BLED WORKSHOPS
IN PHYSICS
VOL. 1, NO. 1

Proceedins of the Mini-Workshop
Few-Quark Problems *(p. 41)*
Bled, Slovenia, July 8-15, 2000

The anomalous $\gamma \to \pi^+\pi^0\pi^-$ form factor and the light–quark mass functions at low momenta*

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Abstract. The $\gamma \to 3\pi$ form factor was calculated in a simple–minded constituent model with a constant quark mass parameter, as well as in the Schwinger-Dyson approach. The comparison of these and various other theoretical results on this anomalous process, as well as the scarce already available data (hopefully to be supplemented by more accurate CEBAF data), seem to favor Schwinger–Dyson modeling which would yield relatively small low–momentum values of the constituent (dynamically dressed) quark mass function.

The Abelian-anomalous $\pi^0 \to \gamma\gamma$ amplitude is exactly [1,2] $T_\pi^{2\gamma}(m_\pi = 0) = e^2 N_c/(12\pi^2 f_\pi)$ in the chiral and soft limit of pions of vanishing mass m_π. On similarly fundamental grounds, the anomalous amplitude for the $\gamma(q) \to \pi^+(p_1)$ $\pi^0(p_2)\pi^-(p_3)$ process, is predicted [3] to be

$$F_\gamma^{3\pi}(0,0,0) = \frac{1}{ef_\pi^2}\, T_\pi^{2\gamma}(0) = \frac{eN_c}{12\pi^2 f_\pi^3}\,, \tag{1}$$

also in the chiral limit and at the soft point, where the momenta of all three pions vanish: $\{p_1, p_2, p_3\} = \{0,0,0\}$. While the chiral and soft limit are an excellent approximation for $\pi^0 \to \gamma\gamma$, the already published [4] and presently planned Primakoff experiments at CERN [5], as well as the current CEBAF measurement [6] of the $\gamma(q) \to \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)$, involve values of energy and momentum transfer which are not negligible compared to typical hadronic scales. This gives a lot of motivation for theoretical predictions of the $\gamma \to 3\pi$ amplitude for non–vanishing $\{p_1, p_2, p_3\}$, *i.e.*, the form factor $F_\gamma^{3\pi}(p_1, p_2, p_3)$. We calculated it as the quark "box"-amplitude (see Fig. 1) in the two related approaches [7,8] sketched below.

In our Ref. [7], the intermediate fermion "box" loop is the one of "simple" constituent quarks with the constant quark mass parameter M. The isospinor $\Psi = (u, d)^T$ of the light constituent quarks couple to the isovector pions π^a through the pseudoscalar Yukawa coupling $g\gamma_5\tau^a$. Its constant quark-pion coupling

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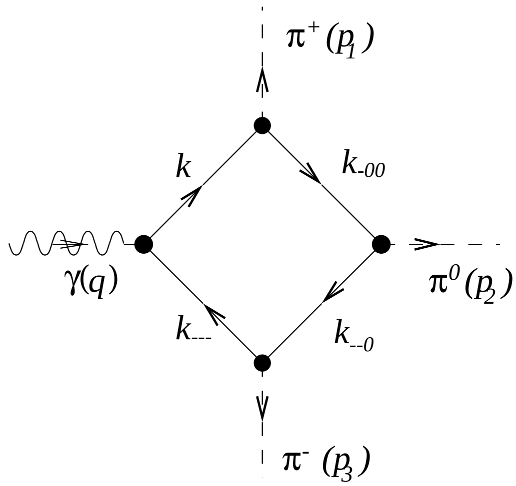

Fig. 1. One of the box diagrams for the process $\gamma(q) \to \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)$. The other five are obtained from this one by the permutations of the vertices of the three different pions.

strength g is related to the pion decay constant $f_\pi = 92.4$ MeV through the quark-level Goldberger-Treiman (GT) relation $g/M = 1/f_\pi$. The result of this calculation also corresponds to the form factor, in the lowest order in pion interactions, of the sigma-model and of the chiral quark model. In Ref. [7], we give the analytic expression for the form factor in terms of an expansion in the pion momenta up to the order $\mathcal{O}(p^8)$ relative to the soft point result, and also perform its exact numerical evaluation. The latter predictions of this quark loop model [7] are given [normalized to the soft-point amplitude (1)] in Fig. 2 by the long-dashed curve for $M = 330$ MeV, by the line of empty boxes for $M = 400$ MeV, and by the line of crosses for the large value $M = 580$ MeV. Note that in the lowest order in pion interactions, they are also the form factors of the σ-model and of the chiral quark model.

Our second Ref. [8] employs the Schwinger-Dyson (SD) approach [9], which is consistent both with the chiral symmetry constraints in the low-energy domain and with the perturbative QCD in the high-energy domain. In this approach, quarks in the fermion loop do not have free propagators with the simple-minded constant constituent mass M. Instead, the box loop amplitude is evaluated with the dressed quark propagator

$$S(k) = \frac{1}{i\not{k}A(k^2) + m + B(k^2)} \equiv \frac{Z(k^2)}{i\not{k} + \mathcal{M}(k^2)} \tag{2}$$

containing the *momentum-dependent*, mostly dynamically generated quark mass function $\mathcal{M}(k^2)$ following from the SD solution for the dressed quark propagator (2). The explicit chiral symmetry breaking $m(\sim 2$ MeV in the present model choice [10,8]) is two orders of magnitude smaller than the quark mass function *at small momenta*, where it corresponds to the notion of the constituent quark mass. Indeed, in Refs. [10,8] as well as in the model choice reviewed in our Ref. [11],

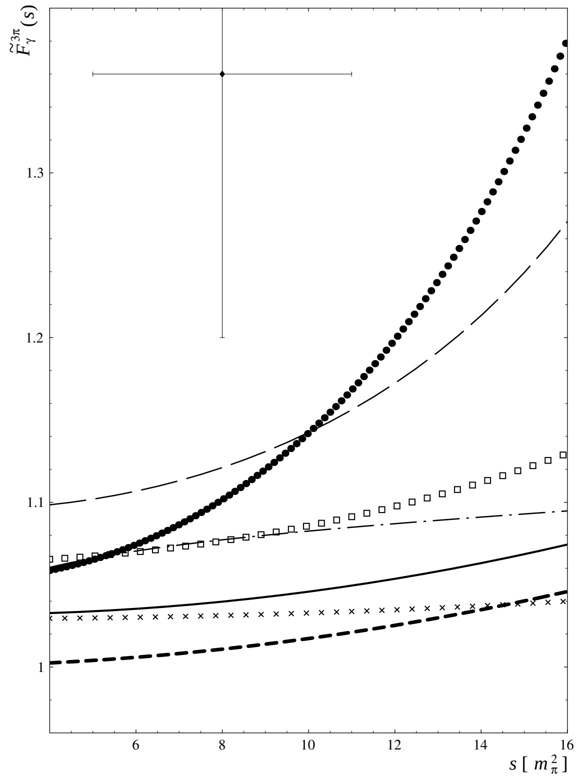

Fig. 2. Various predictions for the dependence of the normalized $\gamma 3\pi$ form factor $\widetilde{F}_\gamma^{3\pi}$ on the Mandelstam variable $s \equiv (p_1 + p_2)^2$. The kinematics is as in the Serpukhov measurement (which provided[4] the shown data point): the photon and all three pions are on shell, $q^2 = 0$ and $p_1^2 = p_2^2 = p_3^2 = m_\pi^2$.

$\mathcal{M}(k^2 \sim 0) \sim 300$ to 400 MeV. On the other hand, since already the present–day SD modeling is well–based [9] on many aspects of QCD, such SD–generated $\mathcal{M}(k^2)$ should be close to the true QCD quark mass function.

SD approach employs the Bethe–Salpeter (BS) bound–state pion–quark–antiquark vertex $\Gamma_{\pi^a}(k, p_{\pi^a})$ (here, in Fig. 1, instead of the aforementioned momentum–independent Yukawa coupling). The propagator (2) is consistent with the solution for the BS solution for $\Gamma_{\pi^a}(k, p_{\pi^a})$, and then, in this approach, the light pseudoscalar mesons are simultaneously the quark-antiquark bound states and the (quasi) Goldstone bosons of dynamical chiral symmetry breaking [9]. Thanks to this, and also to carefully preserving the vector Ward-Takahashi identity in the quark-photon vertex, the *both* fundamental anomalous amplitudes $T_\pi^{2\gamma}(0)$ and $F_\gamma^{3\pi}(0,0,0)$ for the respective decays $\pi^0 \to \gamma\gamma$ and $\gamma \to \pi^+\pi^0\pi^-$, are evaluated analytically and exactly in the chiral limit and the soft limit [10]. (Note that reproducing these results even only roughly, let alone analytically, is otherwise quite problematic for bound-state approaches, as discussed in Ref. [11].)

In Fig. 2, the solid curve gives our $\gamma 3\pi$ form factor obtained in the SD approach for the empirical pion mass, $m_\pi = 138.5$ MeV, while the dashed curve gives it in the chiral limit, $m_\pi = 0 = m$. To understand the relationship between the predictions of these two approaches, one should, besides the curves in Fig. 2, compare also the analytic expressions we derived for the form factors [esp. Eqs. (20)–(21) in Ref. [8] and analogous formulas in Ref. [7]]. This way, one can see, first, why the constant, momentum-independent term is smaller in the SD case, causing the downward shift of the SD form factors with respect to those in the constant constituent mass case. Second, this constant term in the both approaches diminishes with the increase of the pertinent mass scales, namely M in the constant-mass case, and the scale which rules the SD–modeling and which is of course closely related to the resulting scale of the *dynamically generated* constituent mass $\mathcal{M}(k^2 \sim 0)$. Finally, the momentum–dependent terms are similar in the both approaches; notably, the coefficients of the momentum expansions (in powers of $p_i \cdot p_j$) are similarly suppressed by powers of their pertinent scales. This all implies a transparent relationship between $\mathcal{M}(k^2)$ at small k^2 and the $\gamma 3\pi$ form factor, so that the accurate CEBAF data, which hopefully are to appear soon [6], should be able to constrain $\mathcal{M}(k^2)$ at small k^2, and thus the whole infrared SD modeling. Admittedly, we used the Ball–Chiu Ansatz for the dressed quark–photon vertex, but this is adequate since Ref. [12] found that for $-0.4\,\mathrm{GeV}^2 < q^2 < 0.2\,\mathrm{GeV}^2$, the true solution for the dressed vertex is approximated well by this Ansatz plus the vector–meson resonant contributions which however vanish in our case of the real photon, $q^2 = 0$. Therefore, if the experimental form factor is measured with sufficient precision to judge the present SD model results definitely too low, it will be a clear signal that the SD modeling should be reformulated and refitted so that it is governed by a smaller mass scale and smaller values of $\mathcal{M}(k^2 \sim 0)$.

The only already available data, the Serpukhov experimental point [4] (shown in the upper left corner of Fig. 2), is higher than all theoretical predictions and is probably an overestimate. However, the SD predictions are farthest from it. Indeed, in the momentum interval shown in Fig. 2, the SD form factors are lower than those of other theoretical approaches (for reasonable values of their parameters) including vector meson dominance [13] (the dotted curve) and of chiral perturbation theory [14] (the dash-dotted curve). Therefore, even the present experimental and theoretical knowledge indicates that the momentum–dependent mass function in the SD model [10] we adopted [8], may already be too large at small k^2, where its typical value for light u, d quarks is $\mathcal{M}(k^2 \approx 0) \approx 360$ MeV. Note that this value is, at present, probably the lowest in the SD–modeling except for the model reviewed in Ref. [11], which has very similar $\mathcal{M}(k^2)$ at low k^2. (Some other very successful [9] SD models obtain even higher values, $\mathcal{M}(k^2 \approx 0) \approx 600$ MeV and more, which would lead to even lower $\gamma 3\pi$ transition form factors.) It is thus desirable to reformulate SD phenomenology using momentum–dependent mass functions which are smaller at low k^2. This conclusion is in agreement with recent lattice QCD studies of the quark propagator which find [15] $\mathcal{M}(k^2 = 0) = 298 \pm 8$ MeV (for $m = 0$).

Acknowledgment: D. Klabučar thanks the organizers, M. Rosina and B. Golli, for their hospitality and for the partial support which made possible his partic-

ipation at Mini-Workshop Bled 2000: FEW-QUARK PROBLEMS, Bled, Slovenia, 8–15 July 2000.

References

1. S. L. Adler, Phys. Rev. **177**, 2426 (1969).
2. J. S. Bell and R. Jackiw, Nuovo Cim. **A60**, 47 (1969).
3. S. L. Adler *et al.*, Phys. Rev. **D 4** (1971) 3497; M. V. Terent'ev, Phys. Lett. **38B** (1972) 419; R. Aviv and A. Zee, Phys. Rev. **D 5** (1972) 2372.
4. Yu. M. Antipov *et al.*, Phys. Rev. **D 36** (1987) 21.
5. M. A. Moinester *et al.*, e-print `hep-ex/9903017`, and proceedings of 37th International Winter Meeting on Nuclear Physics, Bormio, Italy, 25-29 Jan 1999.
6. R. A. Miskimen, K. Wang and A. Yagneswaran, *Study of the Axial Anomaly using the* $\gamma\pi^+ \to \pi^+\pi^0$ *Reaction ...*, Letter of intent, CEBAF-experiment 94-015.
7. B. Bistrović and D. Klabučar, Phys. Rev. **D 61** (2000) 033006.
8. B. Bistrović and D. Klabučar, Phys. Lett. **B 478** (2000) 127-136.
9. A recent review is: C. D. Roberts, `e-print arXiv:nucl-th/0007054`.
10. R. Alkofer and C. D. Roberts, Phys. Lett. **B 369**, 101 (1996).
11. D. Kekez, B. Bistrović, and D. Klabučar, Int. J. Mod. Phys. **A 14**, (1999) 161.
12. P. Maris and P. Tandy, Phys. Rev. **C 61** (2000) 045202.
13. S. Rudaz, Phys. Lett. **B 145** (1984) 281.
14. B. Holstein, Phys. Rev. **D 53** (1996) 4099.
15. J. I. Skullerud and A. G. Williams, e-print `hep-lat/0007028`.