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Masses of heavy tetraquarks in the relativistic quark model*

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Abstract. The masses of the ground and excited heavy tetraquarks with hidden charm are calculated within the relativistic diquark-antidiquark picture.

Recently, significant experimental progress has been achieved in charmonium spectroscopy. Several new charmonium-like states, such as $X(3872)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z(4248)$, $Z(4430)$, etc., were observed [1] which cannot be simply accommodated in the quark-antiquark ($c\bar{c}$) picture. These states and especially the charged ones can be considered as indications of the possible existence of exotic multiquark states. In this talk we briefly review our papers [2,3] where we calculated masses of the ground and excited states of heavy tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. For our calculations we use the diquark-antidiquark picture to reduce a complicated relativistic four-body problem to the subsequent two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from light and heavy quarks. At the second step, a heavy tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark-gluon interaction in terms of the diquark wave functions. Details of the relativistic quark model and calculations can be found in [2,3].

In the diquark-antidiquark picture of heavy tetraquarks both scalar S (antisymmetric in flavour $(Qq)_{S=0} = [Qq]$) and axial vector A (symmetric in flavour $(Qq)_{S=1} = \{Qq\}$) diquarks are considered. Therefore we get the following structure of the $(Qq)(\bar{Q}\bar{q}')$ ground (1S) states (C is defined only for $q = q'$):

- Two states with $J^{PC} = 0^{++}$:

$$X(0^{++}) = (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=0}$$
$$X(0^{++\prime}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}$$

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[*] Talk presented by D. Ebert

- Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}}[(Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0} + (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1}]$$
$$X(1^{+-}) = \frac{1}{\sqrt{2}}[(Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1} - (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0}]$$
$$X(1^{+-\prime}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}$$

- One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}.$$

The orbitally excited $(1P, 1D\ldots)$ states are constructed analogously. As we find, a very rich spectrum of tetraquarks emerges. However the number of states in the considered diquark-antidiquark picture is significantly less than in the genuine four-quark approach.

The diquark-antidiquark model of heavy tetraquarks predicts the existence of a flavour $SU(3)$ nonet of states with hidden charm or beauty $(Q = c, b)$: four tetraquarks $[(Qq)(\bar{Q}\bar{q}), q = u, d]$ with neither open or hidden strangeness, which have electric charges 0 or ±1 and isospin 0 or 1; four tetraquarks $[(Qs)(\bar{Q}\bar{q})$ and $(Qq)(\bar{Q}\bar{s}), q = u, d]$ with open strangeness $(S = \pm1)$, which have electric charges 0 or ±1 and isospin $\frac{1}{2}$; one tetraquark $(Qs)(\bar{Q}\bar{s})$ with hidden strangeness and zero electric charge. Since we neglect in our model the mass difference of u and d quarks and electromagnetic interactions, the corresponding tetraquarks will be degenerate in mass. A more detailed analysis [4] predicts that the tetraquark mass differences can be of a few MeV so that the isospin invariance is broken for the $(Qq)(\bar{Q}\bar{q})$ mass eigenstates and thus in their strong decays. The (non)observation of such states will be a crucial test of the tetraquark model.

In Table 1 we compare our results (EFG) for the masses of the ground and excited charm diquark-antidiquark bound states with the predictions of Refs. [4] and with the masses of the observed highly-excited charmonium-like states [1]. We assume that the excitations occur only between the bound diquark and antidiquark. Possible excitations of diquarks are not considered. Our calculation of the heavy baryon masses supports such a scheme [5]. In this table we give our predictions only for some of the masses of the orbitally and radially excited states for which possible experimental candidates are observed. The differences in some of the presented theoretical mass values can be attributed to the substantial distinctions in the used approaches. We describe the diquarks dynamically as quark-quark bound systems and calculate their masses and form factors, while in Refs.[4] they are treated only phenomenologically. Then we consider the tetraquark as purely the diquark-antidiquark bound system. In distinction, Maini et al. consider a hyperfine interaction between all quarks which, e.g., causes the splitting of 1^{++} and 1^{+-} states arising from the SA diquark-antidiquark compositions. From Table 1 we see that our dynamical calculation supports the assumption [4] that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed from the scalar and axial vector diquark and antidiquark in the relative $1S$ state. Recent Belle and BaBar results indicate the existence of a second $X(3875)$ particle a

Table 1. Comparison of theoretical predictions for the masses of the ground and excited charm diquark-antidiquark states (in MeV) and possible experimental candidates.

State	Diquark	Theory		Experiment	
J^{PC}	content	EFG	[4]	state	mass[1]
1S					
0^{++}	$S\bar{S}$	3812	3723		
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871	$3872^{\dagger}$	X(3872) X(3876)	3871.4 ± 0.6 $3875.2\pm0.7^{+0.9}_{-1.8}$
1^{+-}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	3871	3754		
0^{++}	$A\bar{A}$	3852	3832		
1^{+-}	$A\bar{A}$	3890	3882		
2^{++}	$A\bar{A}$	3968	3952	Y(3943)	$3943\pm11\pm13$ $3914.3^{+4.1}_{-3.8}$
1P					
1^{--}	$S\bar{S}$	4244	4330 ± 70 $(cs\bar{c}\bar{s})$	Y(4260)	$4259\pm8^{+2}_{-6}$ $4247\pm12^{+17}_{-32}$
1^{-}	$S\bar{S}$	4244		Z(4248)	$4248^{+44+180}_{-29-35}$
0^{-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4267			
1^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4284		Y(4260)	$4284^{+17}_{-16}\pm4$
1^{--}	$A\bar{A}$	4277			
1^{--}	$A\bar{A}$	4350		Y(4360)	$4361\pm9\pm9$ 4324 ± 24
2S					
1^{+}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431		Z(4430)	$4433\pm4\pm2$
0^{+}	$A\bar{A}$	4434			
1^{+}	$A\bar{A}$	4461	~ 4470		
2P					
1^{--}	$S\bar{S}$	4666		Y(4660) X(4630)	$4664\pm11\pm5$ 4634^{+8+5}_{-7-8}

† input

few MeV above X(3872). This state could be naturally identified with the second neutral particle predicted by the tetraquark model [4]. On the other hand, in our model the lightest scalar 0^{++} tetraquark is predicted to be above the open charm threshold $D\bar{D}$ and thus to be broad, while in the model [4] it lies a few MeV below this threshold, and thus is predicted to be narrow. Our 2^{++} tetraquark also lies higher than the one in Ref.[4], thus making the interpretation of this state as Y(3943) less probable, especially if one averages the original Belle result with the recent BaBar value which is somewhat lower.

The discovery in the initial state radiation at B-factories of the Y(4260), Y(4360) and Y(4660) indicates an overpopulation of the expected charmonium 1^{--} states [1]. Maini et al. [4] argue that Y(4260) is the 1^{--} 1P state of the charm-strange diquark-antidiquark tetraquark. We find that Y(4260) cannot be interpreted in this way, since the mass of such ($[cs]_{S=0}[\bar{c}\bar{s}]_{S=0}$) tetraquark is found to be ~ 200 MeV higher. A more natural tetraquark interpretation could be the 1^{--} 1P state

$([cq]_{S=0}[\bar{c}\bar{q}]_{S=0})$ ($S\bar{S}$) which mass is predicted in our model to be close to the mass of $Y(4260)$ (see Table 1). Then the $Y(4260)$ would decay dominantly into $D\bar{D}$ pairs. The other possible interpretations of $Y(4260)$ are the 1^{--} 1P states of $(S\bar{A} - \bar{S}A)/\sqrt{2}$ and $A\bar{A}$ tetraquarks which predicted masses have close values. These additional tetraquark states could be responsible for the mass difference of $Y(4260)$ observed in different decay channels. As we see from Table 1, the recently discovered resonances $Y(4360)$ and $Y(4660)$ in the $e^+e^- \to \pi^+\pi^-\psi'$ cross section can be interpreted as the excited 1^{--} 1P ($A\bar{A}$) and 2P ($S\bar{S}$) tetraquark states, respectively. The peak $X(4630)$ very recently observed by Belle in $e^+e^- \to \Lambda_c^+\Lambda_c^-$ is consistent with a 1^{--} resonance $Y(4660)$ and therefore has the same interpretation in our model.

Recently the Belle Collaboration reported the observation of a relatively narrow enhancement in the $\pi^+\psi'$ invariant mass distribution in the $B \to K\pi^+\psi'$ decay [1]. This new resonance, $Z^+(4430)$, is unique among other exotic meson candidates, since it is the first state which has a non-zero electric charge. Different theoretical interpretations were suggested [1]. Maiani et al. [4] gave qualitative arguments that the $Z^+(4430)$ could be the first radial excitation (2S) of a diquark-antidiquark $X^+_{u\bar{d}}(1^{+-}; 1S)$ state ($A\bar{A}$) with mass 3882 MeV. Our calculations indicate that the $Z^+(4430)$ can indeed be the 1^+ 2S $[cu][\bar{c}\bar{d}]$ tetraquark state. It could be the first radial excitation of the ground state $(S\bar{A} - \bar{S}A)/\sqrt{2}$, which has the same mass as $X(3872)$. The other possible interpretation is the 0^+ 2S $[cu][\bar{c}\bar{d}]$ tetraquark state ($A\bar{A}$) which has a very close mass. Measurement of the $Z^+(4430)$ spin will discriminate between these possibilities.

Encouraged by this discovery, the Belle Collaboration performed a study of $\bar{B}^0 \to K^-\pi^+\chi_{c1}$ and observed a double peaked structure in the $\pi^+\chi_{c1}$ invariant mass distribution. These two charged hidden charm peaks, $Z(4051)$ and $Z(4248)$, are explicitly exotic. We find no tetraquark candidates for the former, $Z(4051)$, structure. On the other hand, we see from Table 1 that $Z(4248)$ can be interpreted in our model as the charged partner of the 1^- 1P state $S\bar{S}$ or as the 0^- 1P state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark.

In summary, we calculated the masses of excited heavy tetraquarks with hidden charm in the diquark-antidiquark picture. In contrast to previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of heavy and light hadron properties. It was found that the $X(3872)$, $Z(4248)$, $Y(4260)$, $Y(4360)$, $Z(4430)$ and $Y(4660)$ exotic meson candidates can be tetraquark states with hidden charm.

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