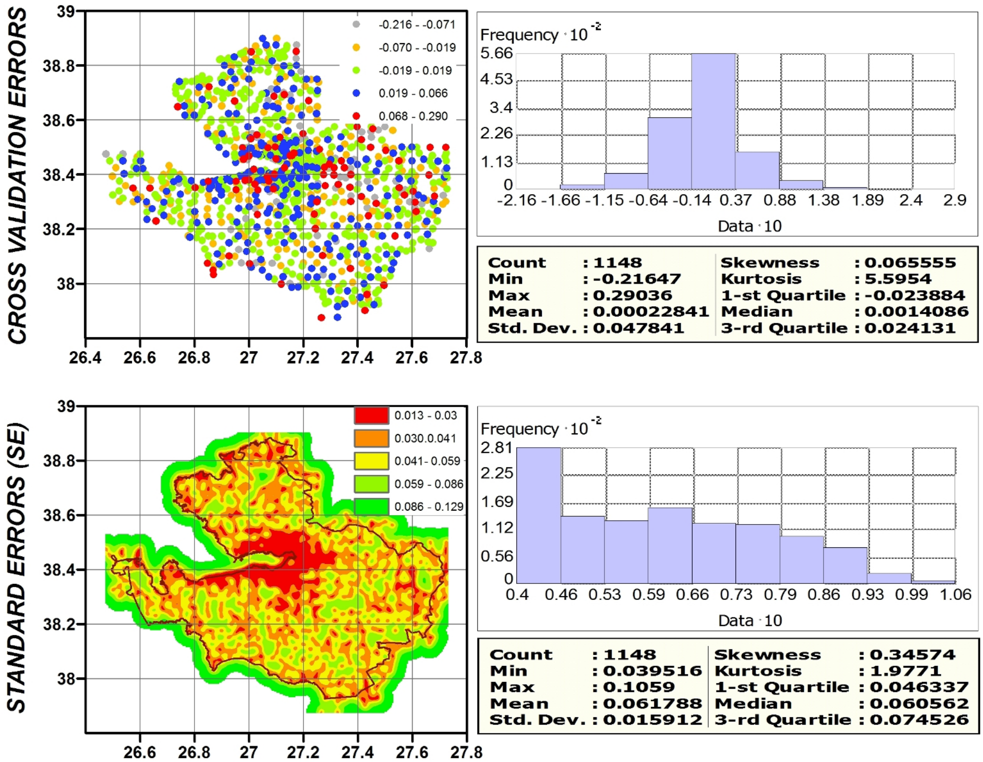

Figure 4: Internal validation results

The left side of Figure 4 shows a map of the discrepancies derived by subtracting the observed and predicted values in scattered point mode and a histogram plot. The colors of the data-points are proportional to the range of the detected discrepancies. There are 11 inconsistent points that were rejected by the ±3σ threshold.

The discrepancies ranged from -0.216 to +0.290 m, with a mean of +0.0002 m. The STD of the discrepancies amounts to 0.048 m. The average of the discrepancies is zero and STD and RMS error are equal so the discrepancies have a normal distribution and they involved random error components (See histogram in Figure 4). We concluded that there were no systematic errors affecting the modeling and the model anymore from this preliminary result. A more comprehensive investigation can be performed with Figure 5.

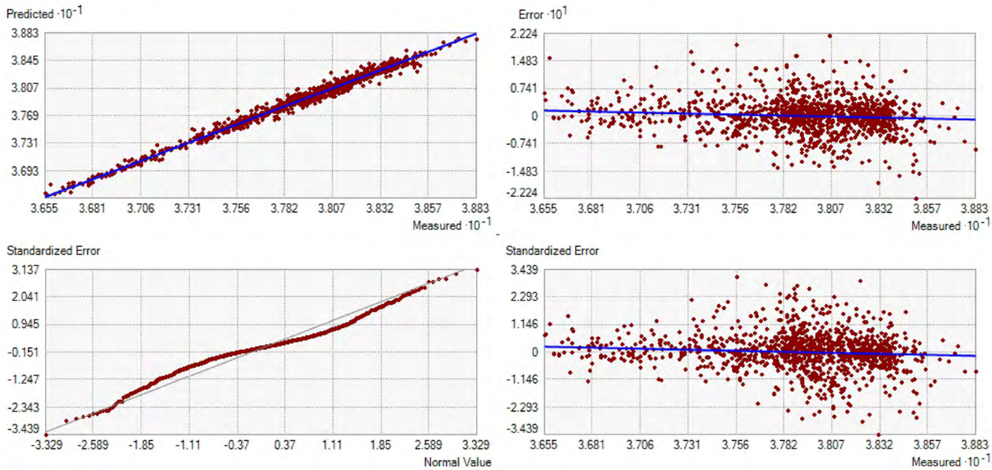

Figure 5: Predicted values versus several variables

Figure 5 shows four different graphs of the prediction results. The first three graphs show how well kriging is predicting.

1. The prediction plot is a scatterplot of measured values versus the predicted values is given in top left as red dots. The fitted line through the scatter of points is given in blue.
2. The measured values are subtracted from the predicted values (in top right) and the error plot can be created from prediction plot. It is expected that the errors are centered on the true values (near zero), so that the prediction will be unbiased.
3. Due to the fact that, error plot depends on the scale of the data we have to standardize them. The measured values are subtracted from the predicted values and divided by the estimated standard errors for the standardized error plot. The mean of these is expected also near zero (in below right).
4. Besides, the points on the Normal QQ plot (Standardized error versus normal value) provide an indication of univariate normality of the dataset (in below left). The QQ plot shows the quantiles of the difference between the predicted and measured values and the corresponding quantiles from a standard normal distribution. Our data is normally distributed and the points are falling on the 45-degree reference line (gray line). A limited number of points are deviating from the reference line.

Samples	Mean	RMS	Mean Standardized	RMSS	Average of the Standard Error
1148	0.0002	0.0478	0.00423	0.8061	0.0606

Table 2: Some quality measures for model (units are meter)

4.2 Validation of the model with Splitted Data (Training and Test Data)

The modeling and the tests performed in the previous section certainly provide useful insight regarding the accuracy of the model over the test area. However, CV approach used for the all reference points may be considered 'simplistic' and time consuming (point by point) for a large data set.

The other validation procedure is the use of independent check points which are not used in the interpolation. This is also known as the jack-knifing technique (Tomczak, 1998; Goncalves, 2006; Soycan and Soycan 2009; Vieira et al., 2010) based on removing of random data from sampling data points, and using the remaining data to perform the interpolation.

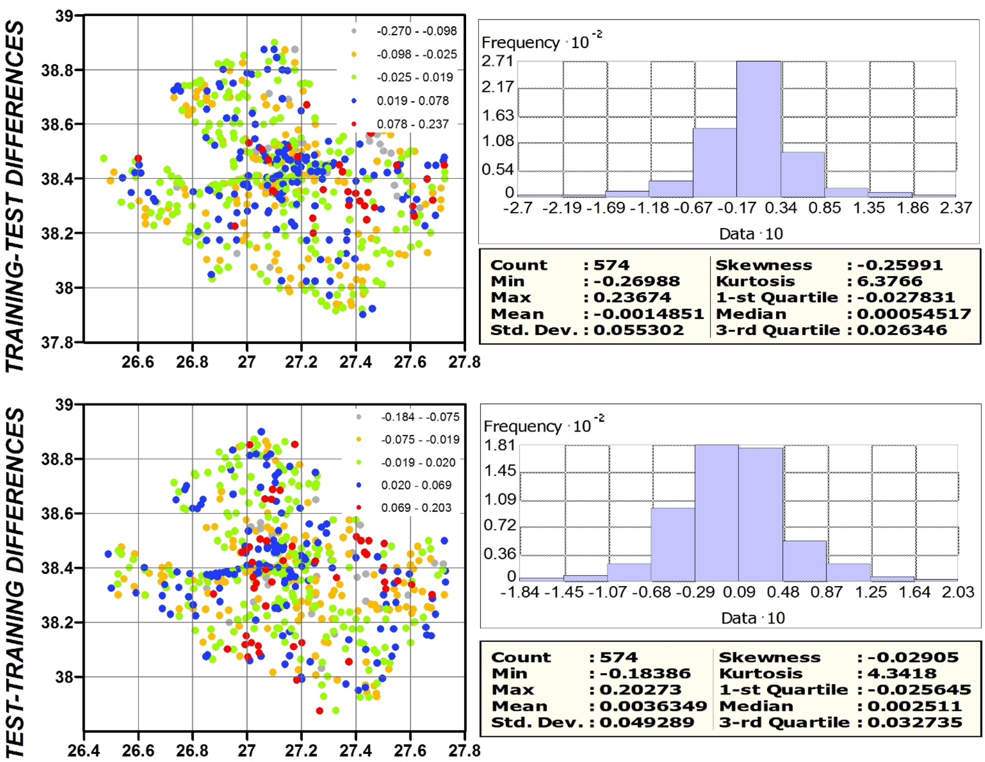

Figure 6: Splitted data validation results

For this purpose, the set of the reference point data were splitted in two sets, test (A) and training (B) data, which have no common observations, but have the same geographical distribution.

In the first step, observations of "data A" were considered as reference points for modeling of surface, then the consistency of the model was checked by values of "data B". The same procedure was applied to set A by using the observation of "data B". A measure of the quality of this procedure is obtained by computing the RMS of discrepancies between observed values and predicted ones. The RMS computed in this manner provides a realistic indication of the accuracy of the model and its performances as a prediction surface for the new point.

As the result of the investigation of the splitted data validation procedure, it is seen that the discrepancies are varying between -0.270 m and 0.237 m, the mean is -0.0015 m, the STD is 0.055 m for training to test data validation. For test to training data validation very similar results were achieved (from -0.184 m up to 0.203 m with 0.0036 m ME and 0.0493 m STD). The accuracy of the achieved model was approximately derived as ± 4-5 cm in this manner. Although, there are some points (red in the color bar), where the discrepancies are much larger than the average, the results of this validation agree reasonably well with CV in general (Figure 6).

4.3 Validation of the model with External Data

Although CV and splitted data validation analysis is a very important tool to identify the outliers and the appropriateness of the model, it assesses internal accuracy of the model. An independent comparison with external data can be made using the ellipsoidal-ortometric height differences determined point-wise by GPS positioning and leveling on other locations. External validation allows evaluating of our predictions by using a dataset that was not involved in creating the prediction model, and thus gives a reasonable indication of the model's external accuracy. For this purpose, the external accuracy of the model was tested by using the external validation points that had not been handled in the interpolation process.

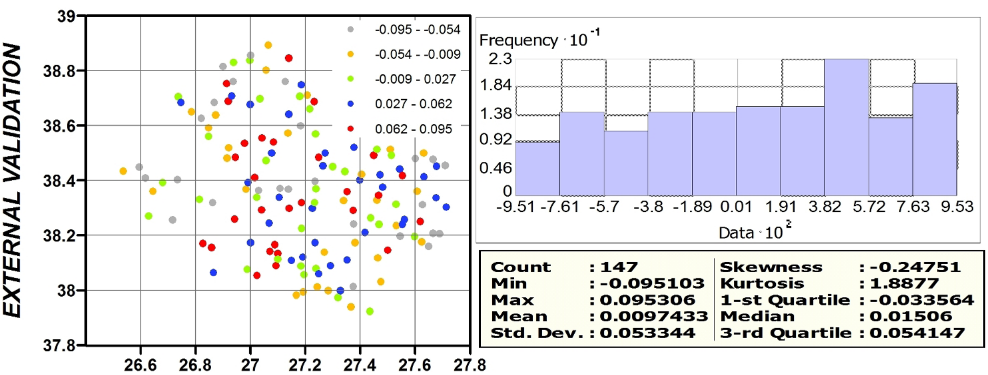

Figure 7: External validation results

The external validation data set was obtained from the independent GPS and levelling observations in different locations of the test area. These points are not from the set of 857 GPS benchmarks + 301 Izjrs2001 points. The comparison was performed in 156 points. The achieved results are presented in Figure 7.

The results are presented in terms of classified color plot, statistics and histograms of the discrepancies between external data and the model. 9 inconsistent points where the discrepancies were much larger than the ±3σ threshold were rejected. In general, the model agrees reasonably well with the remaining (147 points) external data (from -0.095 m up to 0.095 m with 0.010 m mean and 0.053 m STD). Therefore, it can be said that discrepancies are normally distributed and they contain systematic components with a small amount. This may indicate that different geodetic benchmarks for GPS and leveling computation were used at the compilation of the external data.

5 COMPARISON OF THE MODEL WITH REGIONAL AND GLOBAL GEOIDS

This section compares geoid models established using different sources and different methods. Two different models are used in this comparison. The first model is EGM08 (http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008), that is the newest global geopotential model computed from a global 5 arc-minute grid of gravity anomalies from land and satellite-based sources. The model is provided complete to spherical harmonic degree and order 2159, which equates to a grid size of approximately 6.5 km.

The second one is The Turkish Geoid (TG03) national model established by the General Command of Mapping within the borders of Turkey. It was computed in 2003 with heterogeneous data (gravity, topography and geoid heights) were used by Least Squares Collocation (LSC) in a remove-restore procedure. EGM96 was used as the reference model of the Earth's geopotential model. The used data consists of surface gravity anomalies (on ~65000 stations), gravity anomalies derived from ERS1, ERS2 and TOPEX/POSEIDON altimetry data (on ~20000 stations), gravity anomalies derived from ship observations (on ~10000 stations), GPS/leveling data (on 197 stations) and topographic heights. The national report of Turkish National Union of Geodesy and Geophysics (http://www.iugg.org/members/nationalreports/turkey.pdf) states out that the absolute accuracy of TG03 is 8.8 cm.

However, TG03 and EGM08 models provide about 2 times lower accuracy than the local surface model; such a comparison is significant for the examination of the consistency of the local surface model and the other models.

As a result of the investigation of the discrepancies obtained by the EGM08 comparison; it is varying between -0.9072 m and -0.1168 m, the average is -0.5937 m, the STD is 0.1306 m and the RMS error is 0.6070 m. It is easily recognized that, there is a bias between the geoid heights of the EGM08 model and the local surface model. The consistency of our local model with EGM08 is around ±6.5 cm in case of excluding the systematic effect of -0.5937 m. Since the global agreement to GPS-levelling is approximately ~7cm, Turkish proprietary data were not used in EGM08 computations. From the mean discrepancy between EGM08 and GPS/leveling, it is found that the local model (national vertical datum of Turkey) for test area is offset from the global geoid by 60 cm (Kilicoglu et al., 2009).

It is clear that direct comparison of the model with EGM08 does not have the sense. GPS/Levelling derived height differences refer to the GRS80 ellipsoid and their corresponding values computed from EGM08 refer to a mean Earth ellipsoid that does not have the same dimensions as the WGS84 ellipsoid. It is necessary to use a fitting model to minimize the long wavelength geoid errors, the systematic datum discrepancies between the global geoid and the GPS/levelling data and for the real comparison between different models.

It can be concluded from technical literature that successful results can be obtained by using the least-squares adjustment with a four, five or seven parameter transformation model; least squares or robust estimations with polynomial models, least squares collocation (LSC), finite element method (FEM), the Fourier series and similar fitting methods, etc. We used four parameters model in the studies dealing with the combined adjustment of GPS, levelling and geoid data, to remove the systematic errors introduced by the datum discrepancies between the data sets. It is the most commonly used in such adjustments and is given by the following equation;

$$f(\varphi, \lambda)=\Delta N=h_i-H_i-N_i^{EGM08}=N_i^{GPS-LEV.}-N_i^{EGM08}=a_i^T\cdot x+v_i=a_0+a_1\cdot(\lambda_1-\lambda_0)+a_2\cdot(\varphi_1-\varphi_0)+ a_3\cdot(\lambda_i-\lambda_0)$$

where (N_i^{EGM08}) is EGM2008 geoid heights, ($N_i^{GPS-LEV.}$) is the corresponding GPS/levelling-derived height difference, a_0 is the shift parameter between the GPS/levelling datum and the EGM2008 datum a_1, a_2, a_3 are the shift parameters between two parallel' datums and vi denotes residual. λ_0 and φ_0 are appropriate selected arbitrary value for reduction of geographical ellipsoidal coordinates.

# Control Points	Transformation Coefficients: a_0, a_1, a_2, a_3	Statistics for Residuals
29	a_0 = −0.5930±0.0180 a_1 = 0.2717±0.0454 a_2 =−0.1172±0.0589 a_3 = 0.1554±0.0793 (λ_0= 27,14°, φ_0= 38,40°)	Sum: −0.365m. Minimum:0.125 m. Maximum: 0.133 m. Mean: −0.013 m. Std. dev.: 0.066 m. RMS:0.067 m.

Table 3: The results of four parameters transformation

It is found that the agreement between the refinement–EGM08 and the model derived surface approximately ~7–8 cm (from −0.2918 m up to 0.2896 m with −0.0128 m mean and 0.0634 m STD). On the other hand, the discrepancies between the TG03 and local model derived values vary from −0.2696 to 0.2896 m, with a mean of −0.0091 m. The histogram of discrepancies is shown in Figure 8. The histograms of both comparisons in Figure 8 illusturates that the data is closer to be normally distributed with a small bias.

The accuracy of examined model is reasonably better than TG03 and EGM08–derived geoid model. The improvement is 3–4 cm in terms of STD for each case. However, the large discrepancies in local level show that, there are some systematic biases between the EGM08 or TG03 and local model. In Figure 8, we can detect some local and regional details where the discrepancies have the unsystematic nature. It is thought that systematic discrepancies arise from various reasons such as topography, data and methods used in modeling and etc. The discrepancies are quite significant, exceeding a dm level. Clearly, for the further improvement of the regional or global model accuracy the GPS, leveling, gravity surveys and elevation data need to be revised by the alternative approaches.

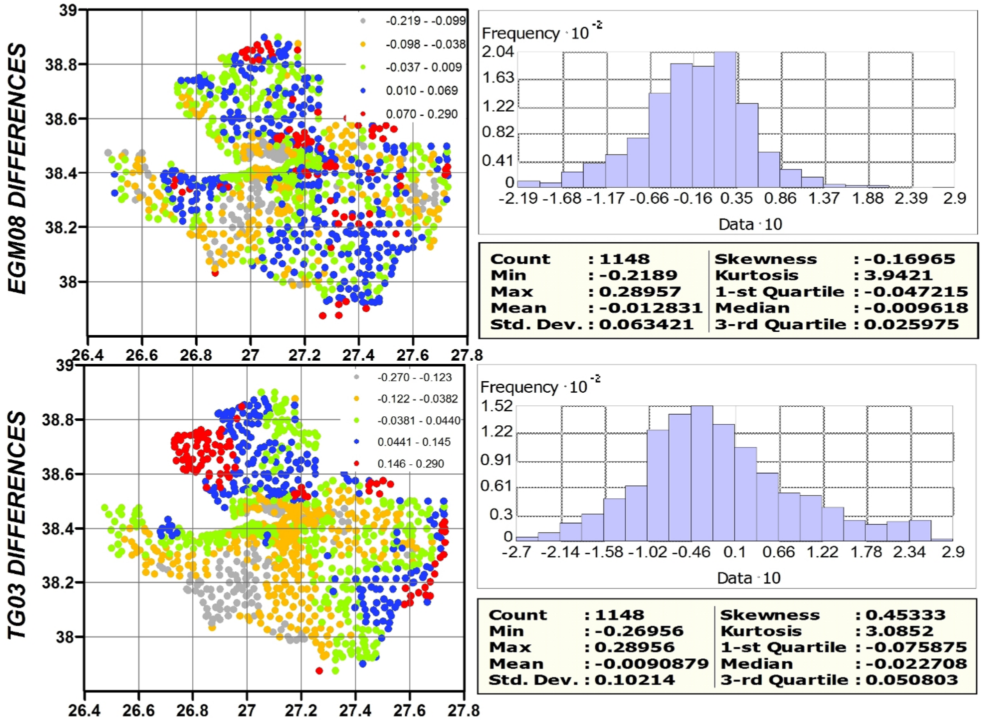

Figure 8: Comparison results for EGM08 and TG03

	Internal Validation		Splitted Data		External Validation	Comparison		
	Standard Errors	Cross Validation	DATA (A)	DATA (B)		TG03	EGM08*	EGM08
Number of values	1148	1148	574	574	147	1148	1148	1148
Minimum	0.0395	-0.2165	-0.2699	-0.1839	-0.0951	-0.2696	-0.2918	-0.9072
Maximum	0.1059	0.2904	0.2367	0.2027	0.0953	0.2896	0.2896	-0.1168
Mean	0.0618	0.0002	-0.0002	0.0036	0.0097	-0.0091	-0.0128	-0.5937
Median	0.0606	0.0014	0.0005	0.0025	0.0151	-0.0227	0.0026	-0.6077
STD	0.0159	0.0478	0.0553	0.0493	0.0533	0.1021	0.0634	0.1306
RMS Error	**0.0638**	**0.0478**	**0.0553**	**0.0494**	**0.0542**	**0.1028**	**0.0647**	**0.6070**

Table 4: Summary of the statistical information for overall validation (refinement-EGM08 with four parameter fitting)*

As a result, the corresponding statistics for overall validation results can be found in Table 4. The splitted data and external data validation results of the model are similar to their preliminary internal validation results. In addition, the model is consistent with the regional and global models within expected level.

6 CONCLUSION

The orthometric height is very important information for all types of engineering projects. It is well-known that by the surveyors, to obtain orthometric heights by geometric levelling or other terrestrial surveying methods are time consuming and expensive, also these methods require extra efforts and field works.

Nowadays, GNSS technology is the most effective positioning methods for all scientific and practical applications. Since GNSS provides ellipsoidal heights instead of orthometric heights, users need a transformation surface model in order to transform ellipsoidal heights to orthometric heights. Consequently, a local GPS&Leveling derived surface model can be used for transformation of GPS heights to orthometric heights in practical geodetic and surveying applications. This solution enables centimeter accuracy in GPS height transformation.

As a result of the geodetic studies conducted in Izmir metropolitan of Turkey by "Geodetic infrastructure for the production of 1 / 5000 scaled digital photogrammetric maps and orthophotos" project, a local surface model of the Izmir region was developed by geostatiscal surface modeling method. The accuracy of the model was evaluated by several validation procedure, the achieved results can be summarized as follows:

- The consistency between the local model with TG03 and EGM08 was 10.3 cm and 60.70 cm, respectively. It is apparent that direct use of EGM08 does not guarantee an accurate transformation of the ellipsoidal heights to the orthometric heights in an absolute form. The systematic discrepancy about 60 cm has to be corrected for the use of EGM08 in practice. The absolute accuracy can be achieved at the level of 6-7 cm in this way for EGM08.
- In a relative form, TG03 and EGM08 give similar accuracy. As long as the baseline length is in both models longer, the errors in terms of orthometric height are also smaller; therefore they can be used at long distances. On the other hand, local model, which has the mean relative accuracy of 1.002 ppm, gives the better performance for the transformation of the ellipsoidal height differences to orthometric height differences. Its accuracy is below 0.5 ppm in all baseline lengths over 50 km.
- The tolerance value for leveling is 12 to $20\sqrt{Skm}$ for the third order leveling application in Turkey. It can be converted into relative accuracy as 3.8 ppm to 1.2 ppm for distances between 10 km and 100 km. So, the resulting surface model will meet this requirement for GPS leveling application in the project area.

Concluding this paper, the most effective methods which can be used in short and ultra-short wavelength component determination seems the GPS&Leveling method by considering the applicability and cost-efficiency criteria. With the GPS&Leveling method, it is possible to determine ellipsoidal-ortometric height differences with absolute accuracies ranging from 1 cm to 2 cm. This method has been globally accepted and adopted due to its ease of measurement and calculation, accuracy and cost efficiency. Moreover, it is mostly recommended for the ellipsoidal to orthometric height transformation studies in Turkey. Although, an adequate number of properly spread out reference points are used for surface modeling, accuracy of our local model is about ±4-5cm. The actual accuracy, of course, depends on the available data, their accuracy, and their spatial distribution. Also, the ellipsoidal and orthometric height accuracies and interpolation method are other significant factors for the accuracy of modeling. The possible vertical deformation in the leveling network and the vertical datum also play an important role on the accuracy.

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