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6 Coupling Electromagnetism to Global Charge

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Abstract. It is shown that an alternative to the standard scalar QED is possible. In this new version there is only global gauge invariance as far as the charged scalar fields are concerned although local gauge invariance is kept for the electromagnetic field. The electromagnetic coupling has the form $j_\mu(A^\mu + \partial^\mu B)$ where B is an auxiliary field and the current j_μ is A_μ independent so that no "sea gull terms" are introduced. In a model of this kind spontaneous breaking of symmetry does not lead to photon mass generation, instead the Goldstone boson becomes a massless source for the electromagnetic field. Infrared questions concerning the theory when spontaneous symmetry breaking takes place and generalizations to global vector QED are discussed. In this framework Q-Balls and other non topological solitons that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC. Finally, we give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

Povzetek. Pokažem, da obstaja alternativa standardni skalarni teroriji kvantne elektrodinamike. V tej novi različici velja za nabita skalarna polja samo globalna umeritvena invarianca, lokalno umeritveno invarianco pa zahtevamo za eletromagnetno polje. Elektromagnetna sklopitev ima obliko $j_\mu(A^\mu + \partial^\mu B)$, kjer je B pomožno polje, tok j_μ je neodvisen od A_μ, zaradi česar so členi tipa "sea gull" v teoriji motenj enaki nič. Pri spontani zlomitvi simetrije ostane foton brez mase, Goldstoneov bozon pa postane brezmasni izvor elektromagnetnega polja. V prispevku obravnavam probleme, ki jih ima ob spontani zlomitvi simetrije ta teorija v infrardečem območju ter njeno posplošitev do globalne vektorske teorije kvantne elektrodinamike. V tem okviru se lahko krogle "Q" in ostali netopološi solitoni, ki dolgujejo svoj obstoj globalni simetriji $U(1)$, sklopijo z elektromagnetnim poljem in bi lahko predstavljali večkratno nabite delce, ki jih trenutno iščejo na LHC. Na koncu podamo primer, kako lahko "porajajočo" globalno skalarno teorija kvantne elektrodinamike izpeljemo iz sistema aksion-foton v zunanjem magnetnem polju.

6.1 Introduction

In this paper it will be shown that an alternative to the standard scalar QED is possible. In this new version there is only global gauge invariance as far as the charged scalar fields are concerned although local gauge invariance is kept for the electromagnetic field, we call this new model Global scalar QED. The electromagnetic coupling has the form $j_\mu(A^\mu + \partial^\mu B)$ where B is an auxiliary field and the current j_μ is A_μ independent so that no "sea gull terms" are introduced. In

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a model of this kind spontaneous breaking of symmetry does not lead to photon mass generation, instead the Goldstone boson becomes a massless source for the electromagnetic field, Infrared questions concerning the theory when spontaneous symmetry breaking takes place and generalizations to global vector QED are discussed.

In this framework Q-Balls [1] and other non topological solitons [2] that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC [3].

Finally, we give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

6.2 Conventional scalar QED and its sea gulls

In conventional scalar QED, we "minimally couple" a globally invariant action (under global phase transformations). To be concrete, for a complex scalar field ψ with mass, m whose Lagrangian density can be represented in relativistic invariant form in the absence of interactions to electromagnetism as

$$\mathcal{L} = \hbar^2 g^{\mu\nu} \frac{\partial\psi^*}{\partial x^\mu} \frac{\partial\psi}{\partial x^\nu} - m^2 c^2 \psi^* \psi \tag{6.1}$$

Then, in the standard scalar QED model we introduce the electromagnetic interaction with scalar charged particles by introducing the minimal coupling in the Lagrangian for charged particles (see Eq. 6.1). As we recall, minimal coupling requires that we let the momentum p_μ be replaced by $p_\mu \to p_\mu - eA_\mu$ where $p_\mu = -i\ \hbar \frac{\partial}{\partial x^\mu}$ and where A_μ is the electromagnetic 4-vector whose Lagrangian is given by

$$\mathcal{L}_{\mathcal{EM}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{6.2}$$

with $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. We can now write the total Lagrangian after using the minimal coupling substitution into Eq. 6.1

$$\mathcal{L}_{\mathcal{T}} = g^{\mu\nu} \left[(\hbar \frac{\partial}{\partial x^\mu} - ieA_\mu)\psi^*][(\hbar \frac{\partial}{\partial x^\nu} + ieA_\nu)\psi \right] - m^2 c^2 \psi^* \psi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{6.3}$$

This leads to the equation of motion for the scalar field ψ

$$(i\ \hbar \frac{\partial}{\partial t} - e\phi)^2 \psi = (\frac{c\ \hbar}{i} \nabla - e\mathbf{A})^2 \psi + m^2 c^4 \psi \tag{6.4}$$

This equation and the lagrangian density from which it is derived are invariant under local gauge transformations:

$$\mathbf{A} \to \mathbf{A}' = \mathbf{A} + \nabla\chi; \quad \phi \to \phi' = \phi - \frac{1}{c} \frac{\partial\chi}{\partial t} \quad \text{with} \quad \psi \to \exp[\frac{ie\chi}{\hbar c}]\psi \tag{6.5}$$

Furthermore the electromagnetic field satisfies the Maxwell's equations where the electric charge density ρ and the current density $\mathbf{j}(x)$ are given by (now set

$c = \hbar = 1$).

$$\rho(x) = i(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) - 2e\phi\psi^*\psi \text{ and } \mathbf{j}(x) = -i(\psi^* \nabla \psi - \psi \nabla \psi^*) - 2e\mathbf{A}\psi^*\psi \tag{6.6}$$

There is an example, the BCS theory of superconductivity [4], where the effective theory in terms of the composite Cooper pairs retains the local gauge invariance which involves the local phase transformations of the composite scalar, however we may ask if this is a general rule, may be not.

When thinking of the electromagnetic interactions of pions, the quadratic dependence of the interactions on the potentials characterises the sea gull behaviour of standard scalar QED. As pointed out by Feynman [5], it is somewhat puzzling that spinor electrodynamics does not lead to any of such sea gulls . Considering that the microscopic description of charged pions is really the spinor electrodynamics of quarks, shouldn't we search for an effective scalar electrodynamics devoid of sea gulls?, is this possible?. In the next section we will see that this can be achieved in global scalar QED. The Global Scalar QED could address other questions as well. like the electromagnetic coupling of Q-balls and can "emerge" as an effective description of a system of axions and photons in an external field.

6.3 Global Scalar QED

There are many possible motivations for departing from the scheme implied by the minimal coupling, which leads to scalar QED. For example, if the complex scalar field is to describe a pion, since the macroscopic hadron is a very non local construction in terms of the fundamental quark fields and gluon fields as has been revealed from both the theoretical point of view [6] and from the experimental point of view [7] and in fact we may have several alternative candidates for the pion wave function (and any such proposal could give rise to a different effective theory), we do not necessarily have to keep a local gauge invariance in terms of the composite scalar fields (that would describe the hadrons), although global phase invariance must be respected. Also local gauge transformations for the photon should be mantained. Other possible use of deviating from the the minimal coupling scheme, as we will see, could be to couple Q-Ball type solitons to electromagnetism. Finally, we will give an example where an "Emergent" Global Scalar QED can appear from an axion photon system in an external magnetic field.

We work therefore with the following lagrangian density

$$\mathcal{L} = g^{\mu\nu} \frac{\partial \psi^*}{\partial x^\mu} \frac{\partial \psi}{\partial x^\nu} - U(\psi^*\psi) - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu (A^\mu + \partial^\mu B) \tag{6.7}$$

where

$$j_\mu = ie(\psi^* \frac{\partial \psi}{\partial x^\mu} - \psi \frac{\partial \psi^*}{\partial x^\mu}) \tag{6.8}$$

and where we have also allowed an arbitrary potential $U(\psi^*\psi)$ to allow for the possibility of spontaneous breaking of symmetry. The model is separately invariant under local gauge transformations

$$A^{\mu} \to A^{\mu} + \partial^{\mu}\Lambda; \quad B \to B - \Lambda \tag{6.9}$$

and the independent global phase transformations

$$\psi \to exp(i\chi)\psi \tag{6.10}$$

The use of a gauge invariant combination $(A^{\mu} + \partial^{\mu}B)$ can be utilized for the construction of mass terms[8] or both mass terms and couplings to a current defined from the gradient of a scalar in the form $(A^{\mu} + \partial^{\mu}B)\partial_{\mu}A$ [9]. Since the subject of this paper is electromagnetic couplings of photons and there is absolutely no evidence for a photon mass, we will disregard such type of mass terms and concentrate on the implications of the $(A^{\mu} + \partial^{\mu}B)j_{\mu}$ couplings.

6.4 A Double Charge Theory

As we will see the scalar QED model has two charge conservation laws associated with it. We see that Maxwell's equations are satisfied with j_{μ} being the source, that is

$$\partial^{\nu}F_{\nu\mu} = j_{\mu} \tag{6.11}$$

of course this implies $\partial^{\nu}\partial^{\mu}F_{\nu\mu} = \partial^{\mu}j_{\mu} = 0$. The same conclusion can be obtained from the equation of motion obtained from the variation with respect to B.

The Noether current obtained from the independent global phase transformations $\psi \to exp(i\chi)\psi$, χ being a constant, is

$$J_{\mu} = ie(\psi^{*}\frac{\partial\psi}{\partial x^{\mu}} - \psi\frac{\partial\psi^{*}}{\partial x^{\mu}}) + 2e(A_{\mu} + \partial_{\mu}B)\psi^{*}\psi \tag{6.12}$$

Therefore

$$j^{B}_{\mu} = J_{\mu} - j_{\mu} = 2e(A_{\mu} + \partial_{\mu}B)\psi^{*}\psi \tag{6.13}$$

is also conserved, that is $\partial^{\mu}((A_{\mu} + \partial_{\mu}B)\psi^{*}\psi) = 0$

6.5 No Klein Paradox

An interesting difference between standard scalar QED and global scalar QED appears in the case of strong fields Consider the global scalar QED equations with an external electromagnetic field potential step-function: $e(A_0 + \partial_0 B) \equiv V(x)$; $eA_i + e\partial_i B = 0$.

$$V(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ V_0 & \text{for} \quad x > 0 \end{cases}$$

The Global QED equation in the presence of this potential is $(\hbar = 1, c = 1)$

$$-\frac{\partial^{2}\psi}{\partial t^{2}} + \nabla^{2}\psi - m^{2}\psi = 0 \tag{6.14}$$

for $x < 0$.

$$-\frac{\partial^2\psi}{\partial t^2} + 2iV_0\frac{\partial\psi}{\partial t} + \nabla^2\psi - m^2\psi = 0 \tag{6.15}$$

for $x > 0$. To solve the equation with this potential, we try solutions of the form:

$$\begin{aligned} \psi_< \equiv \psi &= e^{-iEt}[e^{ipx} + Re^{-ipx}] \text{ for } x < 0 \\ \psi_> \equiv \psi &= Te^{-iEt}e^{ip'x} \text{ for } x > 0 \end{aligned} \tag{6.16}$$

where $\psi_<$ represents a wave like solution for the Klein-Gordon field for $x < 0$ and $\psi_>$ represent the field for wave like solution for $x > 0$. R is the amplitude of that part of wave that is reflected wave while T is that part that is transmitted. We substitute $\psi_<$ and $\psi_>$, Eq. 6.16 into Eqs. 6.14 and 6.15 respectively. We thus find

$$E^2 - p^2 - m^2 = 0 \to E = +\sqrt{p^2 + m^2} \text{ for } x < 0 \tag{6.17}$$

since for incident wave for $x < 0$ we chose the positive sign in the square root as our boundary condition. and

$$E^2 - 2EV_0 - p'^2 - m^2 = 0 \to p' = \pm\sqrt{E(E - 2V_0) - m^2} \text{ for } x > 0 \tag{6.18}$$

We see here that from a certain positive value of V_0, $V_{0crit} = (E^2 - m^2)/2E$ and higher,p' becomes imaginary and therefore there is no transmited wave for large values of V_0, totally opposite to the behaviour of standard scalar QED, where for large enough barrier a transmitted wave is restored once again, leading to the "Klein paradox", the transmitted wave is interpreted there as pair creation process, no such process appears in global scalar QED.

6.6 Behaviour under Spontaneous breaking of symmetry, new couplings of Goldstone Bosons to Electromagnetism and associated infrared problems

The absence of quadratic terms in the vector potential implies that no mass generation for the photon takes place. Furthermore the Goldstone boson that results from this s.s.b. ,writing $\psi = \rho exp(i\theta)$, where ρ is real and positive, we obtain that the phase of the ψ field, is not eaten, it remains in the theory, in fact it couples derivatively to $(A_\mu + \partial_\mu B)$, like the A field studied in [9] and it produces a gradient type charge. In fact under s.s.b. regarding ρ as a constant, $j^\mu = 2e\rho^2\partial^\mu\theta$ the coupling $(A_\mu + \partial_\mu B)j^\mu$ implies the coupling of $(A_\mu + \partial_\mu B)$ to a gradient current, as discussed in [9].

It should be pointed out that this type of gradient current $j^\mu = 2e\rho^2\partial^\mu\theta$ for $\rho = constant$ generates an infrared problem, since the θ field now represents a massless field, which instead of being eaten becomes a source of electromagnetism. The normal way of solving for the electromagnetic field, using the Green's function method does not work straightforwardly, since the source now in Fourier space has support only in the light-cone and the Green's function has a pole like behaviour at the light-cone as well, so we encounter an undefined product of distributions. This is very similar to the solution of a forced harmonic oscillator when the external force has exactly the same frequency to that of the oscillator, that is the resonant case.

To resolve this problem, we note first that considering $F_{\nu\mu}$ as an antisymmetric tensor field (without at first considering whether this field derives from a four vector potential), then a solution of the equation $\partial^\nu F_{\nu\mu} = j_\mu$ is [1]

$$F_{\nu\mu} = \int_0^1 d\lambda\lambda^2 (x_\nu j_\mu(\lambda x) - x_\mu j_\nu(\lambda x)) \tag{6.19}$$

For a generic current the above $F_{\nu\mu}$ does not derive from a potential, however if the current is the gradient of a scalar field, the above $F_{\nu\mu}$ derives from a potential and provides a solution of the problem, where the Green's function method fails. Notice that the similarity with the the resonant case of the forced harmonic oscillator is very close, there the solution is of the form of an oscillating function times time and in the above solution we see the similar x_ν dependence appearing.

The resulting gauge potentials displays also a linear dependence on x_ν, which is interesting, since the central issue in the confinement problem for example is how to obtain potentials with linear dependence on the coordinates, although it is not clear how the very specific solution studied here is relevant to the confinement problem.

Axions are an example of Goldstone bosons with non trivial electromagnetic interactions

6.7 Global Vector QED

In this case we consider a complex vector field W_μ and consider the action

$$\mathcal{L} = -\frac{1}{4} g^{\mu\nu} g^{\alpha\beta} G_{\mu\alpha} G^*_{\nu\beta} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} + j_\mu (A^\mu + \partial^\mu B) + M^2 W_\mu W^{*\mu} \tag{6.20}$$

with $G^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$ and where

$$j_\mu = ie(W^{*\alpha} G_{\alpha\mu} - W^\alpha G^*_{\alpha\mu}) \tag{6.21}$$

This model displays global phase invariance for the complex vector field W_μ and local gauge invariance for the photon and B fields (6.7), as was the case of global scalar QED. Once again, no sea gull terms are present here.

6.8 Q Balls and other global $U(1)$ solitons as electromagnetically charged Particles

An interesting situation could present itself when considering solitons as in the case of Q-Balls [1] or other non topological solitons [2], that depend on the existence of a $U(1)$ symmetry.

[1] I want to thank R. Tabensky for pointing this to me

These solitons have been found using actions like that used in Global scalar QED for the case e=0. The idea is minimizing the energy under the constraint that the charge of the system is given. This leads us to time dependent configurations with time dependence of the form

$$\psi(r,t) = \rho(r)exp(i\omega t) \tag{6.22}$$

We see that if there was a local gauge transformation that involve a local phase transformation of the complex scalar field ψ, then the phase of ψ is a totally unphysical quantity and the above eq. 6.22 becomes totally meaningless. That is not the case in global QED, for which 6.22 is meaningful.

Furthermore, the standard Q-Balls hold in the limit $e \to 0$ and also the small e case can be treated in perturbation theory, The introduction of a non zero e tends to destabilize the soliton as a consequence of the Coulomb repulsion that appears from the Q-ball having an electric charge. This effect is small for the case of small e, so we know there must be a range of parameters for which electrically coupled Q-Ball solitons exist.

6.9 "Emergent" scalar QED from a system of photons and axions in an external magnetic field

In this section we will consider how an "Emergent" scalar QED from a system of photons and axions in an external magnetic field. Such analysis was considered in [10] and in [11], where a "scalar QED analogy" was recognized. As we will discuss here, although the system of photons and axions in an external magnetic field does indeed have features that resemble scalar QED, the more close correspondance is with Global Scalar QED.

The action principle describing the relevant light pseudoscalar coupling to the photon is

$$S = \int d^4x \left[-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{8}\phi\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta} \right]. \tag{6.23}$$

We now specialize to the case where we consider an electromagnetic field with propagation along the y and z directions and where a strong magnetic field pointing in the x-direction is present. This field may have an arbitrary space dependence in y and z, but it is assumed to be time independent.

For the small perturbations, we consider only small quadratic terms in the action for the axion and the electromagnetic fields, considering a static magnetic field pointing in the x direction having an arbitrary y and z dependence and specializing to y and z dependent electromagnetic field perturbations and axion fields. This means that the interaction between the background field , the axion and photon fields reduces to

$$S_I = -\int d^4x \left[\beta\phi E_x\right], \tag{6.24}$$

where $\beta = gB(y,z)$. Choosing the temporal gauge for the photon excitations and considering only the x-polarization for the electromagnetic waves (since only this polarization couples to the axion) we get the following 2+1 effective dimensional action (A being the x-polarization of the photon, so that $E_x = -\partial_t A$)

$$S_2 = \int dydzdt \left[\frac{1}{2}\partial_\mu A \partial^\mu A + \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 + \beta\phi\partial_t A\right]. \tag{6.25}$$

Since we consider only $A = A(t,y,z)$, $\phi = \phi(t,y,z)$, we have avoided the integration over x. For the same reason μ runs over t, y and z only . This leads to the equations

$$\partial_\mu \partial^\mu \phi + m^2\phi = \beta\partial_t A \tag{6.26}$$

and

$$\partial_\mu \partial^\mu A = -\beta\partial_t \phi. \tag{6.27}$$

As is well known, when choosing the temporal gauge the action principle cannot reproduce the Gauss constraint (here with a charge density obtained from the axion photon coupling) and has to be imposed as a complementary condition. However this constraint is automatically satisfied here just because of the type of dynamical reduction employed and does not need to be considered anymore.

Without assuming any particular y and z-dependence for β, but still insisting that it will be static, we see that in the case $m = 0$, we discover a continuous axion photon duality symmetry (these results were discussed previously in the 1+1 dimensional case, where only z dependence was considered in [10] and generalized for the case of two spatial dimensions in [11]), since

1. The kinetic terms of the photon and axion allow for a rotational $O(2)$ symmetry in the axion-photon field space.
2. The interaction term, after dropping a total time derivative, can also be expressed in an $O(2)$ symmetric way as follows:

$$S_I = \frac{1}{2}\int dydzdt\beta \left[\phi\partial_t A - A\partial_t \phi\right]. \tag{6.28}$$

It is easy to see that after introducing an appropriate complex field ϕ, this coupling is exactly of the global scalar QED form. The $U(1)$ axion photon symmetry is (in the infinitesimal limit)

$$\delta A = \epsilon\phi, \delta\phi = -\epsilon A, \tag{6.29}$$

where ϵ is a small number. Using Noether‘s theorem, this leads to the conserved current j_μ, with components given by

$$j_0^N = A\partial_t \phi - \phi\partial_t A - \frac{\beta}{2}(A^2 + \phi^2) \tag{6.30}$$

and

$$j_i^N = A\partial_i\phi - \phi\partial_i A. \tag{6.31}$$

Here $i = y, z$ coordinates. In order to have the exact correspondence with Global scalar QED, we must define the complex field ψ as

$$\psi = \frac{1}{\sqrt{2}}(\phi + iA), \tag{6.32}$$

we see that in terms of this complex field, the Noether charge density takes the form

$$j_0^N = i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) - \beta\psi^*\psi. \tag{6.33}$$

which, as in Global scalar QED does not coincide with the current that enters in the interaction lagrangian, which is

$$j_0 = i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) \tag{6.34}$$

We observe that the correspondance with standard scalar QED is approximate, only to first order in β, since (6.28) which represents the interaction of the magnetic field couples with the "axion photon density" 6.34, that does not contain β dependence.

This interaction has exactly the same form as that of the global scalar QED with an external "electric " field. In fact the magnetic field (or more precisely $\beta/2$) appears to play the role of external electric potential of Global scalar QED $e(A_0 + \partial_0 B) \equiv V(x)$ that couples to the axion photon density,6.34 which plays the role of an electric charge density, exactly as in Global Scalar QED.

From the point of view of the axion-photon conversion experiments, the symmetry (6.29) and its finite form, which is just a rotation in the axion-photon space, implies a corresponding symmetry of the axion-photon conversion amplitudes, for the limit $\omega >> m$.

In terms of the complex field, the Noether current takes the form

$$j_k^N = i(\psi^*\partial_k\psi - \psi\partial_k\psi^*). \tag{6.35}$$

6.10 Discussion and Conclusions

Discussing the new global QED makes sense from both the purely theoretical point of view, since it provides a new type of viewing interactions of charged scalar particles with electromagnetism, as well as from a phenomenological point of view, since standard scalar QED contains the sea gull contributions for which apparently do not represent any known physical process in the electrodynamics of charged pions for example, so it makes sense to build a theory without such sea gulls.

In this framework Q-Balls [1] and other non topological solitons [2] that owe their existence to a global $U(1)$ symmetry can be coupled to electromagnetism and could represent multiply charged particles now in search in the LHC.

Finally we have shown an example of an "Emergent" global scalar QED from a system of photons and axions in an external magnetic field.

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