Description of nucleon excitations as decaying states *

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Abstract. Two methods to describe excited states of baryons as decaying states are presented: the Analytic Continuation in Coupling Constant and the Kohn variational principle for the K-matrix. The methods are applied to a simple model of the Δ resonance consisting of the pion coupled to three valence quarks.

The work has been done in collaboration with Vladimir Kukulin and Simon Širca.

1 Motivation

Baryons are usually computed as bound states neglecting possible decay channels. The inclusion of strongly decaying channels may considerably influence the position of the state as well as some other properties. The aim of the present work is to estimate this effect in a simplified model and to discuss two possible approaches to describe decaying states. The methods determine the position and the width of the resonance, and furthermore, provide a suitable tool to calculate new observables, which cannot be obtained in a bound state calculation, such as non-resonant contributions to production amplitudes. In this work we shall focus on the decay of the Δ resonance.

2 The model

The decay of the Δ resonance into the nucleon and the pion is most naturally described in models with chiral symmetry, such as the linear σ model (LSM), the chromodielectric model (CDM), the cloudy bag model CBM, etc. Here we use a simplified model which contains the main features of these models. It assumes frozen quark profiles and neglects meson-self interaction. Furthermore, it does not take into account additional scalar fields (sigma mesons in the LSM, chromodielectric field and sigma mesons in the CDM, or the bag potential in the CBM) since their main role is to fix the quark profiles and generate a constant energy shift for all baryons. In the present calculation, the quarks profiles are taken over from the ground state calculation in the LSM[1]. We know that the profiles do not change considerably from one model to the other, so this is not a very severe restriction. The inclusion of meson self-interaction may, however, more importantly alter the results.

For the quark-pion interaction we assume the usual pseudoscalar form:

$$H_{\text{quark-meson}} = \mathrm{i}g \int \mathrm{d}\boldsymbol{r}^3 \bar{q} \boldsymbol{\tau} \cdot \hat{\boldsymbol{\pi}} \gamma_5 q \; . \tag{1}$$

In models with spontaneous symmetry breaking, such as the LSM, the parameter g is related to the 'constituent' quark mass by $M_q = g f_\pi$. From 350 MeV< M_q <450 MeV we estimate that physically sensible values for g are $4 < g < 5$.

The model is usually solved at the mean field level. We interpret the solution as a coherent state of pions around the three quark core, and generate physical N and Δ states by the Peierls Yoccoz projection of good spin and isospin. The resulting states are interpreted as a superposition

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of 3 bare quarks plus 3 quarks with one or more pions coupled, respectively, to nucleon or Δ quantum numbers:

$$|\Phi_N\rangle = P^{J=\frac{1}{2},T=\frac{1}{2}}|\Phi\rangle = (3q)_N + [(3q)_N\pi]^{J=\frac{1}{2},T=\frac{1}{2}} + [(3q)_\Delta\pi]^{\frac{1}{2},\frac{1}{2}} + [(3q)_N\pi\pi]^{\frac{1}{2},\frac{1}{2}} + \ldots \quad (2)$$

$$|\Phi_\Delta\rangle = P^{J=\frac{3}{2},T=\frac{3}{2}}|\Phi\rangle = (3q)_\Delta + [(3q)_N\pi]^{J=\frac{3}{2},T=\frac{3}{2}} + [(3q)_\Delta\pi]^{\frac{3}{2},\frac{3}{2}} + [(3q)_N\pi\pi]^{\frac{3}{2},\frac{3}{2}} + \ldots \quad (3)$$

In the Δ channel, the probability of finding one or more pions is higher than in the N channel; as a consequence the Δ lies higher then the nucleon. In the simplified model we obtain $E_\Delta - E_N =$ 84 MeV and 126 MeV for $g = 4.3$ and 5 respectively; including meson self interaction and performing self-consistent calculation increases the splitting by some 40 MeV. Hence, the ΔN splitting due to pions is only roughly one half of the experimental one; an additional hyperfine interaction is needed to bring $E_\Delta - E_N$ to the experimental value (293 MeV). In our simple model we therefore introduce a *phenomenological* form of the interaction:

$$H' = \varepsilon P_{(3q)_\Delta} \quad (4)$$

where $P_{(3q)_\Delta}$ is the projector onto components containing 3 quarks coupled to Δ quantum numbers. Using $\varepsilon = 262$ MeV and 235 MeV for $g = 4.3$ and 5 respectively, increases the splitting to the desired value.

3 The Kohn variational principle for the phase shift

The ansatz for the Δ resonance is taken in the form

$$|\Psi_\Delta\rangle = c\,|\Phi_\Delta\rangle + \int \mathrm{d}k\,\eta(k_0,k)\,\left[a^\dagger_{mt}(k)|\Phi_N\rangle\right]^{J=\frac{3}{2},T=\frac{3}{2}}$$

where $a^\dagger_{mt}(k)$ creates a p-wave pion, m, t are the third components of its spin and isospin, k_0 denotes the pion momentum, while $|\Phi_N\rangle$ and $|\Phi_\Delta\rangle$ correspond to the nucleon and the Δ bound states ((2) and (3)), respectively. Asymptotically, the pion state behaves as

$$\eta(k_0,r) = k_0\,j_1(k_0 r) - \tan\delta\,k_0\,y_1(k_0 r)\ , \quad r \to \infty\ .$$

Here we use standing waves to describe the pion rather than outgoing (and incoming) waves. In k-space this leads to

$$\eta(k_0,k) = \sqrt{\frac{\pi}{2}}\,\delta(k-k_0) + \frac{\chi(k_0,k)}{\omega_k - \omega_0}\ , \quad \tan\delta = \sqrt{2\pi}\,\frac{\omega_0}{k_0}\,\chi(k_0,k_0)$$

The variational principle requires that the Kohn functional[2]

$$\mathcal{F}_K = \tan\delta - \frac{2\omega_0}{k_0\langle\Phi_N|\Phi_N\rangle}\,\langle\Psi_\Delta|H - E|\Psi_\Delta\rangle$$

remains stationary with respect to variation of c and $\chi(k_0,k)$, as well as to variation of the intrinsic pion profile in $|\Psi_\Delta\rangle$.

In the above form only one channel is assumed; if more than one channel is open, $\tan\delta$ is replaced by the K matrix.

Typical results for the phase shift are displayed in Fig. 1 and compared to the experimental values. By varying ε it is possible to reproduce the experimental position of the resonance; using $g = 4.3$ ($\varepsilon = 273$ MeV) the width (i.e. the slope of the curve) is well reproduced while for $g = 5$ ($\varepsilon = 252$ MeV) the width is too large. These results are obtain by optimizing $|\Psi_\Delta\rangle$; if we do not vary the intrinsic pion profile but take it over from the bound state calculation the results change only very slightly provided the value of ε is changed by a few MeV. Hence, the properties of the Δ do not change significantly when the decay channel is open; the main effect is that the energy drops by some 10 MeV (10 MeV for $g = 4.3$ and 13 MeV for $g = 5$).

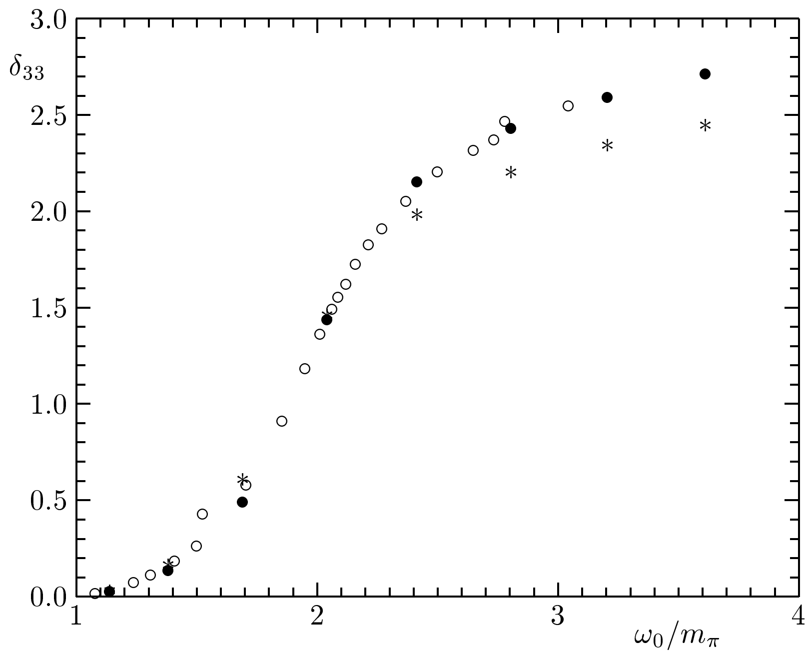

Fig. 1. The phase shift in the P33 channel: ∘ are the experimental values, • values from the variational calculation using $g = 4.3$ and $\varepsilon = 273$ MeV, and $*$ those for $g = 5$ and $\varepsilon = 253$ MeV.

4 The Analytic Continuation in Coupling Constant

Consider the scattering of a non-relativistic particle on an attractive potential $V(r)$ which possesses a quasi bound state in the continuum. Introduce a parameter (coupling constant) λ:

$$H = H_{\text{kin}} + \lambda V(r) \ .$$

For sufficiently large λ, $\lambda > 1$ the state becomes bound. Let's denote the threshold value as λ_{th}. The method [3] is based on the fact that it much easier to solve the bound state problem than the continuum case. It consists of the following steps:

- Determine λ_{th} and calculate E as a function of λ for $\lambda > \lambda_{\text{th}}$.
- Introduce a variable $x = \sqrt{\lambda - \lambda_{\text{th}}}$; calculate $k(x) = \mathrm{i}\sqrt{-2mE}$ in the bound state region.
- Fit $k(x)$ by a polynomial:

$$k(x) = \mathrm{i}(c_0 + c_1 x + c_2 x^2 + \ldots + c_{2M} x^{2M}) \ .$$

- Construct a Padé approximant:

$$k(x) = \mathrm{i}\, \frac{a_0 + a_1 x + \ldots + a_M x^M}{1 + b_1 x + \ldots + b_M x^M} \ . \tag{5}$$

- Analytically continue $k(x)$ to the region $\lambda < \lambda_{\text{th}}$ (i.e. to imaginary x) where $k(x)$ becomes complex.
- Determine the position and the width of the resonance as analytic continuation in λ:

$$E_r = \frac{1}{2m}\, \mathrm{Re}\, \mathrm{cont}_{\lambda\to 1} k^2 \ , \qquad \Gamma = -2\, \frac{1}{2m}\, \mathrm{Im}\, \mathrm{cont}_{\lambda\to 1} k^2 \ . \tag{6}$$

This method does not provide only the position and the width of the resonance; the matrix element of an operator $\mathcal{O}$ between the resonant state $|\Psi_r\rangle$ and a bound state $|\Phi\rangle$ can be calculated as

$$\langle \Psi_r | \mathcal{O} | \Phi \rangle = \mathrm{cont}_{\lambda\to 1} \langle \Psi_b(\lambda) | \mathcal{O} | \Phi \rangle \ .$$

In our implementation of the method, we relate the coupling constant λ to the parameter of the phenomenological hyperfine interaction:

$$\lambda V(r) \to \varepsilon P_{(3q)_\Delta}\ , \qquad x = \sqrt{\varepsilon_{th} - \varepsilon} \tag{7}$$

where ε_{th} is the value of ε at the threshold: $E_\Delta(\varepsilon_{th}) - E_N = m_\pi$. For sufficiently high ε, the real part of the energy eventually reaches the experimental position of the resonance; this value of ε then corresponds to $\lambda = 1$ of the original formulation of the method.

In our very preliminary calculation we treat the pion non-relativistically. For $\varepsilon < \varepsilon_{th}$ we calculate

$$k(x) = \mathrm{i}\sqrt{2m_\pi(E_{th} - E)}, \quad E = E_\Delta(x) - E_N\ ,$$

fit $k(x)$ using a Padé approximant (5) and continue $k(x)$ to the resonance region. The energy difference, $E_\Delta - E_N$, and the width of the resonance are then obtained by (6). The 'physical value' of x (and ε from (7)) is determined as $\mathrm{Re}E(= E_\Delta - E_N)$ reaches the experimental value 293 MeV. The corresponding value of $\mathrm{Im}E(= \Gamma)$ then predicts the width of Δ and is to be compared with the experimental value ~ 120 MeV.

Fig. 2 shows the behaviour of $E_\Delta - E_N$ and Γ as functions of x for two vales of g. For higher order of the Padé approximant, $M \geq 3$, the method becomes numerically instable and the determination of E and Γ is no more reliable. For $g = 4.3$ and $M = 1$ and 2, the experimental splitting is reached for $x^2 \approx 230$ MeV (and corresponding $\varepsilon = 300$ MeV). This yields $\Gamma \approx 60$ MeV which is only half of the experimental value, most probably due to the non-relativistic treatment. For $g = 5$ the value of Γ is larger (in accordance with Fig. 1) but its determination is less reliable.

In order to be able predict reliable results it is necessary formulate the approach relativistically and to understand the origin of numerical instabilities for higher M.

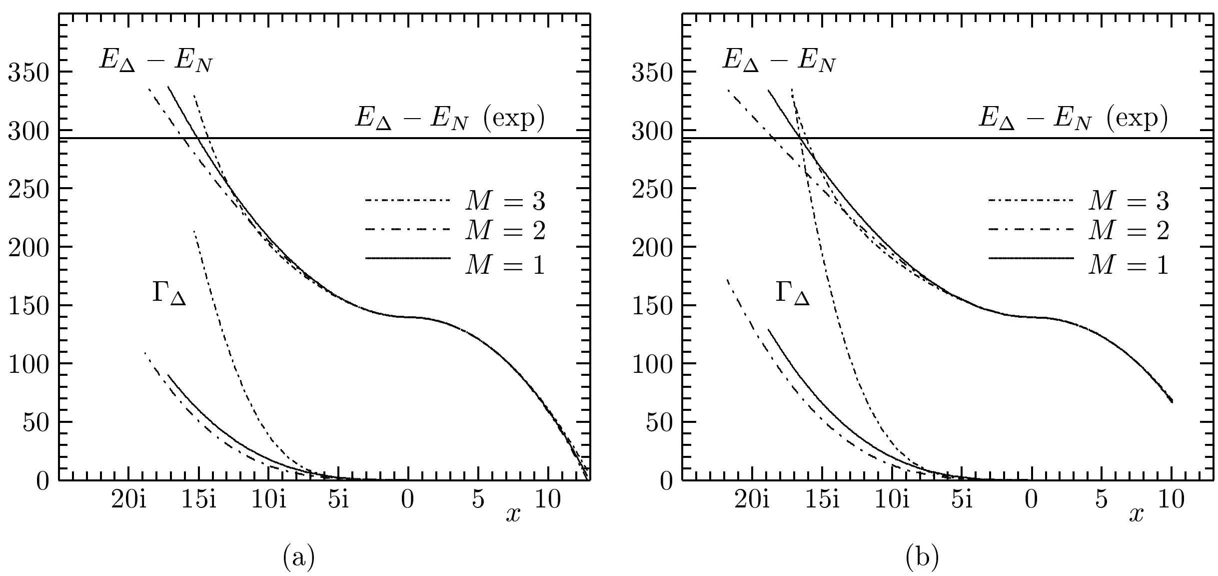

Fig. 2. ΔN splitting and Δ width (in MeV) as functions of x (in units $\sqrt{\mathrm{MeV}}$) for $g = 4.3$ (a), and $g = 5$ (b).

References

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