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Resonance phase shifts from lattice QCD: The pion-nucleon channel*

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We discuss recently published results for the negative parity nucleon channel [1,2] and the problem of baryonic resonances in lattice calculations.

The Euclidean space-time lattice regularisation is the only controllable non-perturbative regularisation of QCD. It defines the quantum field theory as a limit from a finite to an infinite number of points and from non-zero to vanishing lattice spacing. The last decades have shown an enormous progress towards computing hadronic properties that way, mainly by large scale Monte Carlo simulations.

The ground states can be computed from the asymptotic decay of the correlation function of hadronic operators $\langle \mathcal{O}(t)\mathcal{O}(0)\rangle$ in the corresponding quantum channel. In nature, however, most hadrons are unstable. In the lattice world, due to the finiteness of the spatial volume, correlation functions have a discrete energy spectrum – as opposed to the continuous spectral function in the continuum - and one has to retrieve all information on resonances from the measured energy levels. Also the quark mass may be set to unphysically large values and thus some decay thresholds lie higher than in the physical situation and the respective lowest state appears to be (artificially) stable. Towards the physical limit the decay poses a serious problem in the analysis. Two- (and more-) hadron intermediate states should have an impact on the energy spectrum.

In recent years there have been several studies determining baryonic excitations. One common feature was the absence of hadron-hadron decay state signals (except for possibly the s-wave states). This was especially blatant in the $\rho \to \rho$ studies, which on the lattice should exhibit p-wave $\pi\pi$ intermediate states. These should show up as discrete energy levels of the spectral decomposition of the hadron correlation function, but were not observed. In the meson sector the cure was to include meson-meson interpolators in the set of operators of the correlation matrix. In the baryon studies, the baryon interpolators were exclusively three-quark operators and the statements above applied. In order to clarify the situation we studied the negative parity nucleon channel including s-wave $N\pi$ (4+1 quark) operators [1]. Indeed we find significant differences in the energy spectrum.

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* Talk delivered by C. B. Lang

Our study is for gauge configurations with two mass degenerate dynamical quarks. The used parameters correspond to a lattice spacing of 0.1239 fm on a $16^3 \times 32$ lattice with a pion mass of 266 MeV. For the nucleon interpolator we use the standard 3-quark operators, For the $N\pi$ system in the rest frame the leading s-wave contribution comes from the interpolator with both particles at rest, $N\pi(\mathbf{p} = 0) = \gamma_5 N_+(\mathbf{p} = 0)\pi(\mathbf{p} = 0)$, where N_+ denotes the positive parity nucleon and the factor γ_5 ensures negative parity for the interpolator.

We determine the energy levels of the coupled N and $N\,\pi$ system with help of the so-called variational method [3]. For this approach one determines and diagonalizes the cross-correlation matrix between several operators with the given quantum numbers. In order to compute this correlation matrix we had to first compute the Wick decomposition of the correlators in terms of the quark propagators. For $N \leftrightarrow N$ these are of only two types, while the complete $(N_-, N_+\pi) \leftrightarrow (N_-, N_+\pi)$ system requires the evaluation of 29 graphs. Among all these contraction terms there are some involving backtracking propagators. Hence special tools are necessary in order to obtain statistically reliable signals. We used the so-called distillation method [4].

We find that the energy spectrum changes when allowing for the $N\pi$ coupled channel. We find now a clear signal for the lowest $N(0)\pi(0)$ state, lying closely below threshold, as expected. Indeed, also the quality of the energy levels improves. Lüscher [5] derived a relation between the energy levels at finite volume and the phase shifts of the infinite volume, valid in the elastic region. We apply this relation up to the region of the second resonance. More detailed studies would have to include more operators and deal with the coupled channel problem. In a Breit-Wigner fit to the corresponding phase shift values the resonance masses lie approximately 150 MeV above the physical values, similar to the nucleon mass, due to the unphysical pion mass of 266 MeV. Further details can be found in Refs. [1] and [2].

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