BLED WORKSHOPS
IN PHYSICS
VOL. 1, NO. 1

Proceedins of the Mini-Workshop
Few-Quark Problems *(p. 82)*
Bled, Slovenia, July 8-15, 2000

Nucleon-Nucleon Scattering in a Chiral Constituent Quark Model

Floarea Stancu*

Institute of Physics, B.5, University of Liege, Sart Tilman, B-4000 Liege 1, Belgium

Abstract. We study the nucleon-nucleon interaction in the chiral constituent quark model of Refs. [1,2] by using the resonating group method, convenient for treating the interaction between composite particles. The calculated phase shifts for the $^3S_1$ and $^1S_0$ channels show the presence of a strong repulsive core due to the combined effect of the quark interchange and the spin-flavour structure of the effective quark-quark interaction. Such a structure stems from the pseudoscalar meson exchange between quarks and is a consequence of the spontaneous breaking of the chiral symmetry. We perform single and coupled channel calculations and show the role of coupling of the $\Delta\Delta$ and hidden colour CC channels on the behaviour of the phase shifts. The addition of a σ-meson exchange quark-quark interaction brings the $^1S_0$ phase shift closer to the experimental data. We intend to include a tensor quark-quark interaction to improve the description of the $^3S_1$ phase shift.

In this talk I shall mainly present results obtained in collaboration with Daniel Bartz [3,4] for the nucleon-nucleon (NN) scattering phase shifts calculated in the resonating group method.

The study of the NN interaction in the framework of quark models has already some history. Twenty years ago Oka and Yazaki [5] published the first $L = 0$ phase shifts with the resonating group method. Those results were obtained from models based on one-gluon exchange (OGE) interaction between quarks. Based on such models one could explain the short-range repulsion of the NN interaction potential as due to the chromomagnetic spin-spin interaction, combined with quark interchanges between $3q$ clusters. In order to describe the data, long- and medium-range interactions were added at the nucleon level. During the same period, using a cluster model basis as well, Harvey [6] gave a classification of the six-quark states including the orbital symmetries $[6]_O$ and $[42]_O$. Mitja Rosina, Bojan Golli and collaborators [7] discussed the relation between the resonating group method and the generator coordinate method and introduced effective local NN potentials.

Here we employ a constituent quark model where the short-range quark-quark interaction is entirely due to pseudoscalar meson exchange, instead of one-gluon exchange. This is the chiral constituent quark model of Ref. [1], parametrized in a nonrelativistic version in Ref. [2]. The origin of this model is thought to lie in the spontaneous breaking of chiral symmetry in QCD which implies the existence of Goldstone bosons (pseudoscalar mesons) and constituent quarks

* E-mail: fstancu@ulg.ac.be

with dynamical mass. If a quark-pseudoscalar meson coupling is assumed this generates a pseudoscalar meson exchange between quarks which is spin and flavour dependent. The spin-flavour structure is crucial in reproducing the correct order of the baryon spectra [1,2]. The present status of this model is presented by L. Glozman and W. Plessas at this workshop. Hereafter this model will be called the Goldstone boson exchange (GBE) model.

It is important to correctly describe both the baryon spectra and the baryon-baryon interaction with the same model. The model [1,2] gives a good description of the baryon spectra and in particular the correct order of positive and negative parity states, both in nonstrange and strange baryons, in contrast to the OGE model. In fact the pseudoscalar exchange interaction has two parts : a repulsive Yukawa potential tail and an attractive contact δ-interaction. When regularized, the latter generates the short-range part of the quark-quark interaction. This dominates over the Yukawa part in the description of baryon spectra. The whole interaction contains the main ingredients required in the calculation of the NN potential, and it is thus natural to study the NN problem within the GBE model. In addition, the two-meson exchange interaction between constituent quarks reinforces the effect of the flavour-spin part of the one-meson exchange and also provides a contribution of a σ-meson exchange type [8] required to describe the middle-range attraction.

Preliminary studies of the NN interaction with the GBE model have been made in Refs. [9–11]. They showed that the GBE interaction induces a short-range repulsion in the NN potential. In Refs. [9,10] this is concluded from studies at zero separation between clusters and in [11] an adiabatic potential is calculated explicitly. Here we report on dynamical calculations of the NN interaction obtained in the framework of the GBE model and based on the resonation group method [3,4]. In Ref. [3] the $^3S_1$ and $^1S_0$ phase shifts have been derived in single and three coupled channels calculations. It was found that the coupling to the $\Delta\Delta$ and CC (hidden colour) channels contribute very little to the NN phase shift. These studies show that the GBE model can explain the short-range repulsion, as due to the flavour-spin quark-quark interaction and to the quark interchange between clusters.

However, to describe the scattering data and the deuteron properties, intermediate- and long-range attraction potentials are necessary. In Ref. [4] a σ-meson exchange interaction has been added at the quark level to the six-quark Hamiltonian. This interaction has the form

$$V_\sigma = -\frac{g_{\sigma q}^2}{4\pi}\,(\frac{e^{-\mu_\sigma r}}{r} - \frac{e^{-\Lambda_\sigma r}}{r})\ , \tag{1}$$

An optimal set of values of the parametres entering this potential has been found to be

$$\frac{g_{\sigma q}^2}{4\pi} = \frac{g_{\pi q}^2}{4\pi} = 1.24, \quad \mu_\sigma = 0.60 \text{ GeV}\ , \quad \Lambda_\sigma = 0.83 \text{ GeV}\ . \tag{2}$$

As one can see from Fig. 1, with these values the theoretical phase shift for $^1S_0$ gets quite close to the experimental points without altering the good short-range behaviour, and in particular the change of sign of the phase shift at $E_{lab} \approx 260$

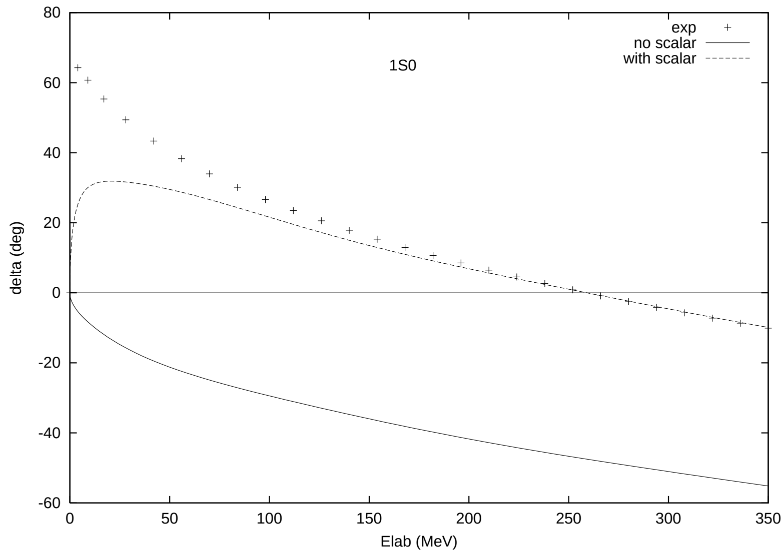

Fig. 1. The $^1S_0$ NN scattering phase shift obtained in the GBE model as a function of E_{lab}. The solid line is without and the dashed line with the σ-meson exchange potential between quarks with $\mu_\sigma = 0.60$ GeV and $\Lambda_\sigma = 0.83$ GeV. Experimental data are from Ref. [12].

MeV. Thus the addition of a σ-meson exchange interaction alone leads to a good description of the phase shift in a large energy interval. One can argue that the still existing discrepancy at low energies could possibly be removed by the coupling of the $^5D_0$ N-Δ channel. To achieve this coupling, as well as to describe the $^3S_1$ phase shift, the introduction of a tensor interaction is necessary.

References

1. L.Ya. Glozman and D.O. Riska, Phys. Rep. **268**, 263 (1996)
2. L.Ya. Glozman, Z. Papp, W. Plessas, K. Varga and R. Wagenbrunn, Nucl.Phys. **A623** (1997) 90c
3. D. Bartz and Fl. Stancu, e-print nucl-th/0009010
4. D. Bartz and Fl. Stancu, e-print hep-ph/0006012
5. M. Oka and K. Yazaki, Phys. Lett. **90B** 41 (1980); Progr. Theor. Phys **66** 556 (1981); ibid **66** 572 (1981).
6. M. Harvey, Nucl. Phys. **A352** (1981) 301; **A481** (1988) 834.
7. M. Cvetic, B. Golli, N. Mankoc-Borstnik and M. Rosina, Nucl. Phys. **A395** (1983) 349
8. D. O. Riska and G. E. Brown, Nucl. Phys. **A653** (1999) 251
9. Fl. Stancu, S. Pepin and L. Ya. Glozman, Phys. Rev. **C56** (1997) 2779; **C59** (1999) 1219 (erratum).
10. D. Bartz, Fl. Stancu, Phys. Rev. **C59** (1999) 1756.
11. D. Bartz and Fl. Stancu, Phys. Rev. **C60** (1999) 055207
12. V. G. J. Stoks, R. A. M. Klomp, M. C. M. Rentmeester and J. J. de Swart, Phys. Rev. **C48** (1993) 792; V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen and J. J. de Swart, Phys. Rev. **C49** (1994) 2950.

BLED WORKSHOPS
IN PHYSICS
VOL. 1, NO. 1

Proceedins of the Mini-Workshop
Few-Quark Problems *(p. 85)*
Bled, Slovenia, July 8-15, 2000

Description of nucleon excitations as decaying states

Bojan Golli*

Faculty of Education, University of Ljubljana, and J. Stefan Institute, Ljubljana, Slovenia

Abstract. Two methods to describe excited states of baryons as decaying states are presented: the Analytic Continuation in Coupling Constant and the Kohn variational principle for the K-matrix. The methods are applied to a simple model of the Δ resonance consisting of the pion coupled to three valence quarks.

The work has been done in collaboration with Vladimir Kukulin and Simon Širca.

1 Motivation

Baryons are usually computed as bound states neglecting possible decay channels. The inclusion of strongly decaying channels may considerably influence the position of the state as well as some other properties. The aim of the present work is to estimate this effect in a simplified model and to discuss two possible approaches to describe decaying states. The methods determine the position and the width of the resonance, and furthermore, provide a suitable tool to calculate new observables, which cannot be obtained in a bound state calculation, such as non-resonant contributions to production amplitudes. In this work we shall focus on the decay of the Δ resonance.

2 The model

The decay of the Δ resonance into the nucleon and the pion is most naturally described in models with chiral symmetry, such as the linear σ model (LSM), the chromodielectric model (CDM), the cloudy bag model CBM, etc. Here we use a simplified model which contains the main features of these models. It assumes frozen quark profiles and neglects meson-self interaction. Furthermore, it does not take into account additional scalar fields (sigma mesons in the LSM, chromodielectric field and sigma mesons in the CDM, or the bag potential in the CBM) since their main role is to fix the quark profiles and generate a constant energy shift for all baryons. In the present calculation, the quarks profiles are taken over from the ground state calculation in the LSM[1]. We know that the profiles do not change considerably from one model to the other, so this is not a very severe restriction. The inclusion of meson self-interaction may, however, more importantly alter the results.

* E-mail: Bojan.Golli@ijs.si

For the quark-pion interaction we assume the usual pseudoscalar form:

$$H_{\mathrm{quark-meson}} = ig \int d\mathbf{r}^3 \bar{q}\boldsymbol{\tau} \cdot \hat{\boldsymbol{\pi}}\gamma_5 q \ . \tag{1}$$

In models with spontaneous symmetry breaking, such as the LSM, the parameter g is related to the 'constituent' quark mass by $M_q = gf_\pi$. From 350 MeV$<$ M_q $<$450 MeV we estimate that physically sensible values for g are $4 < g < 5$.

The model is usually solved at the mean field level. We interpret the solution as a coherent state of pions around the three quark core, and generate physical N and Δ states by the Peierls Yoccoz projection of good spin and isospin. The resulting states are interpreted as a superposition of 3 bare quarks plus 3 quarks with one or more pions coupled, respectively, to nucleon or Δ quantum numbers:

$$\begin{aligned} |\Phi_N\rangle &= P^{J=\frac{1}{2},T=\frac{1}{2}}|\Phi\rangle \\ &= (3q)_N + [(3q)_N\pi]^{J=\frac{1}{2},T=\frac{1}{2}} + [(3q)_\Delta\pi]^{\frac{1}{2},\frac{1}{2}} + [(3q)_N\pi\pi]^{\frac{1}{2},\frac{1}{2}} + \ldots \end{aligned} \tag{2}$$

$$\begin{aligned} |\Phi_\Delta\rangle &= P^{J=\frac{3}{2},T=\frac{3}{2}}|\Phi\rangle \\ &= (3q)_\Delta + [(3q)_N\pi]^{J=\frac{3}{2},T=\frac{3}{2}} + [(3q)_\Delta\pi]^{\frac{3}{2},\frac{3}{2}} + [(3q)_N\pi\pi]^{\frac{3}{2},\frac{3}{2}} + \ldots \end{aligned} \tag{3}$$

In the Δ channel, the probability of finding one or more pions is higher than in the N channel; as a consequence the Δ lies higher then the nucleon. In the simplified model we obtain $E_\Delta - E_N = 84$ MeV and 126 MeV for $g = 4.3$ and 5 respectively; including meson self interaction and performing self-consistent calculation increases the splitting by some 40 MeV. Hence, the ΔN splitting due to pions is only roughly one half of the experimental one; an additional hyperfine interaction is needed to bring $E_\Delta - E_N$ to the experimental value (293 MeV). In our simple model we therefore introduce a *phenomenological* form of the interaction:

$$H' = \varepsilon P_{(3q)_\Delta} \tag{4}$$

where $P_{(3q)_\Delta}$ is the projector onto components containing 3 quarks coupled to Δ quantum numbers. Using $\varepsilon = 262$ MeV and 235 MeV for $g = 4.3$ and 5 respectively, increases the splitting to the desired value.

3 The Kohn variational principle for the phase shift

The ansatz for the Δ resonance is taken in the form

$$|\Psi_\Delta\rangle = c\,|\Phi_\Delta\rangle + \int dk\,\eta(k_0,k)\,\left[a^\dagger_{mt}(k)|\Phi_N\rangle\right]^{J=\frac{3}{2},T=\frac{3}{2}}$$

where $a^\dagger_{mt}(k)$ creates a p-wave pion, m, t are the third components of its spin and isospin, k_0 denotes the pion momentum, while $|\Phi_N\rangle$ and $|\Phi_\Delta\rangle$ correspond to the nucleon and the Δ bound states ((2) and (3)), respectively. Asymptotically, the pion state behaves as

$$\eta(k_0, r) = k_0\, j_1(k_0 r) - \tan\delta\, k_0\, y_1(k_0 r)\ , \quad r \to \infty\ .$$

Here we use standing waves to describe the pion rather than outgoing (and incoming) waves. In k-space this leads to

$$\eta(k_0, k) = \sqrt{\frac{\pi}{2}}\,\delta(k - k_0) + \frac{\chi(k_0, k)}{\omega_k - \omega_0}\,, \quad \tan\delta = \sqrt{2\pi}\,\frac{\omega_0}{k_0}\,\chi(k_0, k_0)$$

The variational principle requires that the Kohn functional[2]

$$\mathcal{F}_K = \tan\delta - \frac{2\omega_0}{k_0\langle\Phi_N|\Phi_N\rangle}\,\langle\Psi_\Delta|H - E|\Psi_\Delta\rangle$$

remains stationary with respect to variation of c and $\chi(k_0, k)$, as well as to variation of the intrinsic pion profile in $|\Psi_\Delta\rangle$.

In the above form only one channel is assumed; if more than one channel is open, $\tan\delta$ is replaced by the K matrix.

Typical results for the phase shift are displayed in Fig. 1 and compared to the experimental values. By varying ε it is possible to reproduce the experimental position of the resonance; using $g = 4.3$ ($\varepsilon = 273$ MeV) the width (i.e. the slope of the curve) is well reproduced while for $g = 5$ ($\varepsilon = 252$ MeV) the width is too large. These results are obtain by optimizing $|\Psi_\Delta\rangle$; if we do not vary the intrinsic pion profile but take it over from the bound state calculation the results change only very slightly provided the value of ε is changed by a few MeV. Hence, the properties of the Δ do not change significantly when the decay channel is open; the main effect is that the energy drops by some 10 MeV (10 MeV for $g = 4.3$ and 13 MeV for $g = 5$).

4 The Analytic Continuation in Coupling Constant

Consider the scattering of a non-relativistic particle on an attractive potential $V(r)$ which possesses a quasi bound state in the continuum. Introduce a parameter (coupling constant) λ:

$$H = H_{\text{kin}} + \lambda V(r)\,.$$

For sufficiently large λ, $\lambda > 1$ the state becomes bound. Let's denote the threshold value as λ_{th}. The method [3] is based on the fact that it much easier to solve the bound state problem than the continuum case. It consists of the following steps:

- Determine λ_{th} and calculate E as a function of λ for $\lambda > \lambda_{\text{th}}$.
- Introduce a variable $x = \sqrt{\lambda - \lambda_{\text{th}}}$; calculate $k(x) = i\sqrt{-2mE}$ in the bound state region.
- Fit $k(x)$ by a polynomial:

$$k(x) = i(c_0 + c_1 x + c_2 x^2 + \ldots + c_{2M} x^{2M})\,.$$

- Construct a Padé approximant:

$$k(x) = i\,\frac{a_0 + a_1 x + \ldots + a_M x^M}{1 + b_1 x + \ldots + b_M x^M}\,. \tag{5}$$

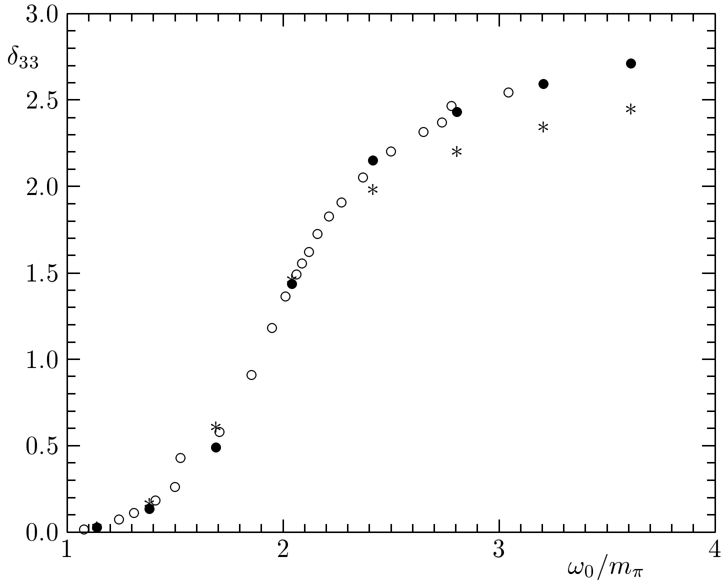

Fig. 1. The phase shift in the P33 channel: ∘ are the experimental values, • values from the variational calculation using $g = 4.3$ and $\varepsilon = 273$ MeV, and $*$ those for $g = 5$ and $\varepsilon = 253$ MeV.

- Analytically continue $k(x)$ to the region $\lambda < \lambda_{\text{th}}$ (i.e. to imaginary x) where $k(x)$ becomes complex.
- Determine the position and the width of the resonance as analytic continuation in λ:

$$E_r = \frac{1}{2m}\,\mathrm{Re}\,\mathrm{cont}_{\lambda\to 1} k^2\ , \qquad \Gamma = -2\,\frac{1}{2m}\,\mathrm{Im}\,\mathrm{cont}_{\lambda\to 1} k^2\ . \tag{6}$$

This method does not provide only the position and the width of the resonance; the matrix element of an operator $\mathcal{O}$ between the resonant state $|\Psi_r\rangle$ and a bound state $|\Phi\rangle$ can be calculated as

$$\langle\Psi_r|\mathcal{O}|\Phi\rangle = \mathrm{cont}_{\lambda\to 1}\langle\Psi_b(\lambda)|\mathcal{O}|\Phi\rangle\ .$$

In our implementation of the method, we relate the coupling constant λ to the parameter of the phenomenological hyperfine interaction:

$$\lambda V(r) \to \varepsilon P_{(3q)_\Delta}\ , \qquad x = \sqrt{\varepsilon_{th} - \varepsilon} \tag{7}$$

where ε_{th} is the value of ε at the threshold: $E_\Delta(\varepsilon_{th}) - E_N = m_\pi$. For sufficiently high ε, the real part of the energy eventually reaches the experimental position of the resonance; this value of ε then corresponds to $\lambda = 1$ of the original formulation of the method.

In our very preliminary calculation we treat the pion non-relativistically. For $\varepsilon < \varepsilon_{th}$ we calculate

$$k(x) = i\sqrt{2m_\pi(E_{th} - E)}, \quad E = E_\Delta(x) - E_N\ ,$$

fit $k(x)$ using a Padé approximant (5) and continue $k(x)$ to the resonance region. The energy difference, $E_\Delta - E_N$, and the width of the resonance are then obtained by (6). The 'physical value' of x (and ε from (7)) is determined as $\mathrm{Re}E(= E_\Delta - E_N)$ reaches the experimental value 293 MeV. The corresponding value of $\mathrm{Im}E(= \Gamma)$ then predicts the width of Δ and is to be compared with the experimental value ~ 120 MeV.

Fig. 2 shows the behaviour of $E_\Delta - E_N$ and Γ as functions of x for two vales of g. For higher order of the Padé approximant, $M \geq 3$, the method becomes numerically instable and the determination of E and Γ is no more reliable. For $g = 4.3$ and $M = 1$ and 2, the experimental splitting is reached for $x^2 \approx 230$ MeV (and corresponding $\varepsilon = 300$ MeV). This yields $\Gamma \approx 60$ MeV which is only half of the experimental value, most probably due to the non-relativistic treatment. For $g = 5$ the value of Γ is larger (in accordance with Fig. 1) but its determination is less reliable.

In order to be able predict reliable results it is necessary formulate the approach relativistically and to understand the origin of numerical instabilities for higher M.

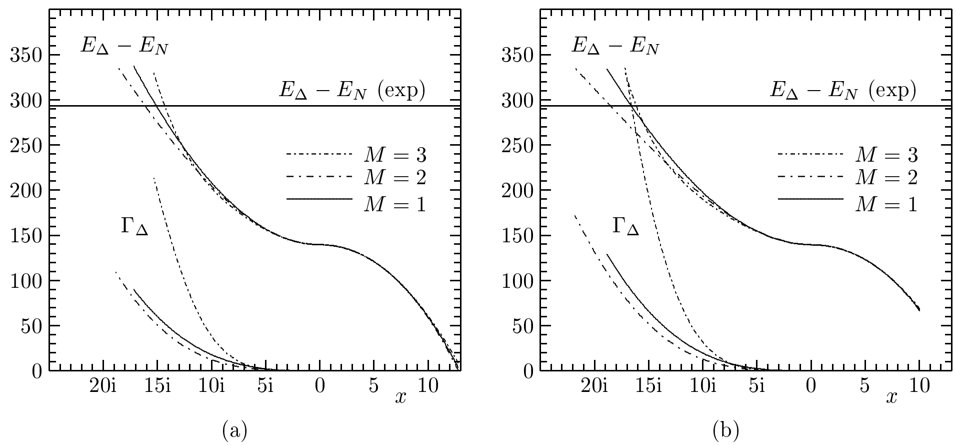

Fig. 2. ΔN splitting and Δ width (in MeV) as functions of x (in units $\sqrt{\mathrm{MeV}}$) for $g = 4.3$ (a), and $g = 5$ (b).

References

1. B. Golli and M. Rosina, Phys. Lett. B 165 (1985) 347; M. C. Birse, Phys. Rev. D 33 (1986) 1934.
2. B. Golli, M. Rosina, J. da Providência, Nucl. Phys. **A436** (1985) 733
3. V.M. Krasnopolsky and V.I.Kukulin, *Phys. Lett*, **69A** (1978) 251, V.M. Krasnopolsky and V.I.Kukulin, *Phys. Lett*, **96B** (1980) 4, N. Tanaka et al. *Phys. Rev.* **C59** (1999) 1391