BLED WORKSHOPS
IN PHYSICS
VOL. 19, NO. 2

Proceedings to the 21st Workshop
What Comes Beyond . . . *(p. 74)*
Bled, Slovenia, June 23–July 1, 2018

4 Emergent Photons and Gravitons

J.L. Chkareuli, J. Jejelava and Z. Kepuladze

Center for Elementary Particle Physics, Ilia State University, 0162 Tbilisi, Georgia
E. Andronikashvili Institute of Physics, 0177 Tbilisi, Georgia

Abstract. Now, it is already not a big surprise that due to the spontaneous Lorentz invariance violation (SLIV) there may emerge massless vector and tensor Goldstone modes identified particularly with photon and graviton. Point is, however, that this mechanism is usually considered separately for photon and graviton, though in reality they appear in fact together. In this connection, we recently develop the common emergent electrogravity model which would like to present here. This model incorporates the ordinary QED and tensor field gravity mimicking linearized general relativity. The SLIV is induced by length-fixing constraints put on the vector and tensor fields, $A_\mu^2 = \pm M_A^2$ and $H_{\mu\nu}^2 = \pm M_H^2$ (M_A and M_H are the proposed symmetry breaking scales) which possess the much higher symmetry than the model Lagrangian itself. As a result, the twelve Goldstone modes are produced in total and they are collected into the vector and tensor field multiplets. While photon is always the true vector Goldstone boson, graviton contain pseudo-Goldstone modes as well. In terms of the appearing zero modes, theory becomes essentially nonlinear and contains many Lorentz and CPT violating interaction. However, as argued, they do not contribute in processes which might lead to the physical Lorentz violation. Nonetheless, how the emergent electrogravity theory could be observationally differed from conventional QED and GR theories is also briefly discussed.

Povzetek. Avtorji so razvili model za elektrogravitacijo, ki vsebuje običajno kvantno elektrodinamiko in tenzorsko polje gravitacije. Slednje predstavlja linearizirano splošno teorijo relativnosti. Spontano kršitev Lorentzove invariance sprožijo s predpisom za vektorska in tenzorska polja: $A_\mu^2 = \pm M_A^2$ in $H_{\mu\nu}^2 = \pm M_H^2$ (M_A in M_H sta predlagani skali zlomitve simetrije). Predpis prinese mnogo višjo simetrijo kot jo ima Lagrangeva gostota modela. Dvanajst Goldstonovih delcev tvori multiplete vektorskih in tenzorskih polj. Foton je vedno pravi vektorski Goldstonov bozon, graviton pa vsebuje tudi psevdo Goldstonove načine. Model postane tako nelinearen in vsebuje vrsto interakcij, ki zlomijo Lorentzovo in CPT simetrijo, ki pa ne vodijo do fizikalne zlomitve Lorentzove simetrije. Avtorji komentirajo, v čem se elektrogravitacija razlikuje od elektrodinamike in gravitacije.

Keywords: Spontaneous symmetry violation, Lorentz invariance violation, emergent field theory.

4.1 Introduction

While Lorentz symmetry looks physically as an absolutely exact spacetime symmetry, the spontaneous Lorentz invariance violation (SLIV) suggests a beautiful scenario where massless vectors and/or tensor fields emerge as the corresponding zero modes which may be identified with photons, gravitons and other gauge fields [1–3]. Though they appear through condensation of the pure gauge degrees of freedom in the starting theory their masslessness are provided by their Nambu-Goldstone nature [4–12] rather than a conventional gauge invariance.

4.1.1 Emergent vector fields theory

In order to violate Lorentz invariance one necessarily needs field(s) being sensitive to the spacetime transformations, as vector or tensor fields are. They can evolve vacuum expectation value which fixes direction of the violation in the spacetime and create the corresponding condensate. Therefore, if there is an interaction with this condensate one could expect Lorentz violation to be manifested physically. If we want to arrange spontaneous Lorentz violation by the vector field, we could start, as usual, with the potential terms in the Lagrangian

$$L = -\frac{1}{4}F_{\mu\nu}^2 - V\,; \qquad V = \lambda\left(A_\mu^2 - n_\mu^2 M_A^2\right)^2 \tag{4.1}$$
$$F_{\mu\nu}^2 = F_{\mu\nu}F^{\mu\nu}\,; \quad A_\mu^2 = A_\mu A^\mu\,; \;\; n_\mu^2 = n_\mu n^\mu$$

where n_μ is an unit constant vector specifying character of Lorentz violation. If n_μ is time-like vector, we have time-like violation breaking $SO(1,3)$ to $SO(3)$. If n_μ is space-like vector, we have space-like violation breaking $SO(1,3)$ to $SO(1,2)$.

We started with gauge invariant kinetic term, but since potential violates gauge invariance anyway, we could have started with general kinetic terms

$$L_k = a\,(\partial_\alpha A_\beta)^2 + b\,(\partial_\alpha A^\alpha)^2 \tag{4.2}$$

but problem arising here is a propagating ghost mode, which we get ride off with the gauge invariant form of kinetic terms.

Such a system of vector field with potential, generally appears not stable, its energy is not bound from below unless phase space is restricted with condition

$$A_\mu^2 - n_\mu^2 M_A^2 = 0 \tag{4.3}$$

While this condition may appear out of the blue, it is actually motivated by the conserved current of (4.1)

$$J_\mu = A_\mu (A_\alpha^2 - n_\alpha^2 M_A^2) \tag{4.4}$$

and if in the initial condition the conserved charge of this current is set to zero , which means (4.3) is always zero, no propagating ghosts, Hamiltonian is positively defined and Coulomb law stays the same [13]. So, basically we arrived to the point where we accept to take λ in (4.1) to infinity as a Lagrange multiplier and get conventional vector field kinetics with the addition of (4.3) condition. This condition still is a cause for spontaneous Lorentz invariance violation, but in contrast now Higgs mode is set to zero. This was Nambu's original idea [14]. It is easy to see, if we write expansion of the vector field A into Goldstone and Higgs modes in the exponential manner, which is

$$A_\mu = (M_A + h)n_\nu \exp J^\nu_\mu \tag{4.5}$$

where h is Higgs mode and Goldstone modes a_μ are sitting in J^ν_μ (generators for Lorentz transformation) and $a_\mu = M_A n_\nu J^\nu_\mu$, $a_\mu n^\mu = M_A n^\mu n_\nu J^\nu_\mu = 0$. So,

$$A^2_\mu = (M_A + h)^2 n^2_\nu = n^2_\alpha M^2_A \implies h = 0 \tag{4.6}$$

Expansion (4.5) is nonlinear with respect to vector Goldstone modes, but $\frac{a_\mu}{M_A}$ is a small parameter and we can expand exponent in the power series and in the second approximation get

$$A_\mu = \left(M_A - \frac{n^2_\alpha a^2_\alpha}{2M_A} \right) n_\mu + a_\mu \tag{4.7}$$

It is clear now that we get nonlinear Lagrangian for vector Goldstone modes, which in the first approximation is

$$L(A) \to L(a) = -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2}\delta(n_\alpha a^\alpha)^2 - \frac{1}{2}\frac{n^2}{M_A} f_{\mu\nu} n^\mu \partial^\nu a^2 \tag{4.8}$$

δ is Lagrange multiplier setting orthogonality condition for the vector Goldstone field, thus treating it as gauge fixing one. In general, we have here pletora Lorentz and CPT violating couplings like $\frac{n^2}{M_A} f_{\mu\nu} n^\mu \partial^\nu a^2$ in the higher orders, especially if charged currents are introduced as well, but it appears in all physical processes (photon-photon, matter-photon, matter-matter interactions) at least in the tree and one loop level, there is no sign of physical Lorentz invariance violation. Looks like Lorentz invariance is realized in nonlinear fashion and Lorentz breaking condition (4.3) is treated like a nonlinear gauge choice for vector field [16,17].

Consideration of the spontaneous Lorentz violation scenarios for non-Abelian vector fields meet same challenges, though consequently lead to the same conclusions as in the Abelian vector field case, despite the fact that there are some significant differences as well. The length fixing constraint adapted for non-Abelian vector fields in fact violates not only Lorentz symmetry, but an accidental symmetry $SO(N, 3N)$ of the constraint itself (here N defines unitary symmetry group of vector fields) which is much higher than symmetry of the theory Lagrangian. This gives extra massless modes which together with the true Lorentzian Goldstone complete the whole gauge multiplet of the non-Abelian theory taken [18].

4.1.2 Emergent tensor field gravity

Actually, for the tensor field gravity we can use the similar nonlinear constraint for a symmetric two-index tensor field

$$H^2_{\mu\nu} = n^2 M^2_H \ , \ H^2_{\mu\nu} \equiv H_{\mu\nu}H^{\mu\nu}, \ n^2 \equiv n_{\mu\nu}n^{\mu\nu} = \pm 1 \tag{4.9}$$

(where $n_{\mu\nu}$ is now a properly oriented unit Lorentz tensor, which supposedly specifies vacuum expectation values, while M_H is the proposed scale for Lorentz violation in the gravity sector) which fixes its length in the same manner as it appears for the vector field (4.3). Again, the nonlinear constraint (4.9) may in principle appear from the standard potential terms added to the tensor field Lagrangian

$$U(H) = \lambda_H (H_{\mu\nu}^2 - n^2 M_H^2)^2 \tag{4.10}$$

in the nonlinear σ-model type limit when the coupling constant λ_H goes to infinity. Just in this limit the tensor field theory appears stable, but doing so, we are effectively excluding corresponding Higgs mode from the theory and it does not lead to physical Lorentz violation [19].

This constraint (4.9), like the non-Abelian vector field, has higher symmetry then the kinetic term, particularly $SO(7,3)$. So, spontaneous symmetry violation breaks not only Lorentz symmetry, but also this $SO(7,3)$ and therefore produces also PGM-s, but in contrast to vector field, when we had only two channels of Lorentz symmetry violation to $SO(3)$ or $SO(1,2)$ and three true Goldstone modes always, for tensor field we have more possibilities. If we write down constraint in more details

$$H_{\mu\nu}^2 = H_{00}^2 + H_{i=j}^2 + (\sqrt{2}H_{i\neq j})^2 - (\sqrt{2}H_{0i})^2 = n^2 M_H^2 = \pm M_H^2 \tag{4.11}$$

we see that if only one component of the tensor field should acquire vacuum expectation value (assuming minimal vacuum configuration) we have following alternatives:

$$\begin{aligned} &(a) \quad n_{00} \neq 0\,, \;\; SO(1,3) \to SO(3) \\ &(b) \quad n_{i=j} \neq 0\,, \;\; SO(1,3) \to SO(1,2) \\ &(c) \quad n_{i\neq j} \neq 0\,, \;\; SO(1,3) \to SO(1,1) \end{aligned} \tag{4.12}$$

for $n^2 = 1$ and

$$(d) \quad n_{0i} \neq 0\,, \;\; SO(1,3) \to SO(2) \tag{4.13}$$

for $n^2 = -1$. For a, b cases we have three true Goldstone modes and for c, d we have five, since only one generator of Lorentz transformations remains unbroken. While in b, c, d cases physical graviton consists, at least partially, from true Goldstone modes, in case a only true goldstones are H_{0i} components, thus physical graviton will be constructed from PGM-s. One should notice that pseudo-Goldstone nature of some components of tensor multiplet poses no threats and generally in contrast to the scalar pseudo-Goldstone modes they do not acquire mass duo to the quantum effects, if diffeomorphism (diff) invariance is present.

So, we are putting (4.9) on the tensor field mimicking linearized general relativity

$$L = L(H) + L_S - \frac{1}{M_P} H_{\mu\nu} T_S^{\mu\nu} \tag{4.14}$$

where

$$L(H) = \frac{1}{2}\partial_\lambda H^{\mu\nu}\partial^\lambda H_{\mu\nu} - \frac{1}{2}\partial_\lambda H_{tr}\partial^\lambda H_{tr} - \partial_\lambda H^{\lambda\nu}\partial^\mu H_{\mu\nu} + \partial^\nu H_{tr}\partial^\mu H_{\mu\nu} \tag{4.15}$$

Here H_{tr} stands for the trace of $H_{\mu\nu}$ ($H_{tr} = \eta^{\mu\nu}H_{\mu\nu}$) and $L(H)$ is invariant under the diff transformations

$$\delta H_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu \,, \quad \delta x^\mu = \xi^\mu(x) \,, \tag{4.16}$$

while L_S and $T_S^{\mu\nu}$ are the Lagrangian and corresponding energy momentum tensor of whatever is gravitating, (vector fields, matter). In case, vector field is considered

$$L(A) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \; T^{\mu\nu}(A) = -F^{\mu\rho}F^\nu_\rho + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{4.17}$$

where $L(H)$ is fully diff invariant, but that is not the case for other parts of Lagrangian and diff invariance is satisfied only proximately, but they become more and more invariant when the tensor field gravity Lagrangian (4.14) is properly extended to GR with higher terms in H-fields included[1].

Once tensor field acquires vacuum expectation value, we can expand it into Goldstone mode

$$H_{\mu\nu} = h_{\mu\nu} + n_{\mu\nu}M_H - \frac{n^2h^2}{2M_H} + O(1/M_H^2), \; n \cdot h = 0 \tag{4.18}$$

Here $h_{\mu\nu}$ corresponds to the pure emergent modes satisfying the orthogonality condition and $h^2 \equiv h_{\mu\nu}h^{\mu\nu}, \; n \cdot h \equiv n_{\mu\nu}h^{\mu\nu}$.

Lets specify once again that $h_{\mu\nu}$ consists of Goldstone and PGM-s. Only case, when physical graviton will consists of only Goldstone mode is when Lorentz invariance is fully broken, we have six emergent goldstone modes and other pseudo Goldstone components is gauged away by fixing remaining gauge freedom (more about supplementary conditions below). Such a scenario can not be achieved by minimal vacuum configuration. Nevertheless, whether tensor field will be defined only by Goldstone modes or by a mixture with PGM-s, hole tensor multiplet always stays strictly massless. A particular case of interest is that of the traceless VEV tensor $n_{\mu\nu}$

$$n_{\mu\nu}\eta^{\mu\nu} = 0 \tag{4.19}$$

in terms of which the emergent gravity Lagrangian acquires an especially simple form (see below). It is clear that the VEV in this case can be developed on several $H_{\mu\nu}$ components simultaneously, which in general may lead to total Lorentz violation with all six Goldstone modes generated. For simplicity, we will use sometimes this form of vacuum configuration in what follows, while our arguments can be applied to any type of VEV tensor $n_{\mu\nu}$.

Alongside to basic emergent orthogonality condition in (4.18) one must also specify other supplementary conditions for the tensor field $h^{\mu\nu}$(appearing eventually as possible gauge fixing terms in the emergent tensor field gravity). We have

[1] Such an extension means that in all terms included in the GR action, particularly in the QED Lagrangian term , $(-g)^{1/2}g_{\mu\nu}g_{\lambda\rho}F^{\mu\lambda}F^{\nu\rho}$, one expands the metric tensors

$$g_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu}/M_P, \; g^{\mu\nu} = \eta^{\mu\nu} - H^{\mu\nu}/M_P + H^{\mu\lambda}H^\nu_\lambda/M_P^2 + \cdots$$

taking into account the higher terms in H-fields.

remaining three degrees of gauge freedom. Usually, spin 1 states in tensor field is gauged away by the conventional Hilbert-Lorentz condition

$$\partial^{\mu} h_{\mu\nu} + q\partial_{\nu} h_{tr} = 0 \tag{4.20}$$

(q is an arbitrary constant, giving for $q = -1/2$ the standard harmonic gauge condition), because spin-1 component always has negative contribution in energy and therefore it is desirable action. However, as we have already imposed the emergent constraint (4.18), we can not use the full Hilbert-Lorentz condition (4.20) eliminating four more degrees of freedom in $h_{\mu\nu}$. Otherwise, we would have an "over-gauged" theory with a non-propagating graviton. In fact, the simplest set of conditions which conform with the emergent condition $n \cdot h = 0$ in (4.18) turns out to be

$$\partial^{\rho}(\partial_{\mu} h_{\nu\rho} - \partial_{\nu} h_{\mu\rho}) = 0 \tag{4.21}$$

This set excludes only three degrees of freedom [2] in $h_{\mu\nu}$ and, besides, it automatically satisfies the Hilbert-Lorentz spin condition as well.

Putting parameterization (4.18) into the total Lagrangian given in (4.14), one comes to the truly emergent tensor field gravity Lagrangian containing an infinite series in powers of the $h_{\mu\nu}$ modes. For the traceless VEV tensor $n_{\mu\nu}$, without loss of generality, we get the especially simple form

$$\begin{aligned} L = {} & \frac{1}{2}\partial_{\lambda} h^{\mu\nu}\partial^{\lambda} h_{\mu\nu} - \frac{1}{2}\partial_{\lambda} h_{tr}\partial^{\lambda} h_{tr} - \partial_{\lambda} h^{\lambda\nu}\partial^{\mu} h_{\mu\nu} + \partial^{\nu} h_{tr}\partial^{\mu} h_{\mu\nu} + \\ & -\frac{n^2}{M_H} h^2 n^{\mu\lambda}\left[\partial_{\lambda}\partial^{\nu} h_{\mu\nu} - \frac{1}{2}\partial_{\mu}\partial_{\lambda} h_{tr}\right] + \frac{n^2}{8M_H^2}\left(\eta^{\mu\nu} - \frac{n^{\mu\lambda} n^{\nu\lambda}}{n^2}\right)\partial_{\mu} h^2 \partial_{\nu} h^2 \\ & + L_S - \left(M_H n_{\mu\nu} + h_{\mu\nu} - \frac{h^2 n_{\mu\nu}}{2M_H}\right)\frac{T_S^{\mu\nu}}{M_P} + O(1/M_H^2) \end{aligned} \tag{4.22}$$

The bilinear field term

$$\frac{M_H}{M_P} n_{\mu\nu} T_S^{\mu\nu} \tag{4.23}$$

in the third line in the Lagrangian (4.22) merits special notice. This term arises from the interaction term with tensor field. It could significantly affect the dispersion relation for the all the fields included in $T_S^{\mu\nu}$, thus leading to an unacceptably large Lorentz violation if the SLIV scale M_H were comparable with the Planck mass M_P. However, this term can be gauged away [19] by an appropriate redefinition of the fields involved by going to the new coordinates

$$x^{\mu} \to x^{\mu} + \xi^{\mu}. \tag{4.24}$$

In fact, with a simple choice of the parameter function $\xi^{\mu}(x)$ being linear in 4-coordinate

[2] The solution for a gauge function $\xi_{\mu}(x)$ satisfying the condition (4.21) can generally be chosen as $\xi_{\mu} = \Box^{-1}(\partial^{\rho} h_{\mu\rho}) + \partial_{\mu}\theta$, where $\theta(x)$ is an arbitrary scalar function, so that only three degrees of freedom in $h_{\mu\nu}$ are actually eliminated.

$$\xi^{\mu}(x) = \frac{M_H}{M_P} n^{\mu\nu} x_{\nu} \; , \tag{4.25}$$

the term (4.23) is cancelled by an analogous term stemming from the kinetic term in L_S. On the other hand, since the diff invariance is an approximate symmetry of the Lagrangian L we started with (4.14), this cancellation will only be accurate up to the linear order corresponding to the tensor field theory. Indeed, a proper extension of this theory to GR[1] with its exact diff invariance will ultimately restore the usual dispersion relation for the vector field and other matter fields involved. We will consider all that in significant detail in the next section.

So, with the Lagrangian (4.22) and the supplementary conditions (4.18) and (4.21) lumped together, one eventually comes to a working model for the emergent tensor field gravity [19]. Generally, from ten components of the symmetric two-index tensor $h_{\mu\nu}$ four components are excluded by the supplementary conditions (4.18) and (4.21). For a plane gravitational wave propagating in, say, the z direction another four components are also eliminated, due to the fact that the above supplementary conditions still leave freedom in the choice of a coordinate system, $x^{\mu} \to x^{\mu} + \xi^{\mu}(t - z/c)$, much as it takes place in standard GR. Depending on the form of the VEV tensor $n_{\mu\nu}$, caused by SLIV, the two remaining transverse modes of the physical graviton may consist solely of Lorentzian Goldstone modes or of pseudo-Goldstone modes, or include both of them. This theory, similar to the nonlinear QED [14], while suggesting an emergent description for graviton, does not lead to physical Lorentz violation [19].

4.1.3 Length Fixing Constraints and Nonlinear Gauge

We have overviewed above the SLIV scenarios for vector and tensor fields and could see that, though the well motivated length fixing constraint for a given field causes spontaneous Lorentz violation, somewhat counterintuitively, in physical processes, Lorentz symmetry appears intact. Therefore we rightfully suspect that the Lorentz breaking constraint condition acts effectively as a gauge fixing condition. To prove or disprove whether this suspicion is reasonable one either should check the SLIV effects in the corresponding physical processes in all orders, that looks unrealistic, or has to find some generic argument, particularly find a solution for gauge function or, at least, prove that such a solution exists.

In case of vector field A_{α} and Lorentz breaking condition $A_{\alpha}^2 = n_{\beta}^2 M_A^2$, the corresponding equation for gauge function S is

$$(A_{\alpha} + \partial_{\alpha} S)^2 = n_{\beta}^2 M_A^2 \tag{4.26}$$

This equation is nonlinear and its exact solution for arbitrary A_{α} is not yet found. However, to our fortune, it is well known that this equation taken for time-like violation case ($n_{\beta}^2 = 1$) is in fact the Hamilton-Jacobi equation for the relativistic particle, which moves in the external electromagnetic field. An action for such a system is given by

$$S = \int M\sqrt{dx_\alpha dx^\alpha} - A_\alpha dx^\alpha$$
$$= \int \left(M\sqrt{u_\alpha u^\alpha} - A_\alpha u^\alpha\right) d\tau \tag{4.27}$$

where τ is evolution parameter and $u_\alpha = \frac{dx_\alpha}{d\tau}$. In this case, even though we do not have exact solution for that, we know that an action S describes a physical system and therefore it has a solution for an arbitrary electromagnetic field A_α.

Analogously, for the space-like n_β ($n_\beta^2 = -1$) our basic equation (4.26) might be considered as the Hamilton-Jacobi equation for a hypothetical tachyon moving in the external electromagnetic field

$$S = \int M\sqrt{-dx_\alpha dx^\alpha} - A_\alpha dx^\alpha = \int \left(M\sqrt{-u_\alpha u^\alpha} - A_\alpha u^\alpha\right) d\tau \tag{4.28}$$

So, though this action can only correspond to a hypothetical particle, which is not discovered so far, theoretically it might exist at least as a free particle state. At this point we are unable to solve (4.26) exactly nor for time-like, neither for space-like cases, but we can check that ultra-relativistic particle and tachyon (in the limit of very large momenta, when particle velocity $\nu_p \longrightarrow c$ from below and tachyon velocity $\nu_t \longrightarrow c$ from above) have somewhat similar equations of motions

$$\frac{d}{dt}p_i = F_{0i} - \frac{p_l}{\sqrt{p_k^2}}F_{li}$$
$$\frac{d}{dt}p_i = -F_{0i} + \frac{p_l}{\sqrt{p_k^2}}F_{li} \tag{4.29}$$

with the electromagnetic field flipped for tachyon (p_i stands for the corresponding three-momenta). No dependent, one believes or not in an existence of charged tachyon one might at least can take this similarity as a hint that in space-like case, similar to a time-like violation, we are dealing with effectively nonlinear gauge fixing condition.

For the tensor field, diff gauge invariance also could only fully be approved, when corresponding gauge function $\xi_\alpha(x_\mu)$ is found, which satisfies the following equation

$$(H_{\alpha\beta} + \partial_\alpha\xi_\beta + \partial_\beta\xi_\alpha)^2 = \pm M_H^2 \tag{4.30}$$

While we do not have a heuristic argument like that we had above for the vector field time-like SLIV case, we can provide some arguments very similar to its space-like violation case leading again to the mainly intuitive suggestion.

So, to conclude, though the above discussion looks highly suggestive towards the vector and tensor field constraints, (4.3) and (4.9), to consider them as the be nonlinear gauge choices, they are not yet, sure, the rigorous proofes. Therefore, presently the only way to check whether these constraints are just gauge choices or not is actually related to seeking of the SLIV efects by a direct analysis of the corresponding physical processes.

4.2 Electrogravity model

Usually, an emergent gauge field framework is considered either regarding emergent photons or regarding emergent gravitons, but in nature they do not exist in separate framework, they are different parts of one picture and therefore the most natural thing is to discuss them as such. For the first time, we consider it regarding them both in the so-called electrogravity theory where together with the Nambu QED model [14] with its gauge invariant Lagrangian we propose the linearized Einstein-Hilbert kinetic term for the tensor field preserving a diff invariance (more details can be found in our recent paper [20]). We show that such a combined SLIV pattern, conditioned by the constraints (4.3) and (4.9), induces the massless Goldstone modes which appear shared among photon and graviton. One needs in common nine zero modes both for photon (three modes) and graviton (six modes) to provide all necessary (physical and auxiliary) degrees of freedom. They actually appear in our electrogravity theory due to spontaneous breaking of high symmetries of our constraints. While for a vector field case the symmetry of the constraint coincides with Lorentz symmetry $SO(1,3)$, the tensor field constraint itself possesses much higher global symmetry $SO(7,3)$, whose spontaneous violation provides a sufficient number of zero modes collected in a graviton. As we understand already these modes are largely pseudo-Goldstone modes since $SO(7,3)$ is symmetry of the constraint (4.9) rather than the electrogravity Lagrangian whose symmetry is only given by Lorentz invariance.

4.2.1 Constraints and zero mode spectrum

Before going any further, let us make some necessary comments. Note first of all that, apart from dynamics that will be described by the total Lagrangian, the vector and tensor field constraints (4.3, 4.9) are also proposed to be satisfied. In principle, these constraints, like in previous cases, could be formally obtained from the conventional potential introduced in the total Lagrangian. The most general potential, where the vector and tensor field couplings possess the Lorentz and $SO(7,3)$ symmetry, respectively, must be solely a function of $A_\mu^2 \equiv A_\mu A^\mu$ and $H_{\mu\nu}^2 \equiv H_{\mu\nu}H^{\mu\nu}$. Indeed, it cannot include any contracted and intersecting terms like as H_{tr}, $H^{\mu\nu}A_\mu A_\nu$ and others which would immediately reduce the above symmetries to the common Lorentz one. So, one may only write

$$U(A,H) = \lambda_A(A_\mu^2 - n^2 M_A^2)^2 + \lambda_H(H_{\mu\nu}^2 - \mathfrak{n}^2 M_H^2)^2 + \lambda_{AH} A_\mu^2 H_{\rho\nu}^2 \qquad (4.31)$$

where $\lambda_{A,H,AH}$ stand for the coupling constants of the vector and tensor fields, while values of $n^2 = \pm 1$ and $\mathfrak{n}^2 = \pm 1$ determine their possible vacuum configurations. As a consequence, an absolute minimum of the potential (4.31) might appear for the couplings satisfying the conditions

$$\lambda_{A,H} > 0\,,\ \lambda_A \lambda_H > \lambda_{AH}/4 \qquad (4.32)$$

However, as in the pure vector field case discussed in section 1, this theory is generally unstable with the Hamiltonian being unbounded from below unless the phase space is constrained just by the above nonlinear conditions (4.3, 4.9). They in turn follow from the potential (4.31) when going to the nonlinear σ-model type limit $\lambda_{A,H} \to \infty$. In this limit, the massive Higgs mode disappears from the theory, the Hamiltonian becomes positive, and one comes to the pure emergent electrogravity theory considered here.

We note again that the Goldstone modes appearing in the theory are caused by breaking of global symmetries related to the constraints (4.3, 4.9) rather than directly to Lorentz violation. Meanwhile, for the vector field case symmetry of the constraint (4.3) coincides in fact with Lorentz symmetry whose breaking causes the Goldstone modes depending on the vacuum orientation vector n_μ, as can be clearly seen from an appropriate exponential parametrization for the starting vector field (4.5). However, in the tensor field case, due to the higher symmetry $SO(7,3)$ of the constraint (4.9), there are much more tensor zero modes than would appear from SLIV itself. In fact, they complete the whole tensor multiplet $h_{\mu\nu}$ in the parametrization (4.18). However, as was discussed in the previous section, only a part of them are true Goldstone modes, others are pseudo-Goldstone ones. In the minimal VEV configuration case, when these VEVs are developed only on the single A_μ and $H_{\mu\nu}$ components, one has several possibilities determined by the vacuum orientations n_μ and $\mathfrak{n}_{\mu\nu}$. There appear the twelve zero modes in total, three from Lorentz violation itself and nine from a violation of the $SO(7,3)$ symmetry that is more than enough to have the necessary three photon modes (two physical and one auxiliary ones) and six graviton modes (two physical and four auxiliary ones). We could list below all possible cases corresponding $n - \mathfrak{n}$ values, the timelike-spacelike SLIV, when $n_0 \neq 0$ and $\mathfrak{n}_{i=j} \neq 0$, the spacelike-timelike (nonzero n_i and $\mathfrak{n}_{00}$), spacelike-spacelike diagonal (nonzero n_i and $\mathfrak{n}_{i=j}$) and spacelike-spacelike nondiagonal (nonzero n_i and $\mathfrak{n}_{i\neq j}$) cases, but for brevity, instead we only list the most interesting cases corresponding to minimal and maximal Lorantz symmetry breaking.

(1) When both $n_\mu \neq 0$ and $\mathfrak{n}_{\mu\mu} \neq 0$, wether μ is time or space component we have minimally broken Lorentz invariance and only three broken generators and therefore three Goldstone modes and all of them is collected into the photon, while components of $h_{\alpha\beta}$ needed for physical graviton and its auxiliary components can be only provided by the pseudo-Goldstone modes following from the symmetry breaking $SO(7,3) \to SO(6,3)$ related to the tensor-field constraint (4.9).

(2) For the case, when $n_i \neq 0$ and $\mathfrak{n}_{\beta\gamma} \neq 0$ (one of the nondiagonal space components of the unit tensor $\mathfrak{n}_{\mu\nu}$ is nonzero), when $i \neq \beta \neq \gamma$ Lorentz symmetry appears fully broken so that the photon a_μ has three Goldstone components , while the graviton is collected by the rest of true Goldstone and PGM–s.

(3) Only case when both physical photon and graviton h_{ij} consists of true Goldstone modes is when $n_0 \neq 0$ and $\mathfrak{n}_{i\neq j} \neq 0$, but some gauge degrees of freedom for a graviton are given by the PGM states stemming from the symmetry breaking of the tensor-field constraint (4.9).

In any case, while photon may only contain true Goldstone modes, some PGM-s appear necessary to be collected in graviton together with some true Goldstone modes to form full tensor multiplet.

4.2.2 The Model

In the previous section and Generally in emergent tensor field gravity theories we considered the vector field A_μ as an unconstrained material field which the emergent gravitons interacted with, but now in electrogravity model we propose that the vector field also develops the VEV through the SLIV constraint (4.3), thus generating the massless vector Goldstone modes associated with a photon. We also include the complex scalar field φ (taken to be massless, for simplicity) as an actual matter in the theory

$$\mathcal{L}(\varphi) = D_\mu \varphi \, (D_\mu \varphi)^* \, , \; D_\mu = \partial_\mu + ieA_\mu \, . \tag{4.33}$$

So, the proposed total starting electrogravity Lagrangian is

$$\mathcal{L}_{tot} = L(A) + L(H) + L(\varphi) + L_{int}(H, A, \varphi) \tag{4.34}$$

where $L(A)$ and $L(H)$ are U(1) gauge invariant and diff invariant vector and tensor field Lagrangians, while the gravity interaction part

$$L_{int}(H, A, \varphi) = -\frac{1}{M_P} H_{\mu\nu}[T^{\mu\nu}(A) + T^{\mu\nu}(\varphi)] \tag{4.35}$$

contains the tensor field couplings with canonical energy-momentum tensors of vector and scalar fields.

In the symmetry broken phase one goes to the pure Goldstone vector and tensor modes, a_μ and $h_{\mu\nu}$, respectively, Which is thoroughly discussed in the previous sections (4.8), (4.22). At the same time, the scalar field Lagrangian $\mathcal{L}(\varphi)$ in (4.34) is going now to

$$\mathcal{L}(\varphi) = \left| \left(\partial_\mu + iea_\mu + ieM_A n_\mu - ie\frac{n^2}{2M_A} a^2 n_\mu \right) \varphi \right|^2 \tag{4.36}$$

while tensor field interacting terms (4.35) in $\mathcal{L}_{int}(H, A, \varphi)$ convert to

$$\mathcal{L}_{int} = -\frac{1}{M_P}\left(h_{\mu\nu} + M_H n_{\mu\nu} - \frac{n^2}{2M_H} h^2 n_{\mu\nu} \right) \left[T^{\mu\nu}\left(a_\mu - \frac{n^2}{2M_A} a^2 n_\mu \right) + T^{\mu\nu}(\varphi) \right] \tag{4.37}$$

where the vector field energy-momentum tensor is now solely a function of the Goldstone a_μ modes.

4.2.3 Emergent electrogravity interactions

To proceed further, one should eliminate, first of all, the large terms of the false Lorentz violation being proportional to the SLIV scales M_A and M_H in the interaction Lagrangians (4.36) and (4.37). Arranging the phase transformation for the scalar field in the following way

$$\varphi \to \varphi \exp[-ieM_A n_\mu x^\mu] \tag{4.38}$$

one can simply cancel that large term in the scalar field Lagrangian (4.36), thus coming to

$$\mathcal{L}(\varphi) = \left| \left(D_\mu - ie\frac{n^2}{2M_A} a^2 n_\mu \right) \varphi \right|^2 \tag{4.39}$$

where the covariant derivative D_μ is read from now on as $D_\mu = \partial_\mu + iea_\mu$. Another unphysical set of terms (4.23) appear from the gravity interaction Lagrangian L_{int} (4.37) where the large SLIV entity $M_H \mathfrak{n}_{\mu\nu}$ couples to the energy-momentum tensor. They also can be eliminated by going to the new coordinates (4.24), as was mentioned in the previous section.

For infinitesimal translations $\xi_\mu(x)$ the tensor field transforms according to (4.16), while scalar and vector fields transform as

$$\delta\varphi = \xi_\mu \partial^\mu \varphi, \ \delta a_\mu = \xi_\lambda \partial^\lambda a_\mu + \partial_\mu \xi_\nu a^\nu \ , \tag{4.40}$$

respectively. One can see, therefore, that the scalar field transformation has only the translation part, while the vector one has an extra term related to its nontrivial Lorentz structure. For the constant unit vector n_μ this transformation looks as

$$\delta n_\mu = \partial_\mu \xi_\nu n^\nu, \tag{4.41}$$

having no the translation part. Using all that and also expecting that the phase parameter ξ_λ is in fact linear in coordinate x_μ (that allows to drop out its high-derivative terms), we can easily calculate all scalar and vector field variations, such as

$$\delta\left(D_\mu \varphi\right) = \xi_\lambda \partial^\lambda (D_\mu \varphi) + \partial_\mu \xi_\lambda D^\lambda \varphi, \ \delta f_{\mu\nu} = \xi_\lambda \partial^\lambda f_{\mu\nu} + \partial_\mu \xi^\lambda f_{\lambda\nu} + \partial_\nu \xi^\lambda f_{\mu\lambda} \tag{4.42}$$

and others. This finally leads to the total variations of the above Lagrangians. Whereas the pure tensor field Lagrangian $L(H)$ (4.15) is invariant under diff transformations, $\delta L(H) = 0$, the interaction Lagrangian L_{int} in (4.34) is only approximately invariant being compensated (in the lowest order in the transformation parameter ξ_μ) by kinetic terms of all the fields involved. However, this Lagrangian becomes increasingly invariant once our theory is extending to GR[1].

In contrast, the vector and scalar field Lagrangians acquire some nontrivial additions

$$\begin{aligned}
\delta L(A) &= \xi_\lambda \partial_\lambda L(A) \\
&\quad -\frac{1}{2}\left(\partial_\mu \xi_\lambda + \partial_\lambda \xi_\mu\right)\left[f^{\mu\nu} f^\lambda_\nu + \frac{n^2}{M_A}\left(f^\lambda_\nu \partial^{\mu\nu} a^2 + \frac{1}{2} f_{\rho\nu} \partial^{\rho\nu}\left(a^\mu a^\lambda\right)\right)\right] \\
\delta L(\varphi) &= \xi_\lambda \partial_\lambda L(\varphi) + \left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right)\left[\left(\mathfrak{D}^\mu \varphi\right)^* \mathfrak{D}^\nu \varphi + \frac{a^\mu a^\nu n^2}{2M_A} n_\lambda J_\lambda\right]
\end{aligned} \tag{4.43}$$

where J_μ stands for the conventional vector field source current

$$J_\mu = ie[\varphi^* D_\mu \varphi - \varphi \left(D_\mu \varphi\right)^*] \tag{4.44}$$

while $\mathfrak{D}_\nu \varphi$ is the SLIV extended covariant derivative for the scalar field

$$\mathfrak{D}_\nu \varphi = D_\nu \varphi - ie\frac{n^2}{2M_A} a^2 n_\nu \varphi \tag{4.45}$$

The first terms in the variations (4.43) are unessential since they simply show that these Lagrangians transform, as usual, like as scalar densities under diff transformations.

Combining these variations with L_{int} (4.37) in the total Lagrangian (4.34) one finds after simple, though long, calculations that the largest Lorentz violating terms in it

$$-\left(\frac{M_H}{M_P}n_{\mu\nu}-\frac{\partial_\mu\xi_\lambda+\partial_\lambda\xi_\mu}{2}\right)\left[-f^{\mu\nu}f^\lambda_\nu-\frac{n^2}{M_A}f^\nu_\lambda\partial^{\mu\lambda}a^2+2\mathfrak{D}^\nu\varphi\left(\mathfrak{D}^\mu\varphi\right)^*\right] \quad (4.46)$$

will immediately cancel if the transformation parameter is chosen exactly as is given in (4.25) in the previous section. So, with this choice we finally have for the modified interaction Lagrangian

$$\mathcal{L}'_{int}(h,a,\varphi)=-\frac{1}{M_P}h_{\mu\nu}T^{\mu\nu}(a,\varphi)+\frac{1}{M_PM_A}\mathcal{L}_1+\frac{1}{M_PM_H}\mathcal{L}_2+\frac{M_H}{M_PM_A}\mathcal{L}_3 \quad (4.47)$$

where

$$\begin{aligned}\mathcal{L}_1&=n^2h_{\mu\nu}\left[f^\nu_\lambda\partial^{\mu\lambda}a^2-n^\mu J^\nu+\eta^{\mu\nu}\left(-\frac{1}{4}f_{\lambda\rho}\partial^{\lambda\rho}a^2+n^\lambda J_\lambda\right)\right]\\ \mathcal{L}_2&=\frac{1}{2}n^2h^2n_{\mu\nu}\left[-f^{\mu\lambda}f^\nu_\lambda+2D^\nu\varphi\left(D^\mu\varphi\right)^*\right]\\ \mathcal{L}_3&=n^2n_{\mu\lambda}\left[\frac{1}{2}f_{\rho\nu}\partial^{\rho\nu}\left(a^\mu a^\lambda\right)-(a^\mu a^\lambda)n^\nu J_\nu\right]\end{aligned} \quad (4.48)$$

Thereby, apart from a conventional gravity interaction part given by the first term in (4.47), there are Lorentz violating couplings in $\mathcal{L}_{1,2,3}$ being properly suppressed by corresponding mass scales. Note that the coupling presented in $\mathcal{L}_3$ between the vector and scalar fields is solely induced by the tensor field SLIV. Remarkably, this coupling may be in principle of the order of a normal gravity coupling or even stronger, if $M_H > M_A$. However, appropriately simplifying this coupling (and using also a full derivative identity) one comes to

$$\mathcal{L}_3\sim n^2\left(n_{\mu\lambda}a^\mu a^\lambda\right)n^\rho\left[\partial^\nu f_{\nu\rho}-J_\rho\right] \quad (4.49)$$

that after applying of the vector field equation of motion turns it into zero. We consider it in more detail in the next section where we calculate some tree level processes.

4.3 The lowest order SLIV processes

The emergent vector field Lagrangian (4.8) and emergent gravity Lagrangian in (4.22) taken separately present in fact highly nonlinear theory which contains lots of Lorentz and CPT violating couplings. Nevertheless, as it was shown in [19,16,17] in the lowest order calculations, they all are cancelled and do not manifest themselves in physical processes. As we talked about earlier, this may mean that the length-fixing constraints (4.3,4.9) put on the vector and tensor fields appear as the gauge fixing conditions rather than a source of an actual Lorentz violation. In the context of electrogravity model, which contains both photon and graviton as the emergent gauge fields, this means that only source of new physics can be (4.47). Even if suspicion that length fixing constraints are nonlinear gauge choices is true, for Lorentz invariance to be realized anyway, $U(1)$ and diff gauge transformations should commute in the symmetry broken phase and then we could claim that $\mathcal{L}_1$ and $\mathcal{L}_2$ in (4.47) will have no physical effects, but there is also (4.48), which is proportional to diff transformation parameter and strictly speaking it is not zero Lagrangian. So, in this picture to be logically sound and consistent we should check all interactions in the (4.47) anyway.

For that one properly derive all necessary Feynman rules and then calculate the basic lowest order processes, such as photon-graviton scattering and their conversion, photon scattering on the matter scalar field and other, that has been throughly carried out in our paper mentioned above [20] where can be found all necessary details. These calculations explicitly demonstrate that all the SLIV effects in these processes are strictly cancelled. This appears due to an interrelation between the longitudinal graviton and photon exchange diagrams and the corresponding contact interaction diagrams. So, physical Lorentz invariance in all processes is left intact. Apart, many other tree level Lorentz violating processes related to gravitons and vector fields (interacting with each other and the matter scalar field in the theory) may also appear in higher orders in the basic SLIV parameters $1/M_H$ and $1/M_A$, by iteration of couplings presented in our basic Lagrangians (4.22, (4.47)) or from a further expansions of the effective vector and tensor field Higgs modes (4.7, 4.18) inserted into the starting total Lagrangian (4.34). Again, their amplitudes appear to cancel each other, thus eliminating physical Lorentz violation in the theory.

Most likely, the same conclusion could be expected for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [16], the corresponding one-loop matrix elements in our emergent electrogravity theory could either vanish by themselves or amount to the differences between pairs of similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external four-momenta of the particles involved) which, in the framework of dimensional regularization, could lead to their total cancellation.

So, after all, it should not come as too much of a surprise that emergent electrogravity theory considered here is likely to eventually possess physical Lorentz invariance provided that the underlying gauge and diff invariance in the theory remains unbroken.

4.4 Conclusion

We have combined emergent photon and graviton into one framework of electrogravity. While photon emerges as true vector Goldstone mode from SLIV, graviton at least partially consists of PGM-s as well, because alongside of Lorentz symmetry much bigger global symmetry of (4.9) $SO(7,3)$ is broken as well. Configuration of true Goldstone and PGM-s inside graviton solely depends on VEV-s of vector and tensor fields. So, in total 12 massless Goldstone modes are born to complete photon and graviton multiplets with an orthogonality conditions $n^{\mu}a_{\mu} = 0$, $n^{\mu\nu}h_{\mu\nu} = 0$ in place. Emergent electrogravity theory is nonlinear and in principal contains many Lorentz and CPT violating interactions, when expressed in terms of Goldstone modes. Nonetheless, all non-invariant effects disappear in all possible lowest order physical processes, which means that Lorentz invariance is intact and hence Lorentz invariance breaking conditions (4.3, 4.9) act as a gauge fixing for photon and graviton, instead of being actual source of physical Lorentz violation in the theory. If this cancellation occurs in all orders (i.e. (4.3, 4.9) are truly nonlinear gauge fixing conditions), then emergent electrogravity is physically indistinguishable from conventional gauge theories and spontaneous Lorentz violation caused by the vector and tensor field constraints (4.3, 4.9) appear hidden in gauge degrees of freedom, and only results in a noncovariant gauge choice in an otherwise gauge invariant emergent electrogravity theory.

From this standpoint, the only way for physical Lorentz violation to take place would be if the above gauge invariance were slightly broken by near Planck scale physics, presumably by quantum gravity or some other high dimensional theory. This is in fact a place where the emergent vector and tensor field theories may drastically differ from conventional QED, Yang-Mills and GR theories where gauge symmetry breaking could hardly induce physical Lorentz violation. In contrast, in emergent electrogravity such breaking could readily lead to many violation effects including deformed dispersion relations for all matter fields involved. Another basic distinction of emergent theories with non-exact gauge invariance is a possible origin of a mass for graviton and other gauge fields (namely, for the non-Abelian ones, see [18]), if they, in contrast to photon, are partially composed from pseudo-Goldstone modes rather than from pure Goldstone ones. Indeed, these PGM-s are no longer protected by gauge invariance and may properly acquire tiny masses, which still do not contradict experiment. This may lead to a massive gravity theory where the graviton mass emerges dynamically, thus avoiding the notorious discontinuity problem [21].

So, while emergent theories with an exact local invariance are physically indistinguishable from conventional gauge theories, there are some principal distinctions when this local symmetry is slightly broken which could eventually allow us to differentiate between the two types of theory in an observational way.

Acknowledgements

We would like to thank Colin Froggatt, Archil Kobakhidze and Holger Nielsen for useful discussions and comments. Z.K. wants to thank participants of the 21st Workshop "What Comes Beyond the Standard Models?" (23-30 June, Bled, Slovenia) for interesting and useful discussions, as well as the organizers for such a productive and working environment. This work is partially supported by Georgian National Science Foundation (grant No. YS-2016-81).

References

1. J.D. Bjorken, Ann. Phys. (N.Y.) **24** (1963) 174.
2. P.R. Phillips, Phys. Rev. **146** (1966) 966.
3. T. Eguchi, Phys.Rev. D **14** (1976) 2755;
4. Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122** (1961) 345;
   J. Goldstone, Nuovo Cimento **19** (1961) 154.
5. J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Phys. Rev. Lett. **87** (2001) 091601; Nucl. Phys. B **609** (2001) 46.
6. J.D. Bjorken, hep-th/0111196.
7. Per Kraus and E.T. Tomboulis, Phys. Rev. D **66** (2002) 045015.
8. A. Jenkins, Phys. Rev. D **69** (2004) 105007.
9. V.A. Kostelecky, Phys. Rev. D **69** (2004) 105009.
10. Z. Berezhiani and O.V. Kancheli, arXiv:0808.3181.
11. V.A. Kostelecky and R. Potting, Phys. Rev. D **79** (2009) 065018.
12. S.M. Carroll, H.Tam and I.K. Wehus, Phys. Rev. D **80** (2009) 025020.
13. R. Bluhm, N.L. Cage, R. Potting and A. Vrublevskis, Phys. Rev. D **77** (2008) 125007.
14. Y. Nambu, Progr. Theor. Phys. Suppl. Extra 190 (1968).
15. S. Weinberg, *The Quantum Theory of Fields*, v.2, Cambridge University Press, 2000.
16. A.T. Azatov and J.L. Chkareuli, Phys. Rev. D **73** (2006) 065026 .
17. J.L. Chkareuli and Z.R. Kepuladze, Phys. Lett. B **644** (2007) 212 .
18. J.L. Chkareuli and J.G. Jejelava, Phys. Lett. B **659** (2008) 754.
19. J.L. Chkareuli, J.G. Jejelava, G. Tatishvili, Phys. Lett. B 696 (2011) 126;
    J.L. Chkareuli, C.D. Froggatt and H.B. Nielsen, Nucl. Phys. B 848 (2011) 498.
20. J.L. Chkareuli, J. Jejelava, Z. Kepuladze, Eur. Phys. J. C78 (2018) 156; e-Print: arXiv:1709.02736 [hep-th].
21. H. van Dam and M. J. G. Veltman, Nucl. Phys. B **22** (1970) 397.