INOVATIVE SOLUTION PRINCIPLES OF WAVE PROBLEMS IN HORIZONTALY LAYERED MEDIAS

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Abstract

The paper represents engineeringly reasonable transformation of surface displacements of horizontaly layered half-space. The latter shows in the half-space present types of waves. It is shown that surface waves are expressed through residues in poles of the integrand and the volume waves are expressed as integrals along corresponding branch cuts. The singularity which always appears in the basic singular solution in elastodynamics is in this case exactly excluded. In the second part of the paper the appearance and behaviour of Stonely waves is investigated in greater detail. It is shown that in the case of layers of finite thickness their appearance and velocities depends not only on the material characteristics of neighbouring layers but also on their thickness.

Keywords

horizontaly layered half-space, volume waves, surface waves, Stonely waves, Green's function

1 INTRODUCTION

Waves, which are generated in the source of vibrations due to natural (f. ex. earthquakes), technical (f. ex. railway traffic, pilot driving) and man-made agitations, are propagated through the soil and disturb the functioning of sensitive instruments, cause human discomfort, and can possibly lead to structural damage. Investigating wave propagation is one of the main interests of applied mechanics and civil engineering because of its importance for dynamic structure-soil interaction, foundation engineering, seismology, and geophysical methods for determining the structure of foundation soils. As a rule, the soil is modeled as a half space in all problems displaying local soil characteristics or soil movement in the immediate vicinity of the selected point. The problematic nature of soil propagation in three-dimensional homogeneous and mainly layered half-space mainly has aroused significant scientific attention.

Knowledge of soil propagation in a layered half-space is thus of key importance and has always attracted much scientific attention. Bromwich [1] was the first to investigate wave propagation in half-continuous solid medium covered with a solid layer of continuous thickness. His work, which refers to standing waves with wave lengths greater than the thickness of the top layer, was continued by Love [2] who also investigated waves with equal or smaller lengths than the thickness of the top layer. Sezawa [3] studied the dispersion of elastic waves which propagate on the surface of layered bodies and on curved surfaces, whilst Thomson [4] investigated the transmission of elastic waves through a layered solid medium. Dispersion of surface waves in a multi-layered medium was studies by Haskell [5]. The classical work by Ewing, Jardetzky and Press [6], which also summarized contributions of several other authors, must also be mentioned in this review.

The authors of this contribution have chosen Green's function as a starting point for wave propagation in horizontally layered half-space. They [7] first formulated Green's function for the elastic layer loaded on the surface with concentrated harmonic force of a general direction. Then, by taking into account the findings of Kobayashi [8], they expressed Green's function of a half-space, i. e. a layer of continuous thickness described with integrals for a half-continuous integration field, with integrals along suitably chosen branch cuts and residues in the poles of integrands. The solutions described in this article lead to the exact solution of Green's function of a horizontally layered half-space described in the article [9].

It results from the above quoted works that it is possible to express the displacements on the surface of a horizontally layered half-space as a sum of products of integrals and corner functions due to a concentrated time-dependent force of general direction acting on the surface. The concentrated force of general direction can namely be presented with two components, i. e. a vertical and a horizontal one. The fact that the solution of integrals using discreet Fourier-Bessel transformation does not provide satisfactory accuracy and convergence near the source point especially motivated the authors to adopt an alternative three-step approach to the solution of the integrals. As shown in this article, after the adoption of three steps, the integrals in the form that does not observe the singularity where it exists are written as integrals along branch cuts and residues in the poles of the integrand. This confirms the facts originating in the theory of elastodynamics, namely that it is possible to express surface waves in an elastic medium with poles and that volume waves are given with integrals along nodal cuts.

This approach has opened completely new possibilities to study surface waves. This article presents the results of the new approach.

In passing from a homogenous half-space to a horizontally layered half-space, Stonely waves can be generated besides Rayleigh waves. The magnitude of the system of equations for determining integration constants quickly increases with the increase of the number of layers, which is evident from the article by [9]. This also means that Stonely poles can occur beside Rayleigh poles in solutions. It is thus reasonable to investigate the phenomenon of singularity in the integrand more in detail. In the continuation, the article will show at which ratios of thickness and shear modulus of two adjoining layers Stonely waves occur and how the thickness of layers affects the speed of Stonely and Rayleigh waves.

2 THREE STEP SOLUTION

The thesis that it is engineeringly reasonable to transform surface displacements of a layered half-space into the from which is easy to calculate and which clearly shows wave types present in the half-space, is emphasized in the works [10], [11], [9]. For an axis-symmetrical example of vertically concentrated load [9] exerted on the surface of a top layer, as well as for a tangentially loaded layered half-space [11], displacements are expressed as a sum of integrals:

$$I(r)=\int_0^\infty F(\eta)\cdot J_n(\eta\cdot r)\cdot d\eta \qquad (1)$$

multiplied with adequate trigonometric functions with a circumferential coordinate as an argument. In these integrals, J is the Bessel function of the first kind of order and n is the order of the function which can adopt the values of 0, 1 and 2. It is known from scientific literature that the evaluation of these integrals with discrete Fourier-Bessel transformation does not lead to required accuracy and stability, especially when the integrals become singular at $r\rightarrow 0$. Mathematical analysis and mechanical understanding of the problem result in an alternative approach to the evaluation of the integrals shown in [11]. The authors evaluated the integrals of type (1) in three steps using the so-called method of the »expected shape«. The first step is the extraction of singularity. In an innovative way and observing the fact that

$$\lim_{r\to 0} I(r)=\lim_{r\to 0}\int_0^\infty F(\eta)\cdot J_0(\eta\cdot r)\cdot d\eta =$$

$$=\int_0^\infty \lim_{\eta\to\infty} F(\eta)\cdot J_0(\eta\cdot r)\cdot d\eta = C\cdot\int_0^\infty J_0(\eta\cdot r)\cdot d\eta=\frac{C}{r} \qquad (2)$$

the integrals which contain singularity are divided into two parts: a regular integral and a singular integral:

$$I(r)=\frac{1}{r}\cdot C+I_1(r)\ , \qquad (2a)$$

where $I_1(r)$ is:

$$I_1(r)=\int_0^\infty\left[F(\eta)-C\right]\cdot J_0(\eta\cdot r)\cdot d\eta\ . \qquad (3)$$

A singular integral has a simple integrand so that it can be analytically solved, whilst regular integrals are better to be calculated using discrete Fourier-Bessel transformation. The next step leads to their even simpler calculation. It is namely evident that the below functions are present in the function $F(\eta)$ of the integrand:

$$\alpha_i = \sqrt{\eta^2 - \gamma_i^2} \quad , \quad \beta_i = \sqrt{\eta^2 - \vartheta_i^2} \quad , \qquad e^{\pm\alpha_i \cdot h_i} \quad , \quad e^{\pm\beta_i \cdot h_i} \tag{4}$$

where

$$\gamma_i = \frac{k_{Li}}{k_{T1}} \quad ; \quad \vartheta_i = \frac{k_{Ti}}{k_{T1}} \tag{4a}$$

k_{Li} or k_{Ti}, respectively, are wave numbers of longitudinal or transversal waves, and h_i is the thickness of the *i*-th layer.

So, it is reasonable to substitute the mechanical understanding of the function $F(\eta)$ with its analytical approach and a suitable selection of branch cuts to make the function uniform. A branch cut, which is shown in Fig. (1), was first selected inventively.

If we wish that the function $F(\eta)$ is an even function of the variable η on the real η-axis we must modify accordingly exponential functions (4) which are neither even nor odd. To make these exponential functions uniform and even they are replaced with their analytical continuation:

$$e^{\pm\alpha_i \cdot h_i} = e^{\pm\frac{\eta}{|\eta|}\alpha_i \cdot h_i} \quad ; \quad e^{\pm\beta_i \cdot h_i} = e^{\pm\frac{\eta}{|\eta|}\beta_i \cdot h_i} \quad . \tag{5}$$

The mentioned modification, which makes the integrands even, allows the Bessel function to be divided into two parts:

$$2J_i(z) = h_i(z) + h_i(-z) \tag{6}$$

and consequently to transforms Hankel's inverse integrands, which are present in the derived components of Green's function, into integrals with the integral range from minus infinite to infinite:

$$\begin{aligned} I_1(r) &= \int_0^\infty \left[F(\eta) - C\right] \cdot J_i(\eta \cdot r) \cdot d\eta \\ &= \frac{1}{2} \cdot \left\{ \int_0^\infty \left[F(\eta) - C\right] \cdot h_i(\eta \cdot r) \cdot d\eta + \int_{-\infty}^0 \left[F(\eta) - C\right] \cdot h_i(\eta \cdot r) \cdot d\eta \right\} \\ &= \frac{1}{2} \cdot \int_{-\infty}^\infty \left[F(\eta) - C\right] \cdot h_i(\eta \cdot r) \cdot d\eta = \frac{1}{2} \cdot I_2(r) \end{aligned} \tag{7}$$

The final or the third step is the evaluation of the integral $I_2(r)$. It is shown in [11] that this integral can be evaluated with a contour integration in the complex plane of the Hankel parameter. Based on the theorem of residua [12] we obtain:

$$I_{2+} + I_{2R} + I_{2-} + I_{2b1} + I_{2r} + I_{2b2} = 2\pi i \sum res \,, \tag{8}$$

where the direction of the integration is evident from Fig. 2.

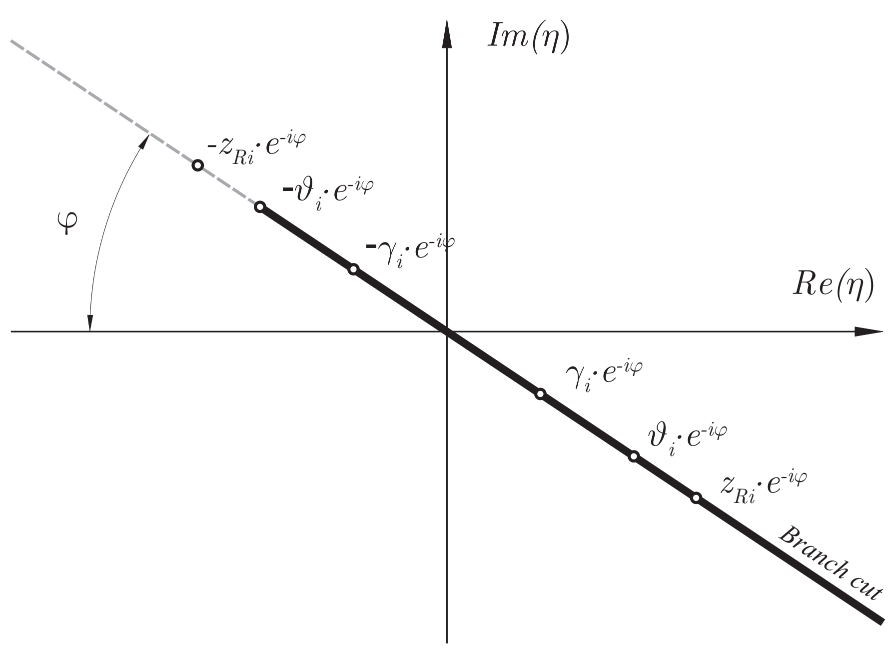

Figure 1. Branch points and poles of expressions α_i and β_i with a branch cut. For clearer presentation material damping, which is expressed with the complex shear modulus $\mu_i = \mu_{0i} \cdot e^{i\varphi}$, is considered.

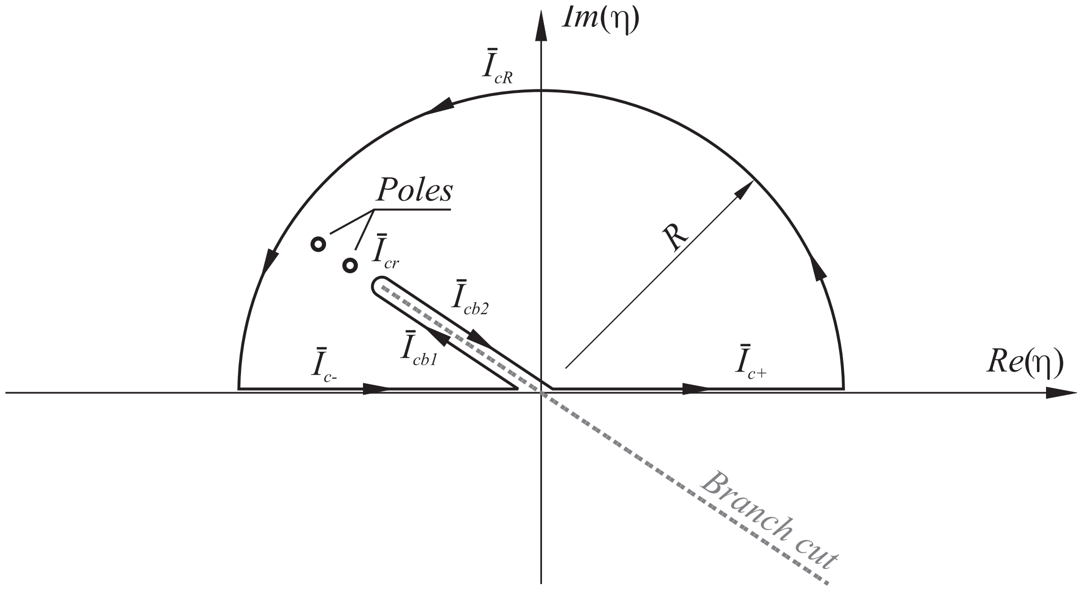

Figure 2. The integration path of the evaluation of Hankel inverse integrals which are present in the components of Green's function. The first member of the c indexes of the expressions in this figure is valid for all displacements of a horizontally layered half-space surface. Material damping is considered for clearer presentation.

Expression (8) is reasonably re-arranged:

$$I_2 = I_{2-} + I_{2+} = 2\pi i \sum res - I_{2b1} - I_{2r} - I_{2b2} \ , \qquad (9)$$

or written in the form:

$$I = \frac{C}{r} + 2\pi i \sum res - I_{2b1} - I_{2r} - I_{2b2} \ . \qquad (10)$$

Inverse Hankel integrals, which occur in the components of Green's function, adopt the below final form using the described alternative three-step approach:

$$I = \frac{C}{r} + 2\pi i \sum res + \sum \int_{a_i}^{b_i} \tilde{F}(\eta) \cdot J_n(\eta \cdot r) \cdot d\eta \ . \qquad (11)$$

Equation (11) provides engineeringly reasonable transformation of displacements of a layered half-space surface because it transparently shows the types of waves present in the half-space. The first member presents singularity which always appears in the basic singular solution in elastodynamics and which is excluded from inverse Henkel integrals that define the components of Green's function. Linking the findings of mathematical physics with the findings in theoretic and applied mechanics leads to the conclusion that surface waves in Eq. (11) are expressed with residues in the poles of the integrand, whilst volume waves are expressed with integrals along branch cuts.

3 STONELY WAVES

In evaluating Green's function it was shown that the contribution of the second member of Eq. (11) is very important. As already said, it manifests the contribution of surface waves which is defined with the poles of integrands. In a homogenous half-space, two conjugated complex poles only appear which define the presence of Rayleigh waves. The system of equations quickly increases with the increase of the number of layers, due to the requirement that continuity conditions on contact planes of individual layers be fulfilled. So, the equation system matrix for determining unknown integration constants increases in accordance with equation $4 \cdot n + 2$ in case of vertical concentrated force acting on the surface. In case of tangential concentrated surface load, the size of the system matrix is dictated with equation $6 \cdot n + 3$. In both cases, n presents the number of layers. Besides Rayleigh waves, which always appear on the surface of a half-space, Stonely waves can also appear on contact surfaces. The presence of surface waves is defined in the system of equations with zeros of the system determinant or with singularities of integrands in integrals for the inverse transformation of expressions for individual components of Green's function. The search for the integrand singularity becomes more demanding when the system increases; yet, numerical calculation of singularity greatly reduces the speed of evaluating Green's function and hinders the automation of the calculation process. Therefore, it is sensible to investigate the process of singularity in integrands more in detail.

Rayleigh's waves always appear when the medium through which waving is propagated has an unobstructed surface. Stonely waves appear on contact surfaces between individual layers, yet only at certain ratios of thicknesses and shear modula of two neighbouring layers. The range of the appearance of Stonely waves is investigated on an example of two half-spaces.

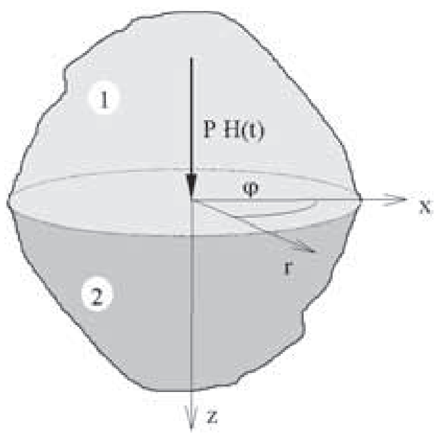

Figure 3. Model of two half-spaces.

As shown in Fig. 3, we take two homogenous elastic isotropic half-spaces with different thicknesses and different shear modula which are loaded at their contact surface with a vertical concentrated force. The problem is mathematically formulated so that the vector of displacement for each half-space is written by using potentials [13] in the form:

$$\vec{u} = \nabla \cdot \varphi + \vec{\nabla} \times \vec{\psi} \qquad (12)$$

The system of so linked partial differential equations, which presents the equations of movement,

$$\mu \cdot \nabla^2 \vec{u} + (\lambda + \mu) \cdot \vec{\nabla} \cdot \left(\vec{\nabla} \bullet \vec{u} \right) = \rho \cdot \frac{\partial^2 \vec{u}}{\partial t^2} \qquad (13)$$

disintegrates into the system of non-linked differential differential equations

$$\nabla^2 \cdot \varphi = \frac{1}{c_L^2} \cdot \frac{\partial^2 \varphi}{\partial t^2} \qquad (14)$$

$$\nabla^2 \cdot \vec{\psi} = \frac{1}{c_T^2} \cdot \frac{\partial^2 \vec{\psi}}{\partial t^2} \;, \qquad (15)$$

where c_L and c_T are the speeds of longitudinal or shear wave front, respectively. They are translated into the frequency domain with the Fourier exponential transformation $t \rightarrow \omega$. The studied elastodynamic problem is axially symmetrical, which dictates the use of the cylindrical coordinate system. Furthermore, the component of the displacement in the direction ϑ equals zero, both potentials, the scalar one φ and the vector one $\vec{\psi}$, must be independent of ϑ , and therefore the components of the vector potential $\vec{\psi}$ in the directions r and z must also equal zero. The individual components of the vector of displacement therefore resume the form:

$$\bar{\vec{u}} = \begin{Bmatrix} \dfrac{\partial \bar{\varphi}_r}{\partial r} - \dfrac{\partial \bar{\psi}_\vartheta}{\partial z} \\ 0 \\ \dfrac{\partial \bar{\varphi}_r}{\partial z} + \dfrac{1}{r} \cdot \dfrac{\partial \left(r \cdot \bar{\psi}_\vartheta \right)}{\partial r} \end{Bmatrix} \qquad (16)$$

and partial differential equations (14) and (15):

$$\frac{\partial^2 \bar{\varphi}_r}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \bar{\varphi}_r}{\partial r} + \frac{\partial^2 \bar{\varphi}_r}{\partial z^2} + \left(\frac{\omega}{c_L} \right)^2 \cdot \bar{\varphi}_r = 0 \qquad (17)$$

$$\frac{\partial^2 \bar{\psi}_\vartheta}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial \bar{\psi}_\vartheta}{\partial r} + \frac{\partial^2 \bar{\psi}_\vartheta}{\partial z^2} - \frac{\bar{\psi}_\vartheta}{r^2} + \left(\frac{\omega}{c_T} \right)^2 \cdot \bar{\psi}_\vartheta = 0 \;. \qquad (18)$$

The relationship between the components of tension and displacements has the form:

$$\bar{\sigma}_z = 2 \cdot \mu \cdot \frac{\partial \bar{w}}{\partial z} - \lambda \cdot \left(\frac{\omega}{c_L} \right)^2 \cdot \bar{\varphi}_r \qquad (19)$$

$$\bar{\tau}_{zr} = 2 \cdot \mu \cdot \frac{\partial \bar{u}}{\partial z} - \mu \cdot \left(\frac{\omega}{c_T} \right)^2 \cdot \bar{\psi}_\vartheta \;. \qquad (20)$$

Partial differential equations (17) and (18) are transformed into ordinary differential equations using Hankel integral transformation $r \rightarrow \xi$:

$$\frac{d^2 \tilde{\bar{\varphi}}^0(\xi)}{dz^2} - \left[\xi^2 - \left(\frac{\omega}{c_L} \right)^2 \right] \cdot \tilde{\bar{\varphi}}^0(\xi) = 0 \qquad (21)$$

$$\frac{d^2 \tilde{\bar{\psi}}^1(\xi)}{dz^2} + \left[\xi^2 - \left(\frac{\omega}{c_T} \right)^2 \right] \cdot \tilde{\bar{\psi}}^1(\xi) = 0 \qquad (22)$$

The transformed components of the vector of displacement have the form

$$\tilde{\bar{u}}^1(\xi) = -\xi \cdot \tilde{\bar{\varphi}}^0(\xi) - \frac{d}{dz} \tilde{\bar{\psi}}^1(\xi) \qquad (23)$$

$$\tilde{\bar{w}}^0(\xi)=\frac{d}{dz}\tilde{\bar{\varphi}}^0(\xi)+\xi\cdot\tilde{\bar{\psi}}^1(\xi) \qquad (24)$$

We then also transform the expressions for normal and shear tensions:

$$\tilde{\bar{\sigma}}_z^0(\xi)=\mu\cdot\left\{\left[2\cdot\xi^2-\left(\frac{\omega}{c_T}\right)^2\right]\cdot\tilde{\bar{\varphi}}^0(\xi)+2\cdot\xi\cdot\frac{d}{dz}\tilde{\bar{\psi}}^1(\xi)\right\} \qquad (25)$$

$$\tilde{\bar{\tau}}_{rz}^1(\xi)=-\mu\cdot\left\{2\cdot\xi\cdot\frac{d}{dz}\tilde{\bar{\varphi}}^0(\xi)+\left[2\cdot\xi^2-\left(\frac{\omega}{c_T}\right)^2\right]\cdot\tilde{\bar{\psi}}^1(\xi)\right\}. \qquad (26)$$

Transformed differential Eqs. (21) and (22) for the potentials $\tilde{\bar{\varphi}}^0(\xi)$ and $\tilde{\bar{\psi}}^1(\xi)$ are ordinary homogeneous differential equations where z is the only variable. Their general solutions are:

$$\tilde{\bar{\varphi}}^0=\Phi_1\cdot\exp\left(\sqrt{\xi^2-k_L^2}\cdot z\right)+\Phi_2\cdot\exp\left(-\sqrt{\xi^2-k_L^2}\cdot z\right) \qquad (27)$$

$$\tilde{\bar{\psi}}^1=\Psi_1\cdot\exp\left(\sqrt{\xi^2-k_T^2}\cdot z\right)+\Psi_2\cdot\exp\left(-\sqrt{\xi^2-k_T^2}\cdot z\right), \qquad (28)$$

where Φ_1, Φ_2, Ψ_1 and Ψ_2 are integration constants dependent on the variable ξ, whilst the coefficients k_L and k_T are wave numbers belonging to the speed of the longitudinal or shear front, respectively. They are defined with the expressions:

$$c_L=\frac{\omega}{k_L}=\sqrt{\frac{\lambda+2\cdot\mu}{\rho}} \qquad (29)$$

$$c_T=\frac{\omega}{k_T}=\sqrt{\frac{\mu}{\rho}}\ . \qquad (30)$$

The positivity of the square root is required in the exponents of expressions (27) and (28). In case of a half-space and considering radiation condition and the selected coordinate system, it holds true that the constants Φ_1 and Ψ_1 must equal zero, or the value of the potentials would increase beyond any limit with an increasing value of the variable z. In case of a half-space, general solutions (27) and (28) for the potentials $\tilde{\bar{\varphi}}^0$ and $\tilde{\bar{\psi}}^1$ therefore equal:

$$\tilde{\bar{\varphi}}^0=\Phi_2\cdot\exp\left(-\sqrt{\xi^2-k_L^2}\cdot z\right) \qquad (31)$$

$$\tilde{\bar{\psi}}^1=\Psi_2\cdot\exp\left(-\sqrt{\xi^2-k_T^2}\cdot z\right). \qquad (32)$$

Radiation damping is considered in the studied case of two half-spaces, and the following substitutions are introduced:

$$\xi=k_{T1}\cdot\eta \qquad (33)$$

$$\frac{c_T}{c_L}=\frac{k_L}{k_T}=\sqrt{\frac{\mu}{\lambda+2\cdot\mu}}=\sqrt{\frac{1-2\cdot\nu}{2\cdot(1-\nu)}}=\gamma \qquad (34)$$

$$\vartheta_2=\frac{k_{T2}}{k_{T1}}\ . \qquad (35)$$

General solutions for individual potentials therefore are:

$$\tilde{\bar{\varphi}}_1^0=C_{1,1}\cdot\exp\left(2\cdot\pi\cdot\sqrt{\eta^2-\gamma_1^2}\cdot\zeta\right)=C_{1,1}\cdot\exp\left(2\cdot\pi\cdot\bar{\alpha}_1\cdot\zeta\right) \quad (36)$$

$$\tilde{\bar{\psi}}_1^1=C_{1,3}\cdot\exp\left(2\cdot\pi\cdot\sqrt{\eta^2-1}\cdot\zeta\right)=C_{1,3}\cdot\exp\left(2\cdot\pi\cdot\bar{\beta}_1\cdot\zeta\right) \quad (37)$$

$$\tilde{\bar{\varphi}}_2^0=C_{2,2}\cdot\exp\left(-2\cdot\pi\cdot\sqrt{\eta^2-\gamma_2^2}\cdot\zeta\right)=C_{2,2}\cdot\exp\left(-2\cdot\pi\cdot\bar{\alpha}_2\cdot\zeta\right) \quad (38)$$

$$\tilde{\bar{\psi}}_2^1=C_{2,4}\cdot\exp\left(-2\cdot\pi\cdot\sqrt{\eta^2-\vartheta_2^2}\cdot\zeta\right)=C_{2,4}\cdot\exp\left(-2\cdot\pi\cdot\bar{\beta}_2\cdot\zeta\right), \quad (39)$$

where ζ is the coordinate z standardized with respect to the wave length of shear waves of the first half-space. Four continuity conditions are available to determine unknown constants in general solutions (36) - (39). The equality of normal and shear tensions, as well as of vertical and horizontal displacements, must be assured on the contact surface of half-spaces.

$$\tilde{\bar{\sigma}}_{z,1}^0\Big|_{z=0}-\tilde{\bar{\sigma}}_{z,2}^0\Big|_{z=0}=0 \qquad (40)$$

$$\tilde{\bar{\tau}}_{zr,1}^1\Big|_{z=0}-\tilde{\bar{\tau}}_{zr,2}^1\Big|_{z=0}=0 \qquad (41)$$

$$\tilde{\bar{w}}_1^0\Big|_{z=0}-\tilde{\bar{w}}_2^0\Big|_{z=0}=0 \qquad (42)$$

$$\tilde{\bar{u}}_1^1\Big|_{z=0}-\tilde{\bar{u}}_2^1\Big|_{z=0}=0\ . \qquad (43)$$

Real zeros of the determinant of the so obtained equation system show the relationships of speeds of shear waves in the first medium and of Stoneley waves. Unlike Rayleigh waves, which are always present on the surface of a half-space, Stonely waves only appear at certain ratios of thicknesses and shear modula. The latter is represented by the graph in Fig. 4, where the abscissa is represented by the ratio of densities (ρ_2/ρ_1), and the ordinate by the ratio of shear modula (μ_2/μ_1). The range where real solutions appear (it is possible to show that there are only two), is limited with two curves. The curve A is defined with the requirement that the

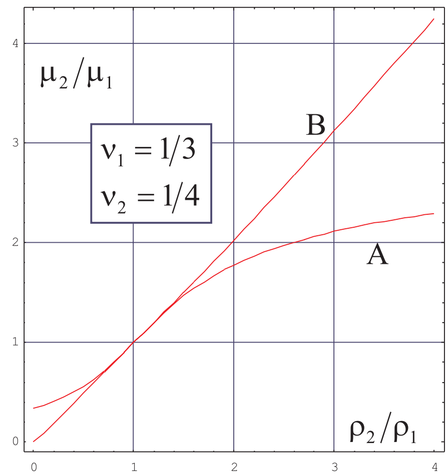

Figure 4. The range of the appearance of Stonely waves.

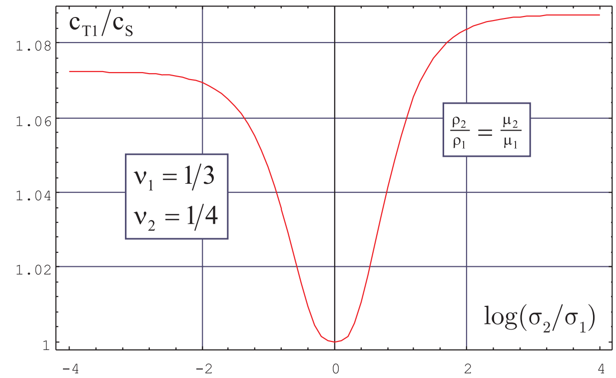

Figure 6. Speed changes of Stonely waves.

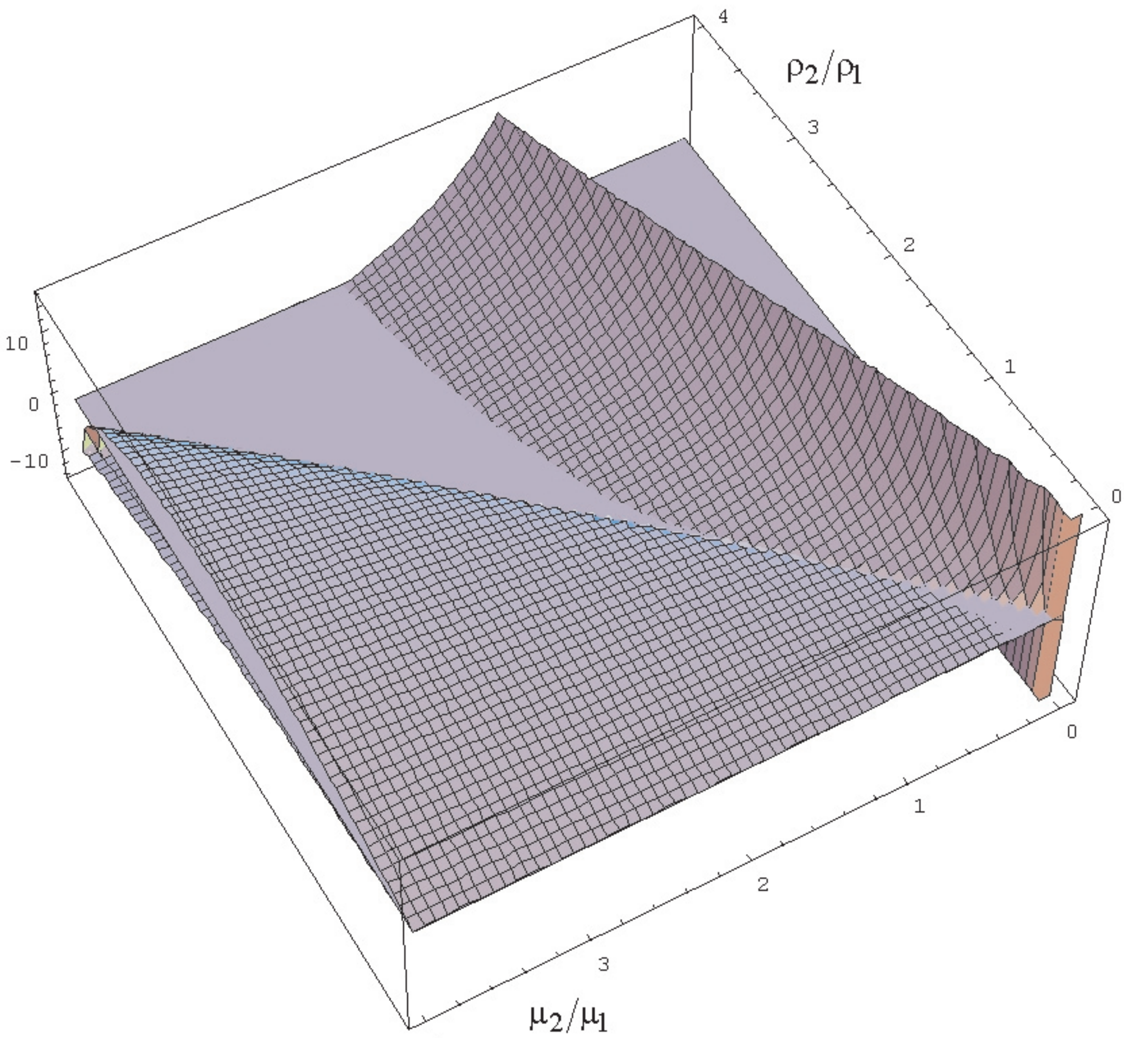

Figure 5. Three dimensional view of finding the appearance range of Stonely waves.

speed of a Stonely wave (c_S) equals the speed of a shear wave in the first medium (c_{T1}), which also physically presents one of the limits of a possible solution range. The curve B is defined with the requirement that the speed of a Stonely wave equals the speed of a shear wave in the second medium. An interesting three-dimensional view (Fig. 5) of finding the mentioned curves is presented in [10], where the author first searches for the intersection of the plane 0 and the surface defined with an equation of a determinant evaluated in $\eta = 1$, and then for the intersection of the plane 0 and the surface defined with an equation of a determinant evaluated in $\eta = \sqrt{\mu_2/\rho_2}$.

The change of speed of Stonely waves in the area where they occur is shown in Fig. 6, in which the abscissa is represented by the logarithm of the ratio of thicknesses or shear modula, respectively, and the ordinate is represented by the ratio of speeds of shear waves in the first medium and Stonely waves. To do this, we first draw a line in Fig. 4 that represents a symmetrical line of the odd quadrants $\rho_2/\rho_1 = \mu_2/\mu_1$; we then draw the change of speeds of Stonely waves along the symmetrical line. When the thickness and the shear modulus of the second medium are infinitely small, a Stonely wave is transformed into a Rayleigh wave for the first medium because this medium behaves as a homogenous half-space with a free surface. Contrary to this, a Stonely wave is transformed into a Rayleigh wave for the second medium when the thickness and the shear modulus of the first medium are infinitely small. When the thicknesses and shear modula of both media are the same, both half-spaces form a homogenous space, and a Stonely wave is transformed into a shear volume wave.

The appearance of Stonely waves does not only depend on characteristics of both media that are in contact but also on their thickness. To investigate this phenomenon it is reasonable to study the layer on an elastic homogenous half-space. In this case, general solutions (27) and (28) for individual potentials can be written in the below form considering a radiation condition and substitutions (33) - (35):

$$\tilde{\bar{\varphi}}_1^0 = C_{1,1} \cdot \exp\left(2 \cdot \pi \cdot \bar{\alpha}_1 \cdot \zeta\right) + C_{1,2} \cdot \exp\left(-2 \cdot \pi \cdot \bar{\alpha}_1 \cdot \zeta\right) \quad (40)$$

$$\tilde{\bar{\psi}}_1^1 = C_{1,3} \cdot \exp\left(2 \cdot \pi \cdot \bar{\beta}_1 \cdot \zeta\right) + C_{1,4} \cdot \exp\left(-2 \cdot \pi \cdot \bar{\beta}_1 \cdot \zeta\right) \quad (41)$$

$$\tilde{\bar{\varphi}}_2^0 = C_{2,2} \cdot \exp\left(-2 \cdot \pi \cdot \bar{\alpha}_2 \cdot \zeta\right) \quad (42)$$

$$\tilde{\bar{\psi}}_2^1 = C_{2,4} \cdot \exp\left(-2 \cdot \pi \cdot \bar{\beta}_2 \cdot \zeta\right), \quad (43)$$

where ζ represents a coordinate z standardized with respect to the length of shear waves of the first half-space.

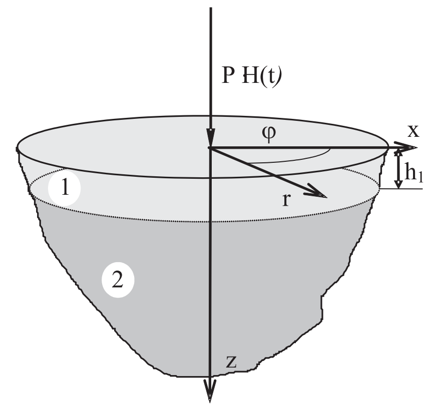

Figure 7. A model of a layer on a homogenous half-space.

To determine unknown constants in general solutions (40) - (43) two boundary conditions on a free surface $(z = 0)$ are available

$$\tilde{\bar{\sigma}}_z^0(\xi)\Big|_{z=0} = -\frac{\bar{P}(\omega)}{2 \cdot \pi} \quad (44)$$

$$\tilde{\bar{\tau}}_{zr}^1(\xi)\Big|_{z=0} = 0 \quad (45)$$

as well as four continuity conditions on the contact surface $(z = h_1)$:

$$\tilde{\bar{\sigma}}_{z,1}^0\Big|_{z_1=h_1} - \tilde{\bar{\sigma}}_{z,2}^0\Big|_{z_2=0} = 0 \quad (46)$$

$$\tilde{\bar{\tau}}_{zr,1}^1\Big|_{z_1=h_1} - \tilde{\bar{\tau}}_{zr,2}^1\Big|_{z_2=0} = 0 \quad (47)$$

$$\tilde{\bar{w}}_1^0\Big|_{z_1=h_1} - \tilde{\bar{w}}_2^0\Big|_{z_2=0} = 0 \quad (48)$$

$$\tilde{\bar{u}}_1^1\Big|_{z_1=h_1} - \tilde{\bar{u}}_2^1\Big|_{z_2=0} = 0\,. \quad (49)$$

The zeros of the determinant of the system of equations (44) - (49) represent poles of solutions of the system of equations in a transformed domain. The mentioned poles define the speeds of surface waves which appear at the surface of the total half-space and on the contact surface between a layer and a half-space.

In the studied case of an elastic layer on a homogenous half-space, we wish to investigate how the occurrence of Stonely waves depends on the thickness of the layer. To do this, we seek the zeros of the determinant of the system for different thicknesses of a layer, with a requirement that the ratio of thicknesses of both media

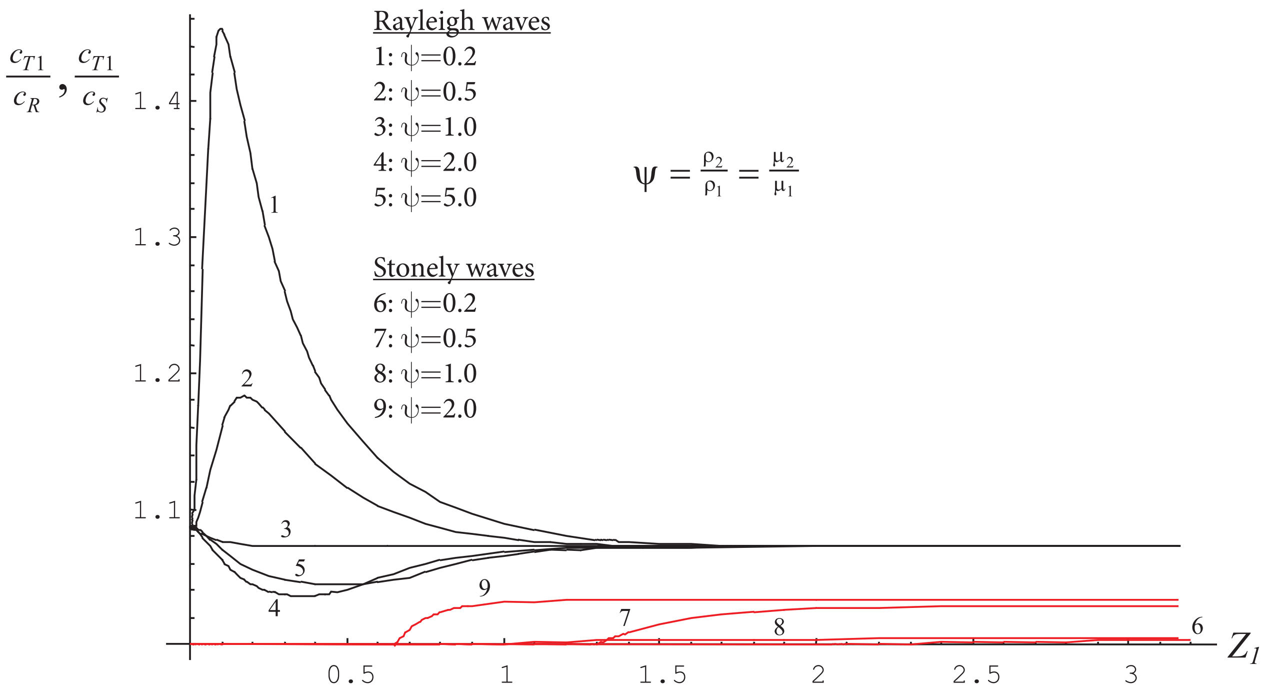

Figure 8. Dispersion diagram.

equals the ratio of their shear modula. We so obtain dispersion curves of the speed of surface waves, which are shown in Fig. 8, in which the abscissa is presented by the ratio of the layer thickness and wave length of a shear wave in the layer (Z_1), and the ordinate by the ratio of the speed of shear waves in a layer and the speed of Rayleigh waves (c_{T1}/c_R) or Stonely waves (c_{T1}/c_S), respectively. As expected, Rayleigh waves always occur without respect to the thickness of a layer. Their speed, however, quickly changes with the layer thickness when the latter is smaller than the wave length of a shear wave in a layer. Contrary to this, Stonely waves only appear at layer thicknesses that are greater than certain limit values depending on the ratio of thicknesses or shear modula, respectively, in both media. In a limit case, when the layer thickness approaches infinity, the speed of Stonely waves stabilizes at values obtained with the interaction of two half-spaces from the previous chapter. The speed of Rayleigh waves, however, approaches the values obtained for the case of a homogenous half-space with layer characteristics.

4 CONCLUSIONS

This article presents and alternative approach to the evaluation of inverse Hankel integrals which appear in the components of Green's function for a layered half-space on the surface of a top layer loaded with a concentrated force of general direction. This approach completely abolishes weak points of past procedures. After three steps of an innovative solution, described in this article, integrals pass into a form which transparently shows types of waves present in a half-space. Besides a better physical interpretation, this form allows a simple, numerically economical, and robust calculation. The results give a better physical insight into a layered half-space, which has been unknown up to the present, and open ways, due to their transparency, to studies of an inverse problem.

This article concentrates on the studies of possible appearance and behavior of Stonely waves. The appearance of Stonely and Rayleigh waves and consequently the finding of the integral poles, which is always numerical due to sophisticated expressions, is namely that segment of the Green's function solving procedure which greatly hinders the introduction of a universal algorithm for calculation. It is shown that the appearance of

Stonely waves only depends on material characteristics of both media, in case when two half-spaces are in contact. As soon as the thickness of an individual half-space becomes final, it is thickness, too, that influences the appearance and behaviour of Stonely waves. On the basis of the above procedures, we can namely fairly accurately limit the ratios of shear modula and material thicknesses of two proximate layers at which Stonely waves appear. This somehow facilitates the calculation procedure, yet the presented semi-analytical method is too sophisticated to be used for calculations in everyday engineering applications. Its advantage, however, is the accuracy of the obtained results. Taken as such, its basic aim and value lie in the comparison of engineering adequacy with other approximate methods for studying dynamic soil-structure interaction based on fundamental solutions. If we wish to obtain practical applicability of the presented method, the calculation process must be inevitably simplified, yet the simplifications must not affect the accuracy of the results. A relatively submissive behavior of roots in expressions for integrands in the complex η-plane further motivates us to consider the change of the integration path as one of possible simplifications in the calculation. In further investigations the integration path should be led so that the system of equations for integration constants could be solved numerically with optional accuracy and no longer with symbols. This would set the starting-points for an extremely fast, accurate and stabile procedure for determining a three-dimensional Green's function which, by considering the fact that it includes both, boundary and continuous conditions as well as radiation condition, presents a basis for calculating a dynamic stiffness soil matrix. This will be shown in our next articles that are in the phase of preparation.

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