



Unquenched quark-model calculation of excited ρ resonances and P-wave $\pi\pi$ phase shifts*

Susana Coito^a, George Rupp^b, Eef van Beveren^c

^aInstitute of Modern Physics, CAS, Lanzhou 730000, China

^bCeFEMA, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal

^cCentro de Física Computacional, Departamento de Física, Universidade de Coimbra, 3004-516 Coimbra, Portugal

Abstract. The $\rho(770)$ vector resonance, its radial recurrences, and the corresponding P-wave $\pi\pi$ phase shifts are investigated in an unquenched quark model with all classes of relevant decay channels included, viz. pseudoscalar-pseudoscalar, vector-pseudoscalar, vector-vector, vector-scalar, axialvector-pseudoscalar, and axialvector-vector, totalling 26 channels. Two of the few model parameters are fixed at previously used values, whereas the other three are adjusted to the $\rho(770)$ resonance and the lower P-wave $\pi\pi$ phases. Preliminary results indicate the model's capacity to reproduce these phases as well as the ρ mass and width. However, at higher energies the phase shifts tend to rise too sharply. A possible remedy is an extension of the model so as to handle resonances in the final states for most of the included decay channels. Work in progress.

1 Introduction

The radial recurrences of the $\rho(770)$ vector resonance play a crucial role in light-meson spectroscopy, owing to the several observed states in the PDG tables [1], up to 1.9 GeV, and the corresponding P-wave pion-pion phase shifts and inelasticities measured in several experiments [1]. These resonances may shed a lot of light on the underlying quark-confinement force as well as the strong-decay mechanism, both assumed to result from low-energy QCD. However, there are two serious problems, one experimental and the other theoretical. First of all, excited ρ states listed in the PDG tables are far from well established, even the generally undisputed $\rho(1450)$ [1] resonance. For example, under the entry of the latter state in the PDG meson listings, experimental observations have been collected with masses in the range 1250–1582 MeV. The state of lowest mass here corresponds to the $\omega\pi^0$ resonance at 1.25 GeV first observed by Aston *et al.* in 1980 [2]. Several other experiments have confirmed such a vector ρ' resonance in the mass interval 1.25–1.3 GeV, both in the $\omega\pi^0$ [3] and $\pi\pi$ [4] channels. More recently, a combined \mathcal{S} -matrix and Breit-Wigner (BW) analysis [5], using P-wave $\pi\pi$ data from the early 1970s, not only confirmed a $\rho(1250)$, but even found it to be much

* Talk delivered by George Rupp

more important for a good fit to the data than the $\rho(1450)$. In view of these findings, it is simply inconceivable that no separate $\rho(1250)$ entry has been created in the PDG tables, and to make things worse, the latter analysis [5] is not even included in the PDG references. The reason for these omissions seems to be based on theory bias, which is the second problem. Indeed, the renowned relativised quark model for mesons by Godfrey & Isgur [6] predicted the first radial ρ excitation at 1.45 GeV. As a matter of fact, practically all quark models employing the usual Coulomb-plus-linear confining potential find a ρ' at about the same mass and cannot accommodate a $\rho(1250)$ (see, however, Ref. [7]).

Unfortunately, no fully unquenched lattice calculations including both $q\bar{q}$ and two-meson interpolating fields have been carried out so far beyond the ground-state $\rho(770)$ [8]. Nevertheless, in the strange-meson sector such a calculation was done recently [9], reproducing the $K^*(892)$ resonance in P-wave elastic $K\pi$ scattering, with mass and width close to the experimental values. Moreover, the first radial excitation was identified as well, tentatively at 1.33 GeV, which should correspond to the $K^*(1410)$ [1] resonance. The latter is thus determined on the lattice to be a normal quark-antiquark resonance. From simple quark-mass considerations, one is led to conclude the same for the ρ' originally found by Aston *et al.* [2] at 1.25 GeV, which contradicts the speculation in Ref. [10] that it “*has necessarily to be an exotic*”. Note that including meson-meson interpolators is absolutely crucial to reliably predict the mass of an excited resonance like the $K^{*'}$, since an also unquenched lattice calculation without considering decay, by partly the same authors [11], predicted a $K^{*'}$ mass a full 300 MeV heavier than in Ref. [9].

In the present work, we analyse the issue of radial ρ recurrences by attempting to describe the elastic and inelastic P-wave $\pi\pi$ phase shifts in the context of an unquenched quark model that has been successfully applied to several problematic mesons (see e.g. Ref. [12] for a very brief recent review). In this so-called Resonance-Spectrum-Expansion (RSE) model, the manifest non-perturbative inclusion of all relevant two-meson channels alongside a confined $q\bar{q}$ sector allows for a phenomenological description of excited meson resonances in the same spirit as the referred lattice calculation [9]. An additional advantage is that the RSE model yields an exactly unitary and analytic \mathcal{S} -matrix for any number of included quark-antiquark and two-meson channels, whereas the lattice still faces serious problems in the case of inelastic resonances and highly excited states.

2 RSE modelling of P-wave $\pi\pi$ scattering

The general expressions for the RSE off-energy-shell \mathcal{T} -matrix and corresponding on-shell \mathcal{S} -matrix have been given in several papers (see e.g. Ref. [13]). In the present case of P-wave $\pi\pi$ scattering, the quantum numbers of the system are $I^G J^{PC} = 1^+ 1^{--}$, which couples to the $I=1$ quark-antiquark state $(u\bar{u} - d\bar{d})/\sqrt{2}$ in the spectroscopic channels 3S_1 and 3D_1 . In the meson-meson sector, we only consider channels allowed by total angular momentum J , isospin I , parity P , and G-parity G . The included combinations are pseudoscalar–pseudoscalar (PP), vector–pseudoscalar (VP), vector–vector (VV), vector–scalar (VS), axialvector–pseudoscalar (AP), and axialvector–vector (AV), with mesons from the lowest-lying pseudoscalar,

vector, scalar, and axialvector nonets listed in the PDG [1] tables. Here, “axialvector” may refer to $J^{PC} = 1^{++}$, $J^{PC} = 1^{+-}$, or $J^P = 1^+$ for mesons with no definite C-parity. This choice of meson-meson channels is motivated by the observed two- and multi-particle decays of the ρ recurrences up to the $\rho(1900)$ [1], which include several intermediate states containing resonances from the referred nonets. For instance, the PDG lists [1] under the 4π decays of the $\rho(1450)$ the modes $\omega\pi$, $\alpha_1(1260)\pi$, $h_1(1170)\pi$, $\pi(1300)\pi$, $\rho\rho$, and $\rho(\pi\pi)_{S\text{-wave}}$, where $(\pi\pi)_{S\text{-wave}}$ is probably dominated by the $f_0(500)$ [1] scalar resonance. By the same token, the 6π decays of the $\rho(1900)$ will most likely include important contributions from modes as $b_1(1235)\rho$, $\alpha_1(1260)\omega$, For consistency of our calculation, we generally include complete nonets in the allowed decays, and not just individual modes observed in experiment. The only exception is the important $\pi(1300)\pi$ P’P mode, because no complete nonet of radially excited pseudoscalar mesons has been observed so far [1]. The included 26 meson-meson channels are given in Table 1.

Nonets	Two-Meson Channels	L
PP	$\pi\pi, KK$	1
VP	$\omega\pi, \rho\eta, \rho\eta', K^*K$	1
VV	$\rho\rho, K^*K^*$	1
VS	$\rho f_0(500), \omega a_0(980), K^*K_0^*(800)$	0, 2
AP	$\alpha_1(1260)\pi, b_1(1235)\eta, b_1(1235)\eta', h_1(1170)\pi, K_1(1270)K, K_1(1400)K$	0
AV	$\alpha_1(1260)\omega, b_1(1235)\rho, f_1(1285)\rho, K_1(1270)K^*, K_1(1400)K^*$	0
P’P	$\pi(1300)\pi$	1

Table 1. Included classes of decay channels containing mesons listed [1] in the PDG tables, with the respective orbital angular momenta. Note that the included P’P decay mode is incomplete (see text above).

Notice that the VS channels count twice, because they can have $L=0$ or $L=2$. However, the S-wave only couples to the 3S_1 $q\bar{q}$ channel and the D-wave only to the 3D_1 . The relative couplings between the $q\bar{q}$ and meson-meson channels are determined using the scheme of Ref. [14], based on overlaps of harmonic-oscillator (HO) wave functions. Special care is due in the cases of flavour mixing (η, η'), and mixing of the C-parity eigenstates 3P_1 and 1P_1 [13], as the strange axialvector mesons $K_1(1270)$ and $K_1(1400)$ have no definite C-parity.

Coming now to describing the data, we have to adjust the model parameters. Two of these, namely the non-strange constituent quark mass and the HO oscillator frequency, are as always fixed [13] at the values $m_u = m_d = 406$ MeV and $\omega = 190$ MeV. This yields a largely degenerate bare ρ spectrum with energy levels 1097 MeV (1^3S_1), 1477 MeV ($2^3S_1/1^3D_1$), 1857 MeV ($3^3S_1/2^3D_1$), 2237 MeV ($4^3S_1/3^3D_1$), This bare spectrum is then deformed upon unquenching, that is, by allowing $q\bar{q}$ pair creation. This results in real or complex mass shifts due to meson-loop contributions, depending on decay channels being closed or open, respectively. Note that these shifts are non-perturbative and can

only be determined, for realistic coupling strengths, by numerically finding the poles of the S -matrix. The adjustable parameters are the overall dimensionless coupling constant λ , the “string-breaking” radius r_0 for transitions between the $q\bar{q}$ and two-meson channels, and a range parameter α for weakening subthreshold contributions via a form factor. The coupling λ is usually in the range 3–5, r_0 should be of the order of 1 fm for systems made of light quarks (with mass m_n), and the α value used in several previous papers is 4 GeV^{-2} (see e.g. Ref. [15]).

In view of these strong limitations, it is quite remarkable that we can obtain a good fit to the P-wave $\pi\pi$ phase shifts up to 1.2 GeV with the choice $\lambda = 5.3$, $r_0 = 0.9 \text{ fm}$, and $\alpha = 4 \text{ GeV}^{-2}$. Moreover, the corresponding $\rho(770)$ pole comes out at the very reasonable energy $E = (754 - i67) \text{ MeV}$. At higher energies, though, the $\pi\pi$ phases tend to rise too fast and no good description has been obtained so far, also due to the very little fitting freedom. As for the poles of the higher ρ recurrences, we find at least four in the energy range 1.2–2.0 GeV, in agreement with the PDG [1] and also Ref. [5], albeit at yet quite different energies. In particular, there are two poles between 1.2 and 1.5 GeV, in qualitative agreement with Ref. [5]. However, these pole positions are extremely sensitive to the precise values of the parameters λ , r_0 , and α , so that they should not be taken at face value as long as no good fit is achieved of the observables above 1.2 GeV.

A possibility to improve the fit is by allowing different decay radii for the several classes of decay channels (PP, VP, VV, VS, AP, AV, P'P), which would be logical in view of the detailed, channel-dependent transition potentials derived in Ref. [14]. Such an additional flexibility will neither affect the exact solvability of the RSE \mathcal{T} -matrix, nor its analyticity.

An additional possible model extension we shall discuss in the next section.

3 Resonances in asymptotic states

The decay channels listed in Table 1 contain several resonances, some of which are even extremely broad, with widths exceeding 300 MeV. So treating the corresponding thresholds as being sharp, at well-defined real energies, is certainly an approximation, which may produce too sudden effects at threshold openings. Ideally, one would like to describe a resonance in the final state via a smooth function of real energy, corresponding to an experimental cross section in which the resonance is observed. By discretising such a function, one could in principle describe each final-state resonance through a large number of effective thresholds. However, this would lead to a proliferation of channels and to a true explosion of Riemann sheets, making the tracing of complex poles impracticable.

An alternative way to handle a resonance in asymptotic states is to replace its real mass by a complex one, on the basis of the PDG [1] resonance mass and total width. However, this inevitably destroys unitarity of the S -matrix. Nevertheless, its symmetry will be unaffected, which can be used to define a new matrix that is unitary again, and so take over the role of S . Here, we closely follow the derivation given in Ref. [16].

An arbitrary symmetric matrix \mathcal{S} can be decomposed, via Takagi [17] factorisation, as

$$\mathcal{S} = \mathcal{V}\mathcal{D}\mathcal{V}^\dagger, \quad (1)$$

where \mathcal{V} is unitary and \mathcal{D} is a real non-negative diagonal matrix. Then we get

$$\mathcal{S}^\dagger\mathcal{S} = (\mathcal{V}^\dagger)^\dagger\mathcal{D}\mathcal{V}^\dagger\mathcal{V}\mathcal{D}\mathcal{V}^\dagger = (\mathcal{V}^\dagger)^\dagger\mathcal{D}^2\mathcal{V}^\dagger = \mathcal{U}^\dagger\mathcal{D}^2\mathcal{U}, \quad (2)$$

where we have defined $\mathcal{U} \equiv \mathcal{V}^\dagger$, which is obviously unitary, too. So the diagonal elements of $\mathcal{D} = \sqrt{\mathcal{U}\mathcal{S}^\dagger\mathcal{S}\mathcal{U}^\dagger}$ are the square roots of the eigenvalues of the positive Hermitian matrix $\mathcal{S}^\dagger\mathcal{S}$, which are all real and non-negative. Moreover, since $\mathcal{S} = \mathbf{1} + 2i\mathcal{T}$ is manifestly non-singular, the eigenvalues of $\mathcal{S}^\dagger\mathcal{S}$ are even all non-zero and \mathcal{U} is unique. Thus, we may define

$$\mathcal{S}' \equiv \mathcal{S}\mathcal{U}^\dagger\mathcal{D}^{-1}\mathcal{U}. \quad (3)$$

Then, using Eq. (1) and $\mathcal{V} = \mathcal{U}^\dagger$, we have

$$\mathcal{S}' = \mathcal{U}^\dagger\mathcal{D}\mathcal{U}\mathcal{U}^\dagger\mathcal{D}^{-1}\mathcal{U} = \mathcal{U}^\dagger\mathcal{U}, \quad (4)$$

which is obviously symmetric. But it is also unitary, as

$$(\mathcal{U}^\dagger\mathcal{U})^\dagger = \mathcal{U}^\dagger(\mathcal{U}^\dagger)^\dagger = \mathcal{U}^{-1}(\mathcal{U}^{-1})^\dagger = (\mathcal{U}^\dagger\mathcal{U})^{-1}. \quad (5)$$

So \mathcal{S}' has the required properties to be defined as the scattering matrix for a process with complex masses in the asymptotic states. Note that this empirical method has been applied very successfully to the enigmatic $X(3872)$ charmonium state in Ref. [16].

4 Conclusions

We have presented preliminary results of an unquenched quark-model study aimed at determining the complex pole positions of vector ρ recurrences up to 2 GeV, motivated by the poor status of these resonances in the PDG tables [1] and their importance for light-meson spectroscopy. The employed RSE model was applied in the past to a variety of problematic mesons, with very good results [12]. The here included classes of meson-meson channels cover most of the observed strong decays. The three adjustable model parameters were fitted to the P-wave $\pi\pi$ phase shifts up to about 1.2 GeV, allowing a good reproduction of these data and a very reasonable $\rho(770)$ resonance pole position.

However, at energies above 1.2 GeV the thus calculated phases rise too fast, and a globally good fit including the higher $\pi\pi$ phases is not feasible with only three parameters. A possible model extension amounts to allowing different decay radii for the different classes of meson-meson channels, which will not spoil the nice model features. Another extension to be considered is the use of complex physical masses for the final-state resonances in nearly all channels of Table 1. This will require a redefinition of the \mathcal{S} -matrix so as to restore manifest unitarity, which can be done with an empirical algebraic procedure, exploiting the symmetry of \mathcal{S} .

All this work is in progress.

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vzbujena vezana stanja, in pokažemo pot do bolj realistične teorije, v kateri nastopijo resonance kot kompleksni poli v ravnini gibalna količina/energija, torej z realno maso in končno širino.

Račun vzbujenih resonanc mezona ρ in faznih premikov $\pi\pi$ v P-valu z razširjenim kvarkovim modelom

Susana Coito^a, George Rupp^b in Eef van Beveren^c

^a Institute of Modern Physics, CAS, Lanzhou 730000, China

^b CeFEMA, Instituto Superior Técnico, Universidade de Lisboa, 1049-001 Lisboa, Portugal

^c Centro de Física Computacional, Departamento de Física, Universidade de Coimbra, 3004-516 Coimbra, Portugal

Raziskujemo vektorsko resonanco $\rho(770)$, njene radialne ponovitve ter ustrezne fazne premike $\pi\pi$ v P-valu z razširjenim kvarkovim modelom, ki vsebuje tudi pare mezonov in pare $q\bar{q}$. Pri tem upoštevamo vse važne razpadne kanale: psevdoskalar-psevdoskalar, vektor-psevdoskalar, vektor-vektor, vektor-skalar, aksialni vektor-psevdoskalar in aksialni vektor-vektor, skupaj 26 kanalov. Dva modelska parametra sta določena s prejšnjimi vrednostmi, trije pa iz resonance ρ in nizkoenergijskih faznih premikov $\pi\pi$ v P-valu. Začasni rezultati že nakazujejo sposobnost modela, da reproducira te fazne premike ter maso in širino mezona ρ . Vendar rastejo fazni premiki pri višjih energijah prestrmo. Možno zdravilo je vključitev resonanc v končnih stanjih v večini kanalov. Raziskava se nadaljuje.

Prispevek pionskega oblaka k elektromagnetni strukturi nukleona

D. Kupelwieser and W. Schweiger

Institute of Physics, University of Graz, A-8010 Graz, Austria

V tem prispevku nadaljujemo in razširimo prejšnje račune elektro-šibkih oblikovnih faktorjev hadronov z uporabo točkovne oblike relativistične kvantne mehanike. Posebej se zanimamo za pionske učinke na elektromagnetno zgradbo nukleona. V ta namen uporabimo hibridni model s konstituentnimi kvarki, ki vsebuje poleg trojice valenčnih kvarkov še komponento $3q+\pi$. S preprosto valovno funkcijo za konfiguracijo $3q$ dobimo smiselne rezultate za oblikovne faktorje nukleona. V skladu z drugimim avtorji ugotovimo, da je učinek pionov znaten le pod $Q^2 \leq 0.5 \text{ GeV}^2$.