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The constituent quark as a soliton in chiral quark models

Bojan Golli

Faculty of Education, University of Ljubljana, 1000 Ljubljana, Slovenia and Jožef Stefan Institute, 1000 Ljubljana, Slovenia

Abstract. We discuss the possibility that the soliton carrying the baryon number 1/3, obtained in the linear σ-model and in the Nambu – Jona-Lasinio model can be identified with the constituent quark. In the linear σ-model we have derived meson exchange potentials between two solitons which turn out to resemble potentials used in constituent quark models.

The mechanism in which a nonstrange constituent quark acquires its mass $\sim 350-400$ MeV is phenomenologically described via spontaneous breaking of chiral symmetry. Yet, such a structureless particle does not agree with a picture of the constituent quark as an extended object in which the quark is surrounded by a cloud of quark-antiquark (meson) and gluon excitations. The fact that the scale for chiral symmetry breaking appears at lower energies than the confinement scale supports a model in which the constituent quark is represented by a current quark surrounded by a chiral field rather than a gluon field, as first suggested by Georgi and Manohar [1], and further elaborated by Cheng and Li [2], and by Baumgartner, Pirner, Königsmann and Povh [3] (see also the contribution of M. Rosina in these Proceedings [4]).

One of the simplest models describing the spontaneous breaking of chiral symmetry is the linear σ-model (LSM). In the non-strange sector it involves u and d quarks, a triplet of pions and the σ-meson [5–8]. The model possesses, for sufficiently strong pion-quark coupling constant g, soliton solutions obtained by putting three quarks in the lowest 1s orbit and allowing for nonzero pion field around the quark source. Below the critical coupling constant only free Dirac particles of mass $M = gf_\pi$ exist, f_π being the pion decay constant. We found [9] another type of non-trivial solutions by putting only one quark in the lowest orbit which we identified with the constituent quark.

In figure 1a) the energy of such a quark soliton is displayed as a function of $M = gf_\pi$. For comparison, the energy of the three quark soliton (the nucleon soliton) divided by 3 is also shown. The three quarks in the nucleon soliton generate a stronger chiral field than in the single quark soliton and the resulting attractive potential lowers more substantially the energy of the valence orbit ($\varepsilon_{\rm val}$) producing a large gap between the two solutions. The energy of the quark soliton is higher than the popular value of the constituent quark mass. In our calculation

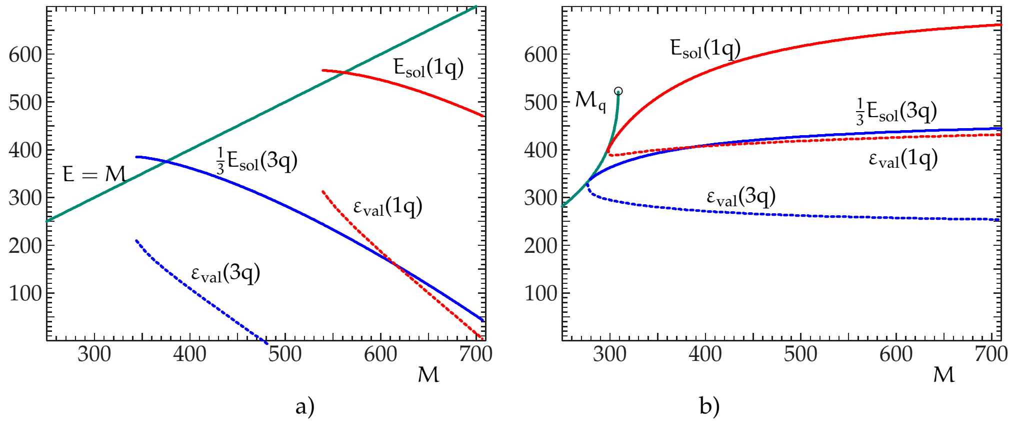

Fig. 1. (a) (b)

we do project the solution onto the subspace of good angular momentum and isospin, but we do not perform the projection onto the states with good linear momentum. The solution is thus interpreted as a wave packet of states with good linear momenta; projecting out the zero-momentum state would further lower the energy of the soliton. Let us notice that for both solutions the valence orbit sinks into the Dirac sea at a sufficiently large coupling constant producing a topological soliton (Skyrmion).

From our solution it is possible to derive a potential between two quark solitons in the framework of the Born Oppenheimer approximation. The interesting part of the interaction is the pion exchange potential. The pion field around the quark soliton with good angular momentum and isospin can be written in the form:

$$\vec{\pi}_b(\mathbf{r}) = \frac{1}{3}\pi(r)\,\hat{\mathbf{r}}\cdot\mathbf{\Sigma}_b\,\vec{T}_b\ , \tag{1}$$

where $\mathbf{\Sigma}$ and $\vec{T}$ act on spin and isospin of the quark soliton, respectively. To obtain the potential between two such solitons, one at the origin and the other one at position $\mathbf{r}$, we evaluate the quark-meson and the meson-meson interaction for such a configuration. For the quark-pion interaction we obtain

$$V_\pi^q(r) = \frac{2g}{3}\langle Q|\sigma_0\tau_0|Q\rangle \int d^3\mathbf{r}'u(r')v(r')\,\pi(|\mathbf{r}-\mathbf{r}'|)\,\hat{\mathbf{r}}'\cdot\mathbf{\Sigma}_a\,(\widehat{\mathbf{r}-\mathbf{r}'})\cdot\mathbf{\Sigma}_b\,\vec{T}_a\vec{T}_b\ , \tag{2}$$

where σ_0 and τ_0 now act on current quark spin and isospin. A similar expression is obtained from the meson self-interaction ("Mexican hat"). The potential (2) contains the scalar as well the tensor part. The scalar part is displayed in figure 2 and compared to a typical one-pion exchange potentials used in the constituent quark model calculations. The potential satisfies $\int dr\, r^2 V_\pi(r) = 0$, a constraint that has to be fulfilled for any pseudo-scalar exchange potentials. It has the correct asymptotic behavior leading to the appropriate form of the pion-exchange potential between two nucleons. The attractive part is too shallow and has a too large range which could be attributed to the spurious center-of-mass motion. We expect that

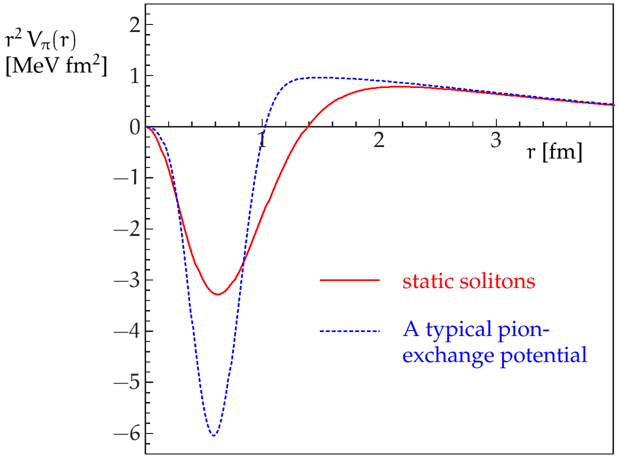

Fig. 2. The pion exchange effective potential (multiplied by r^2) between two quark solitons for $M = 560$ MeV (solid line) compared to a typical OPEP.

linear momentum projection would reduce its range and through the above integral constraint lower the depth of the attractive part, which could finally bring our prediction closer to a realistic OPEP.

Our model of the constituent quark is further supported by our finding that similar quark solitons exist also in a more fundamental chiral model, the Nambu – Jona-Lasinio (NJL) model. In this model the sigma and pion fields are related to the quark-antiquark excitations of the Dirac sea by

$$\sigma(r) = \sum_{\varepsilon_j<0} \bar{q}_j q_j \, \mathcal{R}\,, \qquad \boldsymbol{\pi}(\mathbf{r}) = \sum_{\varepsilon_j<0} \bar{q}_j i\gamma_5 \boldsymbol{\tau} q_j \, \mathcal{R}$$

where $\mathcal{R}$ denotes a regulator which is needed to regularize the ultraviolet divergences, and introduces a new parameter, the cut-off. In our calculation [10, 11] we used a version of the model in which the interaction between quarks is induced by the instantons [12] and has a finite range. The mass of the “bare” constituent quark M which is equal to gf_π in the linear σ-model is now substituted by the 4-momentum-dependent mass $M \to M\mathcal{R}(k^2)$, $k^2 = \mathbf{k}^2 - E^2$; M remains in the model as a (free) parameter measuring the strength of the σ-field in the vacuum. The pole of the quark propagator is obtained by solving the condition $k^2 + M^2\mathcal{R}(k^2) = 0$. The solution exists only below a certain value of M. Using $\mathcal{R}(k^2) = e^{-k^2/\Lambda^2}$ and fixing the cut-off parameter Λ to reproduce the pion decay constant, the critical value of M is around 300 MeV. Above this value only solutions with non-trivial values of chiral fields exist. Similarly as in the linear σ-model, putting three quarks in the valence orbit a soliton corresponding to the nucleon emerges; if we put only one quark in the valence orbit, we obtain a solitonic solution which we identify with the constituent quark.

The energies of both solutions as functions of M are displayed in figure 1b) in the same way as the analogous solutions in the linear σ-model. The energy of the “ bare” constituent quark is denoted by M_q; in contrast to the LSM this solution smoothly continues into the soliton solution. Interestingly, the solutions of both models have similar energies, however, the energies in the NJL model raise with M while those in the LSM lower. This is a consequence of the regularization of the valence orbit which is not performed in the LSM as well in other versions of the NJL model. The regularization used in our approach prevents the orbit to shrink below a certain size and thus makes the soliton absolutely stable without any further ad hoc constraint. The energy of the valence orbit remains almost constant with M and does not sink into the Dirac sea. The presence of the time variable in the regulator does not allow us to perform the exact angular and linear momentum projection which would lower the soliton energy further, and eventually bring it in the ball park of values used in the constituent quark model calculations.

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Izbrani spektroskopski rezultati kolaboracije Belle

Marko Bračko[a,b]

[a] University of Maribor, Smetanova ulica 17, SI-2000 Maribor, Slovenia
[b] Jožef Stefan Institute, Jamova cesta 39, SI-1000 Ljubljana, Slovenia

V prispevku smo poročali o izbranih rezultatih iz spektroskopskih eksperimentov, pred kratkim izvedenih s spektrometrom Belle, ki deluje na energijsko asimetričnem trkalniku elektronov in pozitronov KEKB v laboratoriju KEK, Tsukuba, Japonska.

Konstituentni kvark kot soliton v kiralnih kvarkovskih modelih

Bojan Golli

Pedagoška fakulteta, Univerza v Ljubljani, 1000 Ljubljana in Institut Jožef Stefan, 1000 Ljubljana, Slovenija

Obravnavamo možnost, da lahko soliton z barionskim številom $1/3$, dobljenim v linearnem modelu sigma in v modelu Nambuja in Jona-Lasinija, identificiramo s konstituentnim kvarkom. V linearnem modelu sigma smo izpeljali potencial med dvema solitonoma, ki je podoben potencialom, ki se uporabljajo v modelih s konstituentnimi kvarki.

Mezoni D_s s pozitivno parnostjo in Z_c^+ v kromodinamiki na mreži

Luka Leskovec

Institut Jožef Stefan, Jamova 39, 1000 Ljubljana, Slovenija

Predstavljena sta dva še posebej zanimiva kanala: D_s mezoni s pozitivno parnostjo ter eksotični hadron Z_c^+. V kanalu z D_s je bilo nekaj napetosti med eksperimentom ter teorijo, saj je eksperiment našel stanji $D_{s0}^*(2317)$ in $D_{s1}(2460)$ pod pragom za sipanje mezonov DK in D^*K, medtem ko je teorija napovedala mase teh mezonov nad tem istim pragom. V kromodinamiki na mreži smo simulirali dotični kanal tako, da smo uporabili operatorje $\bar{c}s$ ter tudi $D^{(*)}K$. Upoštevajoč pojave na pragu sipanja smo izločili mase mezonov D_s s pozitivno parnostjo, ki se nahajajo pod pragom za sipanje in se v okviru napak ujemajo z eksperimentalnimi. Simulirali pa smo tudi eksotični kanal v katerem se nahaja Z_c^+. Uporabili smo vse relevantne dvomezonske sipalne operatorje $J/\psi\pi$,$\eta_c\rho$, DD^*, $\psi(2S)\pi$, D^*D^*, $\psi(3770)\pi$, $\psi_{3--}\pi$, kot tudi dodatne operatorje tipa dikvark anti-dikvark. Identificirali smo vse diskretne energijske nivoje, a nismo našli prepoznavnega kandidata za Z_c^+.