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Logit modeling of the Modulation Transfer Function (MTF) of metal/film portal detectors

Tony Falco[1] and Biagio Gino Fallone[2]

[1]McGill University Health Center, Department of Medical Physics, Montreal, Canada
[2]Cross Cancer institute, University of Alberta, Department of Medical Physics, Alberta, Canada

Background. *Logit analysis is used to fit measured Modulation Transfer Function (MTF) data of front-metal/film detectors at megavoltage energies. The detectors consist of double-emulsion portal film placed abutting front metal-plates of Copper or Lead ranging in thickness from 0.39 to 2.40 mm. The MTF data reported by other investigators is also analyzed and authenticates this type of modeling. The logit function predicts the MTF to within experimental uncertainty and the weighted linear regression analysis demonstrates that the fitting is successful with high correlation coefficient: $-0.999 \leq r \leq -0.995$. The logit function parameterizes the MTF with two regression parameters, a and b. These parameters exhibit a linear relationship with the front-plate mass thickness greater than the maximum range of electrons.*

Conclusions. *The logit fitting analysis allows the calculation of the MTF for metal-plates that can be used in the design of the front-end of electronic portal imaging devices.*

Key words: radiotherapy, high-energy; linear models; logit, MTF, megavoltage, portal detectors

Introduction

The modulation transfer function (MTF) is commonly used to describe the resolution capabilities of imaging systems, which often have a metal-plate component. Few MTF's of metal-plate/film or other types of portal detectors are found in the literature since measuring detector MTF's at therapy energies is a very task intensive process that is prone to large systematic errors. Fit modeling may be helpful in the determination of the metal of choice for these systems, as well as, the determination of the metal of choice for the front-end of electronic portal imaging devices (EPID's). Moreover, parameterization can help quantify the dependence of the MTF on physical quantities, such as metal-plate physical density.

MTF modeling of screen/film systems for diagnostic radiological purposes has been performed in the past with varying degrees of success, using exponential, Lorentzian and Gaussian functions.[1-4] It has been shown that the MTF of radiological phosphor screen/film

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Correspondence to: T. Falco, McGill University Health Center, Department of Medical Physics, 1650 Cedar Avenue, Montreal, Quebec H3G 1A4, Canada; Phone: + 1 514 934 8052; Fax: + 1 514 934 8229; E-mail: tfalco@medphys.mcgill.ca

detectors can be accurately modeled by the logistic or logit function with typically high correlation coefficients[5,6] (i.e., r = -0.998). Logit analysis is a straight-line transform method that can effectively parameterize the MTF and is relatively simple to implement. The MTF's of metal/film detectors at megavoltage energies have been measured,[7-9] however, there have not been any reports of the analytical representation or the fit modeling of the MTF for front-metal/film detectors at megavoltage energies. We perform logit analysis of MTF's obtained for front-metal/film detectors irradiated at megavoltage energies. To obtain a comprehensive set of fitting parameters, we use the MTF data we have measured for a large number of metal/film combinations.[9] We also analyze the MTF data reported by other investigators[7] to authenticate this type of modeling.

Logit analysis

The logit analysis transforms sigmoidally shaped functions into straight-line functions[10] that can then be analyzed in terms of the "slope" and "intercept" regression parameters. Following the approach described by Bencomo and Fallone[6] for diagnostic screen/film systems, we can fit the MTF of metal/film detectors by a function $MTF_L(f)$ given by:

$$MTF_L(f) = \frac{1}{1+e^{-(a+b\ln(f/f'))}} \qquad (1)$$

where f is spatial frequency and f' is a constant (typically 1, with units of f) to ensure correct dimensionality. The straight-line logit transform of Eq. 1 is:[10]

$$logit(MTF_L(f)) = \ln\left(\frac{MTF_L(f)}{1-MTF_L(f)}\right). \qquad (2)$$

When the logit of the measured $MTF(f)$ is plotted versus $\ln(f/f')$ a straight line results, which is represented by $a + b \ln (f/f')$ where a and b are the "intercept" and "slope" of the line, respectively.

The constants a and b are regression parameters that were estimated using Berkson's calculated methods[10] which are summarized as:

$$b = s_{lx} / s_x^2 \quad , \qquad (3)$$

$$a = \bar{l} - b\bar{x} \qquad (4)$$

where

$$s_{lx} = \sum_{i=1}^{n} w_i'(l_i - \bar{l})(x_i - \bar{x}) \quad , \qquad (5)$$

$$s_x^2 = \sum_{i=1}^{n} w_i'(x_i - \bar{x})^2 \qquad (6)$$

and

$$l_i = \ln\{MTF(f_i)/[1 - MTF(f_i)]\} \qquad (7)$$

$$x_i = \ln(f_i) \quad , \qquad (8)$$

$$w_i' = w_i / \sum_{i=1}^{n} w_i \quad , \qquad (9)$$

$$w_i = MTF(f_i)\{1 - MTF(f_i)\} \qquad (10)$$

where $\bar{l}$ and $\bar{x}$ are mean values of l and x respectively, and $MTF(f_i)$ is the value of the MTF, at the spatial frequency f_i averaged over the three or four measurements. The summation is over the n frequency components of the MTF.

Logit analysis has been most widely used for accurate modeling of bio-assay dose survival curves. Berkson assumed that when a system is exposed to a dose y the fractional response P which measures the observed portion p affected out of m exposed, is a random variable that is binomially distributed.[11-13] From binomial statistical theory, the variance of the distribution p is $s_p^2 = P[1 - P]/m$. We can view the MTF as the fractional intensity response of the front-metal/film system to an input composed of sinusoidals of equal intensity for all spatial frequencies f. In our case, $P = MTF(f)$, and the regression parameters a and b can be obtained from a simple least squares calculation which minimizes the weighted-square difference between the observed $MTF(f)$ and the estimated $MTF_L(f)$. The weighting w_i, of Eq. 10, equals ms_p^2.

The goodness-of-fit of the logit function to the $MTF(f)$ data can be specified with the regression correlation coefficient r, and the uncertainties in a and b can be specified by s_a and s_b, respectively. The best fit regression correlation coefficient is given by:

$$r = b\left(\frac{s_x}{s_l}\right) = \frac{s_{lx}}{s_x s_l} \tag{11}$$

where s_x and s_l are the standard deviations on x and l, respectively.[14] The standard deviations of a and b are

$$s_a = \sqrt{s^2\left(\frac{1}{n} + \bar{x}^2 s_b^2\right)} \quad , \tag{12}$$

$$s_b = \sqrt{\frac{s^2}{\sum_{i=1}^{n}(x_i - \bar{x})}} \quad , \tag{13}$$

with

$$s^2 = \frac{\sum_{i=1}^{n}(l_i - a - bx_i)^2}{n-2} \tag{14}$$

and the data consists of h spatial frequency observations.[15] The experimental uncertainty in the individual $MTF(f_i)$ is not taken into account in the logit model.

Results and discussion

The detectors from Falco and Fallone[9] having front-plates only, are listed in Table 1 with their best fit a and b regression parameters. Plots of the logit fits to the measured $MTF(f)$ are shown in Figures 1 and 2 for detectors irradiated by the 10 MV and Co-60 spectrum, respectively. The correlation coefficients r (Table 1) range from -0.995 to -0.999, and for a particular metal, the parameter a decreases with front plate thickness (or mass thickness). The decrease of a with front plate thickness corresponds to the decrease of the MTF with front plate thickness for a given metal.

To further demonstrate the fitting capabilities of the logit technique, the technique was also applied to the MTF's of front-metal/film detectors measured by other investigators. Table 2 shows the logit best-fit parameters for the MTF's reported by Munro *et al.*[7] for the 18 MV and Co-60 spectra. The correlation coefficients range between -0.994 and -0.999 except for one value at -0.991. Plots of the logit regression fits to these data are shown in Figure 3. To avoid clutter, some of the curves in these figures have been offset vertically. The decrease in parameter a with beam energy cannot be verified with the Munro *et al.* data because they only used one thickness for each of the front-metal plates. The data of Droege and Bjarngard[8] were not fitted because of a flaw in their technique as was discussed by Munro *et al.*[7]

In Figure 4, our measured $MTF(f)$'s are compared to the fitted $MTF_L(f)$ for the (a) typical and the (b) worst case. For the worst case, the $MTF_L(f)$ is within the uncertainty of the measured $MTF(f)$ for the whole spatial frequency range.

The regression parameters a and b for the detectors in Table 1, are plotted in Figure 5 as a function of front-plate mass thickness. The plots exhibit a linear relationship between the regression parameters and the mass thick-

Table 1. Regression coefficients *a* and *b* for the metal-plate/film detectors studied. The correlation coefficient *r*, is also shown

Front-Plate Thickness (mm)	"Intercept" *a*		"Slope" *b*		Correlation Coefficient *r*	
	Co-60	10 MV	Co-60	10 MV	Co-60	10 MV
0.95 Cu	-0.150 ± 0.010	-0.480 ± 0.014	-0.834 ± 0.009	-0.982 ± 0.012	-0.995	-0.996
1.75 Cu	-0.369 ± 0.010	-0.782 ± 0.015	-0.750 ± 0.009	-0.927 ± 0.013	-0.996	-0.995
2.40 Cu	-0.617 ± 0.008	-0.969 ± 0.007	-0.700 ± 0.009	-0.809 ± 0.007	-0.997	-0.998
0.39 Pb	0.108 ± 0.005	-0.142 ± 0.005	-1.047 ± 0.004	-0.991 ± 0.005	-0.999	-0.999
1.10 Pb	0.033 ± 0.008	-0.331 ± 0.007	-0.932 ± 0.007	-0.968 ± 0.006	-0.998	-0.999
1.31 Pb	-0.046 ± 0.007	-0.415 ± 0.010	-0.895 ± 0.006	-0.949 ± 0.008	-0.998	-0.998
2.05 Pb	-0.174 ± 0.007	-0.586 ± 0.011	-0.810 ± 0.008	-0.917 ± 0.010	-0.997	-0.997

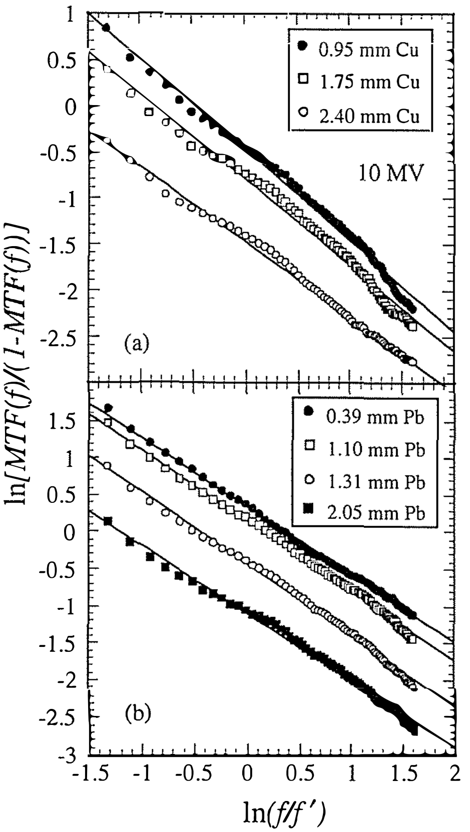

Figure 1. Logit fits to the *MTF*(*f*) data collected with the 10 MV spectrum for the detectors with (a) Cu, and (b) Pb front-plates. The straight lines are the logit MTF's calculated using the regression parameters in Table 1. For clarity, the curves corresponding to the 2.40 mm Cu, 0.39, and 1.10, 2.05 mm Pb front-plates were displaced vertically by -0.5, 0.5, 0.5, and -0.5, respectively.

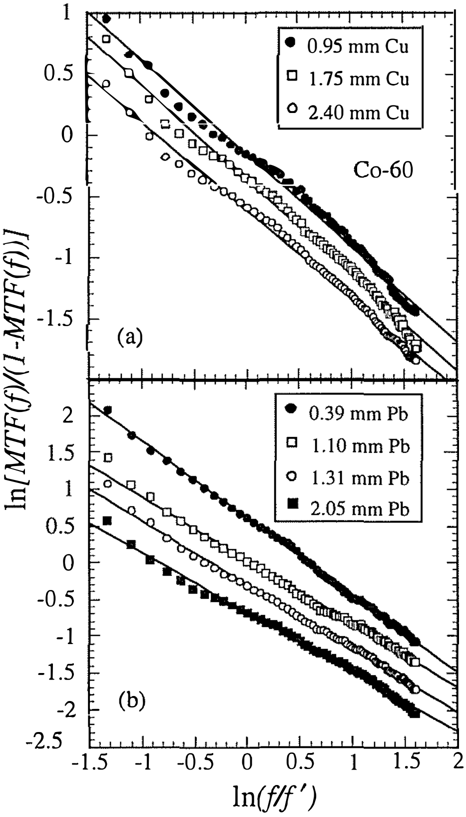

Figure 2. Logit fits to the *MTF*(*f*) data collected with the Co-60 spectrum for the detectors with (a) Cu, and (b) Pb front-plates. The straight lines are the logit MTF's calculated using the regression parameters in Table 1. For clarity, the curves corresponding to the 0.39, 1.10, and 2.05 mm Pb front-plates were displaced vertically by 0.5, -0.25, and -0.5, respectively.

Table 2. Regression coefficients for data from Munro et al.[7] using the 18 MV and Co-60 spectra.

Front-Plate Thickness (mm)	Energy Spectrum	"Intercept" a	"Slope" b	Correlation Coefficient r
1.0 Cu	18 MV	-0.233 ± 0.016	-0.881 ± 0.016	-0.997
1.0 Pb	18 MV	-0.160 ± 0.012	-0.902 ± 0.013	-0.998
1.5 W	18 MV	-0.001 ± 0.006	-0.880 ± 0.007	-0.999
1.5 W	Co - 60	0.281 ± 0.034	-0.689 ± 0.031	-0.991

Table 3. Slope and intercept of the lines in Figure 5.

Energy	Metal	Figure 5 (a & b)		Figure 5 (c & d)	
		slope	intercept	slope	intercept
10 MV	Cu	-0.38 ± 0.03	-0.17 ± 0.05	0.13 ± 0.02	-1.10 ± 0.03
10 MV	Pb	-0.24 ± 0.01	-0.04 ± 0.02	0.04 ± 0.01	-1.01 ± 0.01
Co - 60	Cu	-0.36 ± 0.04	0.17 ± 0.06	0.11 ± 0.01	-0.92 ± 0.01
Co - 60	Pb	-0.15 ± 0.02	0.19 ± 0.03	0.13 ± 0.01	-1.09 ± 0.02

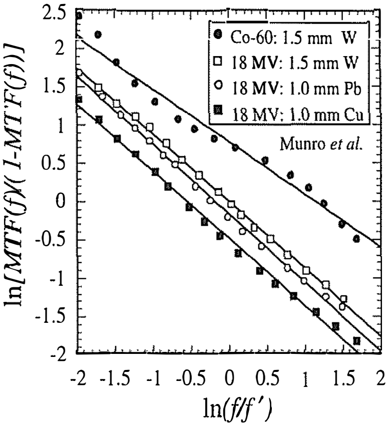

Figure 3. Logit fits to the $MTF(f)$ data from the literature [Munro et al. ref(7)]. The straight lines are the logit MTF's calculated using the regression parameters in Table 2. For clarity, the curves for 1.5 mm W at Co-60 and 1.0 mm Cu at 18 MV were displaced vertically by 0.5 and -0.25, respectively.

ness for a given metal. The slopes and intercepts describing the straight lines are shown in Table 3 for both Cu and Pb, and can be used to calculate the regression parameters (and consequently the MTF) for any other front plate thickness. The data in Table 1 were

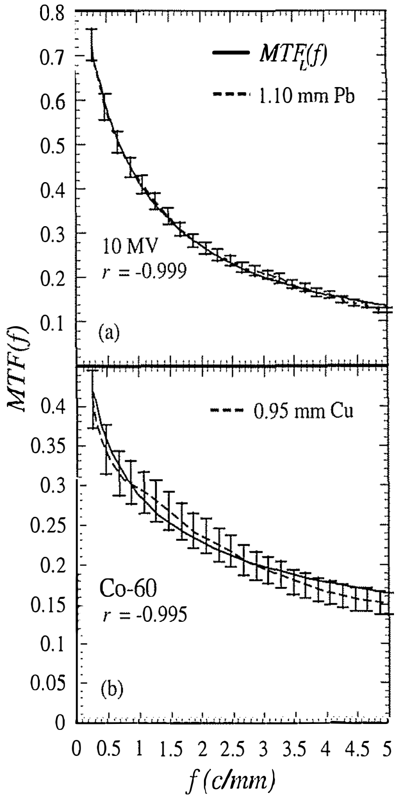

Figure 4. The $MTF(f)$ data and the calculated ($MTF_L(f)$) are shown for (a) typical case with r = -0.999 and for the (b) worst case with r = -0.995.

obtained with double-emulsion film. The Co-60 data from Munro *et al.* listed in Table 2 was not added to that of Figure 5 because they were obtained with a single-emulsion film.

Conclusion

The logit function predicts the MTF to within experimental uncertainty and the weighted linear regression analysis demonstrates that the fitting is successful with high correlation coefficient: $-0.999 \leq r \leq -0.995$. Fitting the MTF data with the logit function allowed the parameterization of the MTF with two regression parameters: *a* and *b*. We have shown that *a* and *b* exhibit a linear relationship with detector front-plate mass thickness greater than the maximum range of electrons in the plate.

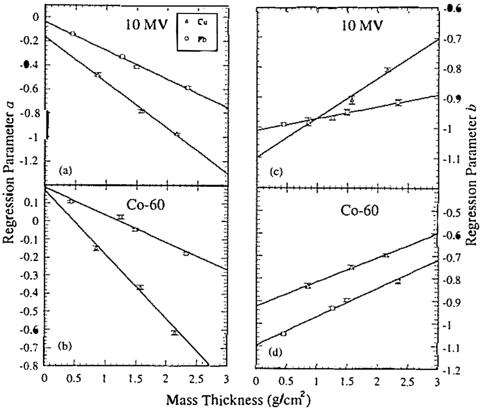

Figure 5. Plots of regression parameters *a* and b versus detector front-plate mass thickness. A linear relationship exists between the regression parameters and the mass thickness for a given metal, and parameters for the linear fits are given in Table 3.

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