



# Chiral Quark Soliton Model and Nucleon Spin Structure Functions

M. Wakamatsu

Department of Physics, Faculty of Science, Osaka University, Toyonaka, Osaka 560-0043, JAPAN

## 1 Introduction

What is the CQSM like? To answer this question, it is instructive to ask another simpler question. What is, or what was, the Skyrme model? In a word, the famous Skyrme model is Bohr's model in baryon physics. The simplest microscopic basis of Bohr's collective model of rotational nuclei is provided by the deformed Hartree-Fock theory supplemented with the subsequent cranking quantization. Very roughly speaking, the relation between the CQSM and the Skyrme model is resembling the relation between these two theories in nuclear physics. Let us start with a brief history of the CQSM.

- The model was first proposed by Diakonov, Petrov and Pobylitsa based on the instanton picture of the QCD vacuum in 1988 [1].
- In 1991 [2], we have established a basis of numerical calculation, which enables us to make nonperturbative estimate of nucleon observables with full inclusion of the deformed Dirac-sea quarks, by extending the method of Kahana, Ripka and Soni [3],[4]. Also derived and discussed in this paper is the nucleon spin sum rule, which reveals the important role of quark orbital angular momentum in the nucleon spin problem.
- In 1993, we noticed the existence of novel  $1/N_c$  correction to some isovector observables, which is totally missing within the framework of the Skyrme model, but it certainly exists within the CQSM, so that it resolves the long-standing  $g_A$ -problem inherent in the hedgehog soliton model [5] (see also [6]).
- The next important step is an application of the model to the physics of parton distribution functions of the nucleon, initiated by Diakonov et al. [7],[8] and also by Tübingen group [9],[10].

## 2 Main achievements of the CQSM for low energy observables

Skipping the detailed explanation of the model, I just summarize below several noteworthy achievements of the CQSM for low energy baryon observables.

- First of all, it reproduces unexpectedly small quark spin fraction of the nucleon [2],[11]-[13] in conformity with the famous EMS observation [14] :

$$\Delta\Sigma \simeq 0.35. \quad (1)$$

- Secondly, it reproduces fairly large pion-nucleon sigma-term favored in the recent phenomenological determination [15] (see also [16]) :

$$\Sigma_{\pi N} \simeq 60 \text{ MeV}. \quad (2)$$

- Furthermore, it resolves the famous  $g_A$ -problem of the Skyrme model as [5],[6]

$$g_A^{(\text{Skyrme})} = g_A(\Omega^0) + g_A(\Omega^1) \simeq 0.8 + 0.0 = 0.8, \quad (3)$$

$$g_A^{(\text{CQSM})} = g_A(\Omega^0) + g_A(\Omega^1) \simeq 0.8 + 0.4 = 1.2. \quad (4)$$

Unfortunately, most baryon observables are quite insensitive to the differences of low energy models, which results in masking the potential ability of the CQSM as compared with the others. It turns out, however, that that the superiority of the CQSM as a field theoretical model of baryons manifests most drastically in its predictions for the internal partonic structure of the nucleon.

### 3 On the role and achievements of CQSM in DIS physics

The standard approach to the DIS (deep-inelastic-scattering) physics is based on the so-called *factorization theorem*, which states that the DIS amplitude is factorized into two part, i.e. the *hard part* which can be handled by the perturbative QCD and the *soft part* which contains information on the nonperturbative quark-gluon structure of the nucleon. The soft part is usually treated as a blackbox, which should be determined via experiments. This is a reasonable strategy, since we have no simple device to solve nonperturbative QCD. We however believe that, even if this part is completely fixed by experiments, one still wants to know why those parton distribution functions (PDFs) take the form so determined ! Nonstandard but complementary approach to DIS physics is necessary here to understand hidden chiral dynamics of soft part, based on models or on lattice QCD.

There are several merits of the CQSM over many other effective model of baryons. First, it is a relativistic mean-field theory of quarks, consistent with the large  $N_c$  QCD supplemented with the  $1/N_c$  expansion. Secondly, the field theoretical nature of the model, i.e. nonperturbative inclusion of polarized Dirac-sea quarks, enables reasonable estimation not only of quark distributions but also of *antiquark* distributions. Finally, only 1 parameter of the model, i.e. the dynamical quark mass  $M$ , was already fixed from low energy phenomenology, which means that we can make *parameter-free* predictions for parton distribution functions. As a matter of course, the biggest default of the model is the lack of the explicit gluon degrees of freedom.

In Fig.1, we summarize parameter-free predictions of the CQSM for the three fundamental twist-2 PDFs. They are the unpolarized PDF with isoscalar and isovector combinations, the longitudinally polarized PDF with isoscalar and isovector combinations, and finally the transversities with isoscalar and isovector combinations. Noteworthy here is totally *different behavior* of the Dirac-sea contributions in *different PDFs*.

The crucial importance of the Dirac-sea contribution can most clearly be seen in the isoscalar unpolarized PDF. First, I recall that the distribution function in the negative  $x$  region should be identified with the antiquark distribution with the extra minus sign.

$$\bar{q}(x) = -q(-x), \quad (0 < x < 1). \quad (5)$$

Then, one can see that the positivity of the antiquark distribution  $\bar{u}(x) + \bar{d}(x)$  is satisfied only after including the Dirac-sea contribution. It is also seen to generate sea-like soft component in the quark distribution in the small  $x$  region, as required in the GRV analysis even at the low energy scale [17].

Turning to the isovector unpolarized PDF, I point out that the  $u(x) - d(x)$  is positive with sizable magnitude in the negative  $x$  region due to the effect of Dirac-sea contribution. Because of the charge conjugation property of this distribution, it means that  $\bar{u}(x) - \bar{d}(x)$  is negative or  $\bar{d}(x) - \bar{u}(x)$  is positive in consistency with the famous NMC observation [18]-[20]. One can also confirm that the model prediction for the  $\bar{d}(x)/\bar{u}(x)$  ratio is consistent with the Fermi-Lab Drell-Yan data at least qualitatively [13].

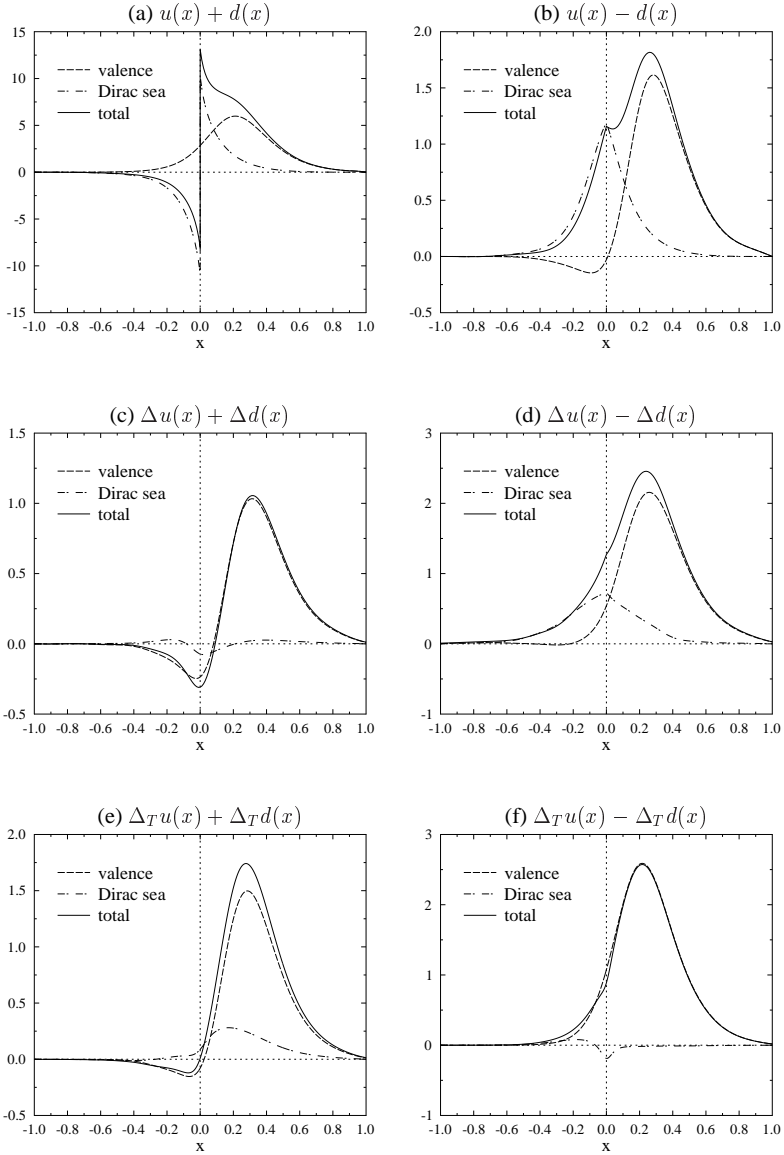
Although we do not have enough space to go into the detail, we can also show that the model also reproduces all the characteristic features of the longitudinally polarized structure functions of the proton, neutron and the deuteron without introducing any additional parameters [11],[13].

## 4 Chiral-odd twist-3 distribution function $e(x)$

The distribution function  $e(x)$  is one of the three twist-3 distribution functions of the nucleon. Why is it interesting? Firstly, its first moment is proportional to the famous  $\pi N$  sigma term. Secondly, within the framework of perturbative QCD, it was noticed that this distribution function may have a delta-function type singularity at  $x = 0$  [21]. However, the physical origin of this delta-function type singularity was left unclear within the perturbative consideration.

By utilizing the advantage of the CQSM, in which the effects of Dirac-sea quarks can be treated nonperturbatively, we have tried to clarify the physical origin of this delta-function type singularity [22],[15]. We first verified that, because of the spontaneous chiral symmetry breaking of the QCD vacuum, the scalar quark density of the nucleon does not damp as the distance from the nucleon center becomes large, but it approaches a nonzero negative constant, which is nothing but the vacuum quark condensate. (See Fig.2.)

It was shown further that this extraordinary nature of the scalar quark density in the nucleon, i.e. the existence of the *infinite range* quark-quark correlation



**Fig. 1.** The CQSM predictions for the fundamental twist-2 PDFs of the nucleon : isoscalar and isovector unpolarized PDFs ((a) and (b)), isoscalar and isovector longitudinally polarized PDFs ((c) and (d)), and isoscalar and isovector transversity distributions ((e) and (f)).

of scalar type, is the physical origin of the delta-function singularity in the chiral-odd twist-3 distribution  $e(x)$ . This singularity of  $e(x)$  will be observed as the violation of  $\pi N$  sigma-term sum rule. To confirm this interesting possibility, we need very precise experimental information for  $e(x)$  through the semi-inclusive DIS scatterings.

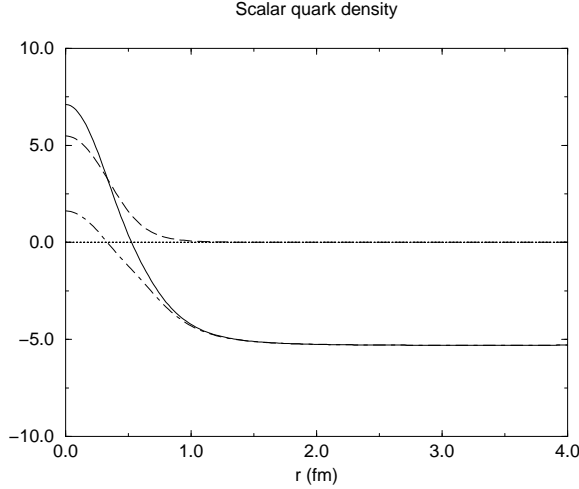


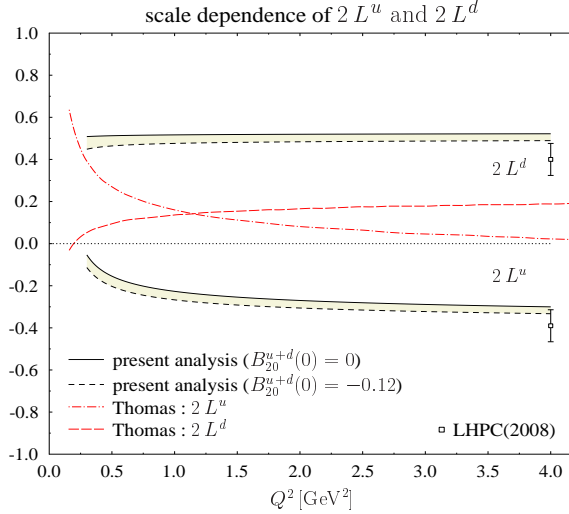
Fig. 2. The scalar quark density predicted by the CQSM.

## 5 Proton spin problem revisited : current status and resolution

Now, we come back to our biggest concern of study, i.e. the nucleon spin problem. Recent two remarkable progresses may be worthy of mention. First, the quark polarization  $\Delta\Sigma$  has been fairly precisely determined, through the high-statistics measurements of deuteron spin structure function by the COMPASS and HERMES groups [23],[24]. Second, a lot of evidences have been accumulated, which indicate that the gluon polarization is likely to be small or at least it cannot be large enough to resolve the puzzle of the missing nucleon spin based on the  $U_A(1)$  anomaly scenario. A general consensus now is therefore as follows. About  $1/3$  of the nucleon spin is carried by the intrinsic quark spin, while the remaining  $2/3$  should be carried by  $L^Q$ ,  $\Delta g$ , and  $L^g$ .

Recently, Thomas advocates a viewpoint that the modern spin discrepancy can well be explained in terms of standard features of the nonperturbative structure of the nucleon, i.e. relativistic motion of valence quarks, the pion cloud required by chiral symmetry, and an exchange current contribution associated with the one-gluon-exchange hyperfine interaction [25]-[27]. His analysis starts from an estimate of the orbital angular momenta of up and down quarks based on the improved (or fine-tuned) cloudy bag model taking account of the above-mentioned effects. Another important factor of his analysis is the observation that the angular momentum is not a renormalization group invariant quantity, so that the above predictions of the model should be associated with a very low energy scale, say,  $0.4\text{ GeV}$ . Then, after solving the QCD evolution equations for the up and down quark angular momenta, first derived by Ji, Tang and Hoodbhoy [28], he was led to a remarkable conclusion that the orbital angular momenta of up and down quarks cross over around the scale of  $1\text{ GeV}$ . This crossover of  $L^u$  and  $L^d$  seems absolutely necessary for his scenario to hold. Otherwise, the prediction  $L^u - L^d > 0$  of the improved cloudy bag model given at the low energy scale is

incompatible with the current empirical information or lattice QCD simulations at the high energy scale, which gives  $L^u < 0, L^d > 0$ .



**Fig. 3.** Our semi-phenomenological predictions of the orbital angular momenta of up and down quarks in the proton are compared with the corresponding results of Thomas' analysis [27]. Also shown for comparison are the predictions of the LHPC lattice simulations for  $2L^u$ , and  $2L^d$  given at the scale  $Q^2 = 4 \text{ GeV}^2$  [34].

On the other hand, we have recently carried out a semi-empirical analysis of the nucleon spin contents based on Ji's angular momentum sum rule, and extracted the orbital angular momentum of up and down quarks as functions of the scale [32]. (See also [33].) Remarkably, we find no crossover of  $L^u$  and  $L^d$  when  $Q^2$  is varied, in sharp contrast to Thomas' analysis. This difference is remarkable, since if there is no crossover of  $L^u$  and  $L^d$ , Thomas' scenario for resolving the proton spin puzzle is seriously challenged.

We show in Fig.3 the results of our semi-empirical analysis for  $L^u$  and  $L^d$  in comparison with the corresponding predictions by Thomas. As already mentioned, Thomas' results show that the orbital angular momenta of up and down quarks cross over around the scale of 1 GeV. In contrast, no crossover of  $L^u$  and  $L^d$  is observed in our analysis:  $L^d$  remains to be larger than  $L^u$  down to the scale where the gluon momentum fraction vanishes. Comparing the two, the cause of this difference seems obvious. Thomas claims that his results are qualitatively consistent with the empirical information and the lattice QCD data at high energy scale. (We recall that the sign of  $L^{u-d}$  at the high energy scale is constrained by the asymptotic condition  $L^{u-d}(Q^2 \rightarrow \infty) = -\Delta\Sigma^{u-d}$ , which is a necessary consequence of QCD evolution [32],[25].) However, the discrepancy between his results and the recent lattice QCD predictions seems more than qualitative.

In any case, our semi-phenomenological analysis, which is consistent with the empirical information and/or the lattice QCD data for  $J^u$  and  $J^d$ , indicates that  $L^u - L^d$  remains fairly large and negative even at the low energy scale of non-perturbative QCD. If this is confirmed, it is a serious challenge to any low energy models of nucleon, since they must now explain small  $\Delta\Sigma^Q$  and large and negative  $L^{u-d}$  *simultaneously*. The refined cloudy bag model of Thomas and Myhrer obviously fails to do this job, since it predicts  $2L^u \simeq 0.64$  and  $2L^d \simeq -0.03$  at the model scale. (See Table.1 of [27]. Shown in this table should be  $2L^u$  and  $2L^d$  not  $L^u$  and  $L^d$ .) Is there any low energy model which can pass this examination? Interestingly, the CQSM can explain both of these peculiar features of the nucleon observables. It has been long known that it can explain very small  $\Delta\Sigma^Q$  ( $\Delta\Sigma^Q \simeq 0.35$  at the model scale) due to the very nature of the model [2],[35]. Besides, its prediction for  $L^{u-d}$  given in [36], i.e.  $L^{u-d} \simeq -0.327$  at the model scale, perfectly matches the conclusion obtained in the present semi-empirical analysis.

## 6 Concluding remarks

To conclude, the CQSM is a unique model of baryons, which has an intimate connection with more popular Skyrme model. Although the former is an effective quark theory, while the latter is an effective meson theory, they share a lot of common features. In spite of many strong similarities, a crucial difference between the two theories was noticed already in the study of ordinary low energy observables of the nucleon. It is a novel  $1/N_c$  correction, or more concretely, the 1st order rotational correction, which was found to exist within the framework of the CQSM, while it is totally missing in the Skyrme model. An immediate consequence of this finding is breakdown of the so-called "Cheshire Car principle" or the fermion-boson correspondence. We can show that the origin of this breakdown of fermion-boson equivalence can eventually be traced back to the *noncommutativity* of the two procedures, i.e. the *bosonization* and the *collective quantization* of the rotational motion. Alternatively, we can simply say that an important information buried in the original fermion theory is lost in the process of approximate bosonization. (See [37] for more detail.) After all, the fact is that one is an effective quark (fermion) theory, while the other is an effective pion (meson) theory in  $3+1$  dimension.

Superiority or wider applicability of the CQSM over the Skyrme model becomes even more transparent if one extends the object of research from low energy observables to the internal partonic structure of the nucleon (or more generally of any baryons). Since the parton distribution functions measure non-local light-cone correlation between quarks (and gluons) inside the nucleon, there is no way to describe them within the framework of effective meson theories like the Skyrme model. In contrast, this is just the place where the potential power of the CQSM manifest most dramatically. In this talk, we have shown, through several concrete examples, that the CQSM provide us with an excellent tool for theoretically understanding the nonperturbative aspect of the internal partonic structure of the nucleon. In particular, we have given a very plausible solution

to the longstanding “nucleon spin problem”. We strongly believe that the proposed solution to this famous puzzle is already close to the truth, and it will be confirmed by experiments to be carried out in the near future.

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