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ACCURATE CLOSED-FORM EXPRESSION FOR THE FREQUENCY-DEPENDENT MUTUAL IMPEDANCE OF ON-CHIP INTERCONNECTS ON LOSSY SILICON SUBSTRATE

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Abstract: A new analytic model is presented (based on the induced current density distribution inside silicon substrate) to calculate the frequency dependent distributed mutual series impedance per unit length for coupled interconnects on lossy silicon substrate in CMOS technology. The proposed analytic model is shown to be very effective for a variety of on-chip interconnect structures in all three modes of propagation (skin-effect, slow-wave, and dielectric quasi-TEM), for both series mutual impedance components (resistance and inductance) over a very wide range of dimension, substrate conductivity, and frequency. The validity of the proposed model has been checked by a comparison with a quasi-TEM spectral domain approach and equivalent-circuit modeling procedure.

Točen zaključen izraz za frekvenčno odvisno vzajemno impedanco povezav na izgubni silicijevi tabletki

Ključne besede: polprevodniki, mikroelektronika, substrati polprevodniški silicijevi, povezave medsebojne na čipu, impedanca serijska vzajemna porazdeljena, impedanca vzajemna frekvenčno odvisna, substrati silicijevi izgubni, modeli analitični, obrazci matematični

Izvleček: V prispevku predstavljamo nov analitičen model (temelji na porazdelitvi gostote induciranih tokov znotraj silicijevega substrata) za izračun frekvenčno odvisne porazdeljene vzajemne serijske impedance na enoto dolžine v CMOS tehnologiji za sklopljene povezave na izgubnem silicijevem substratu. Izkaže se, da je predlagani model zelo učinkovit za obravnavo različnih struktur povezav na tabletki (čipu) za vse tri načine širjenja signala (skin efekt, počasni - val in dielektrični quasi - TEM), za obe serijski komponenti vzajemne impedance (upornost in induktanca) znotraj širokega intervala dimenzij, prevodnosti substrata in frekvence. Veljavnost predlaganega modela smo preverili s primerjavo s quasi-TEM spektralno metodo in metodo modeliranja nadomestnih vezij.

1. Introduction

High frequency RF integrated circuits in CMOS technology are crucial components of today's integrated system. As the density, complexity, and speed of VLSI circuits are continuing to increase, the management of the on-chip interconnects becomes of paramount concern to the IC designer, especially with respect to the internal parasitics parameters immunity /1/. In order to accomplish this, it is necessary to analyze and model the broadband characteristics /2 - 5, 8/ of the silicon IC interconnects since the signals tend to exhibit both the short rising and falling times. For the case of silicon the effect of high lossy substrate (CMOS technology) on the distributed mutual inductance and resistance per unit length of coupled interconnects has not been modeled well with analytical closed form expressions. In this letter (based on silicon substrate induced current distribution) we suggest an analytical model that can accurately predict frequency dependent mutual inductance and resistance of silicon substrate IC interconnects, with good agreement with the quasi-TEM spectral domain approach and full wave numerical simulation /8/, respectively, over a wide range of dimensions, substrate conductivity, and frequency.

2. Analysis

In order to investigate the influence of the longitudinal current distribution in the silicon substrate on the mutual inductance and resistance per unit length of the general coupled interconnects, the structure depicted in Fig. 1a,b has been analyzed. To model actual rectangular conductors, we define an equivalent diameter $2r_{ieq}$ ($i = 1,2$) as the mean of the diameter of the circles inscribed in the conductors ($2r_{ieq} = (w_i + T_i)/2$). The other geometrical dimensions H, h and s are consequently redefined as $H_{eq} = H + (T_2-w_2)/4$, $h_{eq} = h + (T_1-w_1)/4 + (T_2-w_2)/4$ and $s_{eq} = s + (w_1-T_1)/4 + (w_2-T_2)/4$ (see Fig. 1b).

Due to the impressed field that interconnect lines radiates in presence of lossy silicon substrate, an unknown current density $\mathbf{J_s}$ is induced in the substrate: because of the particular geometry, this current has only the component along the z-axis, which is a function of x and y, namely $\mathbf{J_s} = J$ (x, y)1_z. Results obtained from the full-wave analysis /2/ have shown that the influence of the finite substrate thickness d can be neglected for practical dimensions (d >> w_1, w_2, s, t_{ox}). Therefore in the following analysis we have assumed the silicon substrate to be infinitely thick. In order to derive the expression for mutual impedance Z_m of coupled interconnects a straight current filament parallel to a silicon semi-space will be first analyzed (see Fig. 1b). In the two regions depicted in Fig. 1b, the governing equations for the magnetic vector potential is

$$\nabla^2 A_i = j\omega\mu_i\sigma_i A_i \quad i=1,2 \quad (\sigma_1 = 0) \tag{1}$$

A general solution of eq. (1) may be looked for in the form /6/

$$A_1(x,y) = \int_0^\infty [C_{11}(\lambda)e^{\lambda y} + C_{12}(\lambda)e^{-\lambda y}]\cos(\lambda x)d\lambda, \quad for \quad y \geq 0 \tag{2a}$$

$$A_2(x,y) = \int_0^\infty [C_2(\lambda)e^{my}\cos(\lambda x)d\lambda, \quad for \quad y \leq 0 \tag{2b}$$

where m = $(\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon))^{1/2}$.

The integration coefficients may be determined by imposing, at the interface silicon-SiO_2 (y = 0), the continuity of the tangential components of the magnetic field and of the normal component of the magnetic flux density. The following expressions are then obtained:

$$A_1(x,y) = \frac{\mu_0 I}{2\pi}\ln\frac{\sqrt{x^2+(y-b)^2}}{\sqrt{x^2+(y+b)^2}} + \frac{\mu I}{\pi}\int_0^\infty \frac{e^{-(b+y)\lambda}}{\mu_r\lambda + \sqrt{\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon)}}\cos(\lambda x)d\lambda \tag{3a}$$

$$A_2(x,y) = \frac{\mu I}{\pi}\int_0^\infty \frac{e^{-b\lambda}e^{y\sqrt{\lambda^2+j\omega\mu(\sigma+j\omega\varepsilon)}}}{\mu_r\lambda + \sqrt{\lambda^2 + j\omega\mu(\sigma + j\omega\varepsilon)}}\cos(\lambda x)d\lambda \tag{3b}$$

In above expressions the magnetic potential is introduced in Maxwell's equations in order to find easy the current density in silicon substrate as $J_s(x,y) = -j\omega\sigma A_2(x,y)$. This leads to the quasi-static voltage drop $\partial V/\partial z$ in the z-direction that also appears in the classical line equations,

$$\frac{\partial V}{\partial z} = -[Z]I \tag{4}$$

The magnetic vector potential is used in order to find the quasi-static potential drop $\partial V/\partial z$ at any point (x,y) in the space along the lines parallel to the z direction. This allows the mutual impedance per unit length, eq. (4), to be evaluated.

The axial electric field intensity along the lossy silicon substrate is

$$E_{zs}(x, y=0) = -j\omega A_1(x, y=0) - \frac{\partial V(x, y=0)}{\partial z}, \tag{5}$$

and at any point (x,y) above the silicon substrate

$$E_{za}(x,y) = -j\omega A_1(x,y) - \frac{\partial V(x,y)}{\partial z}, \tag{6}$$

Subtracting eq. (5) from eq. (6), the axial electric field intensity at any point above the lossy silicon substrate can be expressed as

$$E_{za}(x,y) = E_{zs}(x,y=0) - j\omega[A_1(x,y) - A_1(x,y=0)] - \frac{\partial}{\partial z}[V(x,y) - V(x,y=0)] \tag{7}$$

The last term in eq. (7) represents the total scalar voltage drop, in axial z-direction, of the distributed parameter circuit consisting of interconnect conductors and silicon substrate (as return), eq. (4).

In expression for mutual impedance per unit length of coupled interconnects the integral parts contain the term of the form $\sqrt{\lambda^2+\gamma^2} - \lambda/\sqrt{\lambda^2+\gamma^2} - \lambda$, where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$. Introducing the following approximation for this term /7/

$$\frac{\sqrt{\lambda^2+\gamma^2} - \lambda}{\sqrt{\lambda^2+\gamma^2} + \lambda} \approx e^{-2\frac{\lambda}{\gamma}}\left[1 + \frac{\lambda^3}{3\gamma^3} - \frac{3\lambda^5}{20\gamma^5} + \ldots\right] \tag{8}$$

the closed form evaluation of these integrals can be done (the integration in the complex plane)/7/ and the following closed-form formula for mutual impedance is obtained:

$$Z_m = \frac{j\omega\mu_0\mu_r}{2\pi}\left[\ln\left(\sqrt{\frac{D_{kn}^2 + \left(h_k + h_n + \frac{2}{jk_s}\right)^2}{D_{kn}^2 + (h_k - h_n)^2}}\right) - \frac{2\left(h_k + h_n + \frac{2}{jk_s}\right)}{3(jk_s)^3}\frac{\left(h_k + h_n + \frac{2}{jk_s}\right)^2 - 3D_{kn}^2}{\left[\left(h_k + h_n + \frac{2}{jk_s}\right)^2 + D_{kn}^2\right]^3}\right] \tag{9}$$

where $k_s = \sqrt{-j\omega\mu(\sigma+j\omega\varepsilon)}$, being j the imaginary unit, $h_k = H_{eq} + h_{eq} + 2r_{2eq} + r_{1eq}$, $h_n = H_{eq} + r_{2eq}$ and $D_{kn} = s_{eq} + r_{1eq} + r_{2eq}$, respectively.

3. Results and discussions

In order to validate the derived new formulas for mutual impedance per unit length ($Z_m = R_m + j\omega L_m$), an asymmetric coupled interconnect structure on a 300 mm silicon substrate (resistivity $\rho_{si} = 0.01$ Ωcm) with a 3 μm oxide layer is considered. The cross sections of the conductors are 2 μm by 1 μm and 1 μm by 1 μm, respectively. The spacing between the two conductors is 2 μm. Fig. 2 shows the variation in the distributed mutual resistance per unit length, $R_m(\omega)$, as a function of a frequency. At higher frequencies, the increase of mutual resistance is enormous. The cause for this phenomenon can be found in the generation of the eddy currents in the silicon substrate. Similarly, Fig. 3 shows the variation of the distributed mutual inductance per unit length, $L_m(\omega)$, as the function of a frequency. When the substrate conductivity is high, a skin-effect arises in substrate and the return current flows more in the silicon substrate. The variation of the mutual inductance per unit length decreases rapidly as a function frequency in Fig. 3 since most of the induced current is confined in a limited zone of the silicon substrate just beneath the source lines (the skin-effect mode). At low frequencies the mutual inductance is high due to the slow-wave mode. It is observed that the values of the mutual inductance and resistance per unit length, calculated from new derived formulas, are found to be in good agreement with those of /8/ (equivalent-circuit models and quasi-TEM approach).

4. Conclusion

In this letter we have developed a simple, highly accurate and low time consuming analytic model for frequency-dependent distributed mutual series impedance per unit length (mutual resistance and inductance per unit length) of coupled interconnects on lossy silicon (CMOS) substrate. The calculated results show very good agreement with those calculated using full-wave analysis and equivalent circuit model procedure over wide range of dimension, substrate conductivity, and frequency. Due to the simplicity of the calculation, this model should be very useful in the computer-aided design of silicon-based RF and microwave integrated circuits.

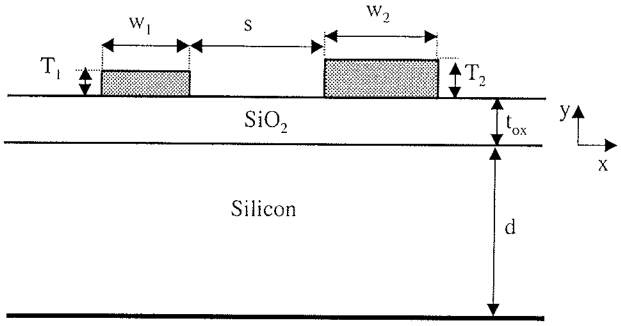

Fig. 1a. Cross section of coupled interconnects on an oxide-semiconductor substrate.

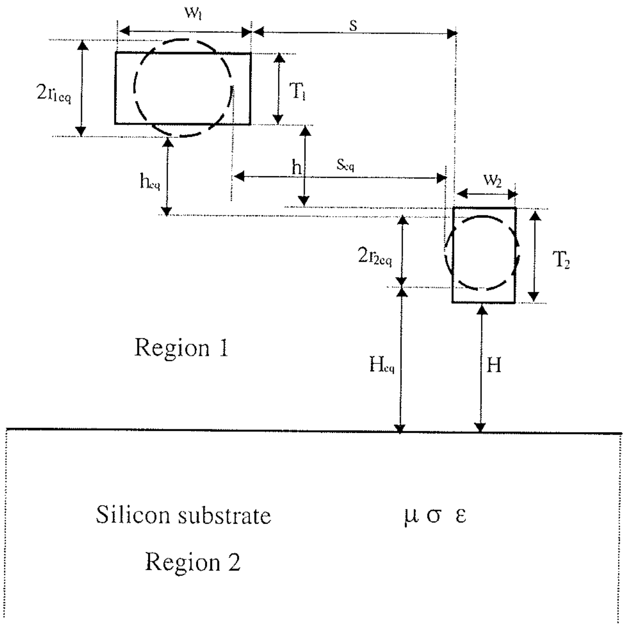

Fig. 1b. Round-sectioned interconnect lines over infinite thick silicon substrate.

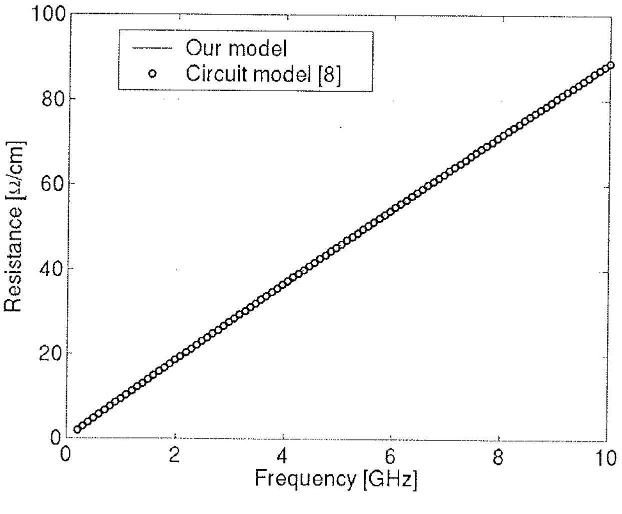

Fig. 2. Mutual resistance per unit length R_m as the function of a frequency.

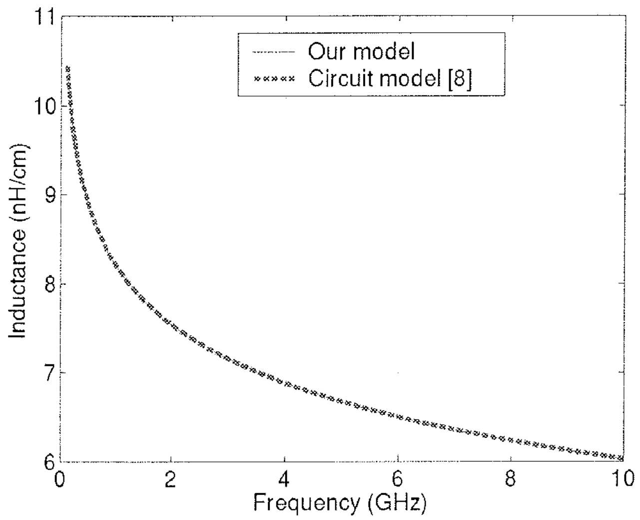

Fig. 3. Mutual inductance per unit length L_m as the function of a frequency.

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