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Pion electro-production in a dynamical model including quasi-bound three-quark states*

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Abstract. We present a method to calculate the pion electro-production amplitude in a framework of a coupled channel formalism incorporating quasi-bound quark-model states.

1 Introduction

In our previous work ([1] and [2]) we have developed a general method to incorporate excited baryons represented as quasi-bound quark-model states into a coupled channel calculation using the K matrix. The method has been applied to calculate pion scattering amplitudes in the energy region of low-lying P11 and P33 resonances. In addition to the elastic channel we have included the $\pi\Delta$ and σN ($\sigma\Delta$) channels where the σ-meson models the correlated two-pion decay. We have been able to explain a rather intriguing behaviour of the scattering amplitudes in these two partial waves in the range of invariant energies from the threshold up to $W \sim 1700$ MeV. In this work we show how the formalism can be extended to the calculation of electro-production amplitudes.

2 Incorporating quark-model states into multi-channel formalism

We consider a class of chiral quark models in which mesons (the pion and the sigma meson in our case) couple linearly to the quark core:

$$H_{\text{meson}} = \int dk \sum_{lmt} \left\{ \omega_k \, a^\dagger_{lmt}(k) a_{lmt}(k) + \left[V_{lmt}(k) a_{lmt}(k) + V^\dagger_{lmt}(k) \, a^\dagger_{lmt}(k) \right] \right\} ,$$

where $a^\dagger_{lmt}(k)$ is the creation operator for a meson with angular momentum l and the third components of spin m and isospin t. In the case of the pion, we include only $l = 1$ pions, and $V_{mt}(k) = -v(k) \sum_{i=1}^{3} \sigma^i_m \tau^i_t$ is the general form of the pion source, with the quark operator, $v(k)$, depending on the model. It includes also

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the possibility that the quarks change their radial function which is specified by the reduced matrix elements $V_{BB'}(k) = \langle B\|V(k)\|B'\rangle$, where B are the bare baryon states (e.g. the bare nucleon, Δ, Roper, ...)

We have shown that in such models it is possible to find an exact expression for the K matrix without explicitly specifying the form of the asymptotic states. In the basis with good total angular momentum J and isospin T, the elements of the K matrix take the form:

$$K^{JT}_{H'H} = -\pi\mathcal{N}_H\langle\Psi^H_{JT}\|V(k)\|\widetilde{\Psi}_{B'}\rangle\,, \qquad \mathcal{N}_H = \sqrt{\frac{\omega E_{B'}}{kW}}\,, \tag{1}$$

where ω and k are the energy and momentum of the meson. Here Ψ^H_{JT} is the principal value state corresponding to channel H specified by the meson (π, σ, ...) and the baryon B (N, Δ, ...):

$$|\Psi^H_{JT}\rangle = \mathcal{N}_H\left\{\sum_{\mathcal{R}} c^H_{\mathcal{R}}|\Phi_{\mathcal{R}}\rangle + [a^\dagger(k)|\widetilde{\Psi}_B\rangle]^{JT} + \sum_{H'}\int\frac{dk\,\chi^{H'H}_{JT}(k)}{\omega_k + E(k) - W}\,[a^\dagger(k)|\widetilde{\Psi}_{B'}\rangle]^{JT}\right\}. \tag{2}$$

The first term is the sum over *bare* tree-quark states $\Phi_{\mathcal{R}}$ involving different excitations of the quark core, the next term corresponds to the free meson and the baryon (N or Δ) and defines the channel, the third term introduces meson clouds around different isobars. The sum in the latter term includes also inelastic channels in which case the integration over the mass of unstable intermediate hadrons (σ-meson, Δ-isobar, ...) is implied. The state $\widetilde{\Psi}_{B'}$ in Eqs (1) and (2) represents either the nucleon or the intermediate Δ with invariant mass M; in the latter case it is equal to (2) with $H = (\pi, N)$ and normalized as $\langle\widetilde{\Psi}_\Delta(M')|\widetilde{\Psi}_\Delta(M)\rangle = \delta(M-M')$, $E(k)$ is the energy of the recoiled baryon (nucleon or Δ). The on-shell meson amplitudes $\chi^{H'H}_{JT}$ are proportional to the corresponding matrix elements of the on-shell K matrix

$$K_{H'H} = \pi\mathcal{N}_{H'}\mathcal{N}_H\,\chi^{H'H}_{JT}(k_{H'})\,. \tag{3}$$

From the variational principle for the K matrix it is possible to derive the integral equation for the amplitudes which is equivalent to the Lippmann-Schwinger equation for the K matrix. The resulting expression for $\chi^{H'H}_{JT}$ can be written in the form

$$\chi^{H'H}_{JT}(k) = -\sum_{\mathcal{R}} c^H_{\mathcal{R}}\mathcal{V}_{H'\mathcal{R}}(k) + \mathcal{D}^{H'H}_{JT}(k) \tag{4}$$

where $\mathcal{V}_{H\mathcal{R}}$ are the dressed matrix elements of the interaction V_{lmt} between the resonant state and the baryon state in channel H, and $\mathcal{D}^{H'H}_{JT}$ is the background contribution.

3 T and K matrices for πN electro-production

We start with the definition of the T matrix for the pion electro-production on the nucleon:

$$T_{\pi N\gamma N} = -\pi\langle\Psi^{(+)}(m_s, m_t; \mathbf{k}_0, t)|H_\gamma|\Psi_N(m'_s, m'_t; \mathbf{k}_\gamma, \mu)\rangle\,, \tag{5}$$

where m_s and m_t are the third components of baryon spin and isospin, $\mathbf{k}_0$ and t are the outgoing pion momentum and the third component of isospin, and $\mathbf{k}_\gamma$ and μ the momentum (along the coordinate z-axis) and the polarization of the incident photon. The interaction Hamiltonian is taken in the form

$$H_\gamma = \int d\mathbf{k}_\gamma \sum_\mu \left[V_\gamma(\mu, \mathbf{k}_\gamma) a_\mu(\mathbf{k}_\gamma) + \text{h.c.}\right] ,$$

$$V_\gamma(\mu, \mathbf{k}_\gamma) = \frac{1}{\sqrt{2\pi}^3} \tilde{V}_\gamma(\mu, \mathbf{k}_\gamma) , \qquad \tilde{V}_\gamma(\mu, \mathbf{k}_\gamma) = \frac{e_0}{\sqrt{2\omega_\gamma}} \int d\mathbf{r}\, \varepsilon_\mu \cdot \mathbf{j}(\mathbf{r}) e^{i\mathbf{k}_\gamma \cdot \mathbf{r}} . \tag{6}$$

The state representing the photon-nucleon system reads

$$|\Psi_N(m_s', m_t'; \mathbf{k}_\gamma, \mu)\rangle = \mathcal{N}_\gamma a_\gamma^\dagger(\mathbf{k}_\gamma) |\Psi_N(m_s' m_t')\rangle , \qquad \mathcal{N}_\gamma = \sqrt{k_\gamma \omega_\gamma} \sqrt{\frac{E_N^\gamma}{W}} . \tag{7}$$

Here $\omega_\gamma = (W^2 - M_N^2 - Q^2)/2W$, $k_\gamma^2 = \omega_\gamma^2 + Q^2$, $E_N^\gamma = W - \omega_\gamma$, with Q^2 measuring the photon virtuality. We perform the spin-isospin decomposition of the outgoing state

$$|\Psi^{(+)}(m_s, m_t; \mathbf{k}_0, t)\rangle = \sum_{lmJT} i^l Y_{lm}^*(\hat{k}_0) |\Psi_{JT}^{(+)}(M_J, M_T; k_0, l, m, t)\rangle C^{JM_J}_{\frac{1}{2}m_s lm} C^{TM_T}_{\frac{1}{2}m_t 1t} . \tag{8}$$

Commuting $a_\gamma^\dagger$ in (5) to the left and using the expansion (8), we can write the T matrix in the JT basis as

$$T^{JT}_{\pi N \gamma N} = -\pi \mathcal{N}_\gamma \langle \Psi_{JT}^{(+)}(M_J M_T; k_0, l) | V_\gamma(\mu, \mathbf{k}_\gamma) | \Psi_N(m_s' m_t') \rangle . \tag{9}$$

The electro-production amplitude is proportional to (9) through $T = \sqrt{k_0 k_\gamma / 8\pi}\, \mathcal{M}$, hence

$$\mathcal{M}^{JT}_{\pi N} = -\frac{\mathcal{N}_\gamma}{\sqrt{k_0 k_\gamma}} \langle \Psi_{JT}^{(+)}(M_J M_T; k_0, l) | \tilde{V}_\gamma(\mu, \mathbf{k}_\gamma) | \Psi_N(m_s' m_t') \rangle . \tag{10}$$

The amplitudes proportional to the elements of the K matrix are obtained by replacing the state $\Psi_{JT}^{(+)}$ by the corresponding principal value state:

$$\mathcal{M}^{K\,JT}_{H} = -\frac{\mathcal{N}_\gamma}{\sqrt{k_0 k_\gamma}} \langle \Psi_{JT}^{H}(M_J M_T; k_0, l) | \tilde{V}_\gamma(\mu, \mathbf{k}_\gamma) | \Psi_N(m_s' m_t') \rangle . \tag{11}$$

The procedure to calculate the electro-production amplitudes in our formalism is the following: we first evaluate (11) using (2) as obtained in pion scattering, and then compute (10) using $\mathcal{M} = \mathcal{M}^K + iT\mathcal{M}^K$. (This equation trivially follows from the Heitler's equation $T = K + iTK$ since the proportionality factor between T and $\mathcal{M}$ is the same as between K and $\mathcal{M}^K$.) In principle, this equation involves also the matrix elements corresponding to Compton scattering. They can be neglected since they are orders of magnitude smaller than those containing strong interaction. In the P11 case we have

$$\mathcal{M}_{\pi N}(W) = \mathcal{M}^K_{\pi N}(W) + i\Big[T_{\pi N \pi N}(W) \mathcal{M}^K_{\pi N}(W) + \overline{T}_{\pi N \pi \Delta}(W) \overline{\mathcal{M}}^K_{\pi \Delta}(W)$$
$$+ \overline{T}_{\pi N \sigma N}(W) \overline{\mathcal{M}}^K_{\sigma N}(W) \Big] . \tag{12}$$

We have further simplified the equation by using averaged values for amplitudes involving the $\pi\Delta$ and the σN channels and thus avoiding integration over the corresponding invariant masses. In the P33 case we have also added the $\pi N(1440)$ channel, while the σN channel has been replaced by the $\sigma\Delta$ channel.

4 The behaviour of the amplitudes close to a resonance

From (3) and (4) it follows that close to a resonance, denoted by $\mathcal{R}$, the K matrix element between the elastic channel and the πB channel can be cast in the form

$$K_{\pi B\pi N} = -\pi\sqrt{\frac{\omega_0\omega_B E_N E_B}{k_0 k_B W^2}}\, c_{\mathcal{R}}^{B}\mathcal{V}_{N\mathcal{R}}(k_0) + K_{\pi B\pi N}^{\text{background}}\,.$$

After some rearrangements, the principal value states (2) take the form

$$|\Psi^H\rangle = -K_{\pi B\pi N}\sqrt{\frac{k_0 W}{\pi^2\omega_0 E_N}}\frac{\sqrt{\mathcal{Z}_{\mathcal{R}}}}{\mathcal{V}_{N\mathcal{R}}}|\widehat{\Psi}^{\text{res}}\rangle + |\Psi^{H\,\text{non-res}}\rangle$$

with

$$|\widehat{\Psi}_{\mathcal{R}}^{\text{res}}\rangle = \frac{1}{\sqrt{\mathcal{Z}_{\mathcal{R}}}}\left\{|\Phi_{\mathcal{R}}\rangle - \int dk\,\frac{\mathcal{V}_{N\mathcal{R}}(k)[a^\dagger(k)|\Psi_N\rangle]^{JT}}{\omega_k + E_N(k) - M} - \sum_B \int dk\,\frac{\mathcal{V}_{B\mathcal{R}}(k)[a^\dagger(k)|\widehat{\Psi}_B\rangle]^{JT}}{\omega_k + E_B(k) - M}\right\}.$$

(the inclusion of the σN channel in the P11 case is straightforward). We can now split the K-matrix type amplitudes (11) into the resonant part containing the pole and the "non-resonant" part:

$$\mathcal{M}_H^K = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\sqrt{\frac{k_0 W}{\pi^2\omega_0 E_N}}\frac{\sqrt{\mathcal{Z}_{\mathcal{R}}}}{\mathcal{V}_{N\mathcal{R}}}K_{NH}\langle\widehat{\Psi}_{\mathcal{R}}^{(\text{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle + \mathcal{M}_H^{K\,(\text{non})}\,. \qquad (13)$$

We see that the resonant matrix elements depend on a particular channel (H) only through the K matrix element referring to that channel. Next we plug (13) into (12) and take into account the relation between the T and the K matrix ($T = K + iTK$). The resonant part of the electro-production amplitudes then reads

$$\mathcal{M}_N^{(\text{res})} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{k_0 W}}\sqrt{\frac{k_0 W}{\pi^2\omega_0 E_N}}\frac{\sqrt{\mathcal{Z}_{\mathcal{R}}}}{\mathcal{V}_{N\mathcal{R}}}\langle\widehat{\Psi}_{\mathcal{R}}^{(\text{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle\, T_{\pi N\pi N}\,, \qquad (14)$$

while the non-resonant part satisfies

$$\mathcal{M}_N^{(\text{non})} = \mathcal{M}_N^{K\,(\text{non})} + i\left[T_{\pi N\pi N}\mathcal{M}_N^{K\,(\text{non})} + \overline{T}_{\pi N\pi\Delta}\overline{\mathcal{M}}_\Delta^{K\,(\text{non})} + \overline{T}_{\pi N\sigma N}\overline{\mathcal{M}}_\sigma^{K\,(\text{non})}\right].$$

Let us note that $\langle\widehat{\Psi}_{\mathcal{R}}^{(\text{res})}(W)|\tilde{V}_\gamma|\Psi_N\rangle$ is the electro-excitation amplitude for the resonance $\mathcal{R}$. For a sufficiently weak meson field the state $\widehat{\Psi}$ is dominated by the bare-three quark core surrounded by a cloud of pions, which is a familiar form of a baryon state in chiral-quark models. The relation (14) can be rewritten in a more familiar form by noting that the elastic part of the K matrix can be written as

$$K_{\pi N\pi N} = \pi \frac{\omega_0 E_N}{k_0 W} \frac{\mathcal{V}_{N\mathcal{R}}^2}{\mathcal{Z}_{\mathcal{R}}(M_{\mathcal{R}})} = \frac{\frac{1}{2}\Gamma_{el}}{M_{\mathcal{R}} - W}\,, \tag{15}$$

where Γ_{el} is the elastic width of the resonance. Expressing $\mathcal{V}_{N\mathcal{R}}$ from (15) we get

$$\mathcal{M}_N^{(res)} = i\sqrt{\frac{\omega_\gamma E_N^\gamma \Gamma_{el}}{2\pi k_0 W \Gamma_{tot}^2}}\,\langle \widehat{\Psi}_{\mathcal{R}}^{(res)}(W)|\tilde{V}_\gamma|\Psi_N\rangle\,, \tag{16}$$

where we have taken into account that at the resonance $T_{\pi N\pi N} = i\Gamma_{el}/\Gamma_{tot}$.

5 Multipole decomposition for the P11 and P33 wave

Expanding (6) into multipoles, we have in the P33 case:

$$M_{1+}^{(3/2)} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{6k_0 W}}\langle \Psi_{JT}^{(+)}||\tilde{V}_\gamma^{M1}||\Psi_N\rangle\,, \qquad E_{1+}^{(3/2)} = -\sqrt{\frac{\omega_\gamma E_N^\gamma}{30k_0 W}}\langle \Psi_{JT}^{(+)}||\tilde{V}_\gamma^{E2}||\Psi_N\rangle\,, \tag{17}$$

and in the P11 ($J = T = \frac{1}{2}$) case

$$M_{1-}^{(1/2)} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{6k_0 W}}\langle \Psi_{JT}^{(+)}||\tilde{V}_{\gamma\,(IV)}^{M1}||\Psi_N\rangle\,, \quad M_{1-}^{(0)} = \sqrt{\frac{\omega_\gamma E_N^\gamma}{18k_0 W}}\langle \Psi_{JT}^{(+)}||\tilde{V}_{\gamma\,(IS)}^{M1}||\Psi_N\rangle\,,$$

related to π^0 production amplitude on the proton as $M_{1-}^p = M_{1-}^{(0)} + \frac{1}{3} M_{1-}^{(1/2)}$, and on the neutron as $M_{1-}^n = M_{1-}^{(0)} - \frac{1}{3} M_{1-}^{(1/2)}$. Here IV and IS denote the isovector and the isoscalar part of the interaction, respectively. The same formulas apply to the $\mathcal{M}^K$ amplitudes. (Similar relations can be derived for the scalar amplitudes.)

The transverse electro-excitation amplitudes are defined in terms of the helicity amplitudes A_{M_J}. In the P33 case we separate them into the magnetic dipole and the electric quadrupole part:

$$M1 = -\tfrac{1}{2}\left[\sqrt{3}\,A_{\frac{3}{2}} + A_{\frac{1}{2}}\right] = -\frac{\sqrt{8}}{3}\,\langle \widehat{\Psi}_{\mathcal{R}}^{(res)}||\tilde{V}_\gamma(M1)||\Psi_N\rangle\,, \tag{18}$$

$$E2 = \frac{1}{2\sqrt{3}}\,\left[A_{\frac{3}{2}} - \sqrt{3}\,A_{\frac{1}{2}}\right] = \sqrt{\frac{8}{45}}\,\langle \widehat{\Psi}_{\mathcal{R}}^{(res)}||\tilde{V}_\gamma(E2)||\Psi_N\rangle\,. \tag{19}$$

Taking into account (17) and (16) we reproduce the familiar relation

$$M_{1+}^{(3/2)} = if\,M1\,, \qquad E_{1+}^{(3/2)} = if\,E2\,, \qquad f = \sqrt{\frac{3\omega_\gamma E_N^\gamma \Gamma_{el}}{8\pi\, k_0 W \Gamma_{tot}^2}}\,.$$

In the P11 case only one transverse helicity amplitude appears and we find

$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{2}{3}}\left[\langle \widehat{\Psi}_{\mathcal{R}}^{(res)}||\tilde{V}^{M1}(IS)||\Psi_N\rangle \pm \frac{1}{\sqrt{3}}\,\langle \widehat{\Psi}_{\mathcal{R}}^{(res)}||\tilde{V}^{M1}(IV)||\Psi_N\rangle\right]$$

(the reduced matrix elements appear only in the angular momentum, the third component of the isospin are $M_T = m_t' = \frac{1}{2}$).

6 Preliminary results in the N(1440) sector

The P33 wave amplitudes in the region of the $\Delta(1232)$ have been extensively investigated in our previous works (see e.g. [3] and [4]). Since the electro-production amplitudes are dominated by the resonant contribution, they follow the shape of the elastic T matrix accordingly to (14).

This is not the case in the P11 wave. In Fig. 1 we show some preliminary results (without including the $\pi\Delta$ and the σN channels) for the electro-production amplitude in the region of the N(1440) resonance showing the important role of the background processes. These are dominated by the nucleon pole contribution, the contribution from the second term in (2) (t-channel), and by a u-channel-type process with the Δ in the intermediate state. Below the resonance, the contribution of the resonant term is almost negligible. The resonant contribution itself is dominated by the pion cloud and the admixture of the nucleon component which considerably reduces the contribution. This point is still under investigation; we expect that inclusion of higher resonances may cure this deficiency.

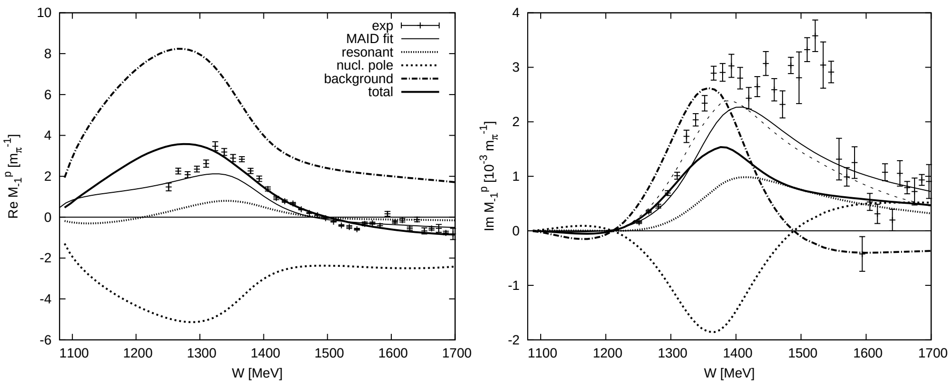

Fig.1. The real (left panel) and the imaginary (right panel) parts of the electro-production amplitudes M^{p}_{1-} for the P11 partial waves. The MAID result is taken from [5]; the experimental points from [6]. The thin dashed curve in the right panel shows the effect of omitting the nucleon component in the resonant state.

References

1. B. Golli and S. Širca, to be published in Eur. Phys. J. A, arXiv:0708.3759 [hep-ph]
2. B. Golli and S. Širca in: B. Golli, M. Rosina, S. Širca (eds.), *Proceedings of the Mini-Workshop "Hadron Structure and Lattice QCD"*, July 9–16, 2007, Bled, Slovenia, p. 61.
3. M. Fiolhais, B. Golli, S. Širca, Phys. Lett. B **373**, 229 (1996)
4. P. Alberto, L. Amoreira, M. Fiolhais, B. Golli, and S. Širca, Eur. Phys. J. A **26** (2005) 99.
5. D. Drechsel, S. S. Kamalov, L. Tiator, Eur. Phys. J. A **34** (2007) 69.
6. R. Arndt et al., Phys. Rev. C **52** (1995) 2120; R. Arndt et al., Phys. Rev. C **69** (2004) 035213.