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An inventory model with METRIC approach in location-routing-inventory problem

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ABSTRACT

In this paper, the stochastic location-routing-inventory problem is considered in which retailers' demands and lead-times are stochastic. Demand quantities follow Poisson distribution and lead-times are functions of the shortage quantity. It is also assumed that both retailers and distributors hold inventory and follow (S-1, S) inventory policy. According to these assumptions, we use METRIC (i.e., Multi-Echelon Technique for Recoverable Item Control) approach to model the problem. For this purpose, a mixed integer stochastic programming model is developed based on extending the basic location-inventory-routing model by adding METRIC stochastic relations into the model. Since solving the model with the exact method is very difficult, the Meta-heuristics are used in solving process. Specially, to empower the solution process, a hybrid method consists of simulated annealing and genetic algorithm is developed. The output results along with sensitivity analysis represent the capability of the model in taking to account the METRIC concepts in this type of supply chain problems. Meanwhile, the performance of developed hybrid Meta-heuristic method was checked and approved.

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1. Introduction

In recent decades, integrated models in supply chain have been under attention of many researchers. The main decisions of any supply chain are location and assignment, routing and inventory control in production and distribution centers. Since these decisions are mutually dependent, considering them simultaneously in supply chain can influence greatly on reducing the costs and increasing supply chain efficiency. Integration of these three decisions is done in location-routing-inventory problem. Liu and Lee [1] proposed the basic location-routing-inventory problem in 2003. They used a heuristic method based on improvement search to tackle the problem, but however the solution might be trapped in local optimum. Therefore, Liu and Lin [2] developed a new solution method for the problem. Their proposed method was able to find global optimum instead of local one. In 2007, Shen and Qi [3] embedded routing costs in the location-routing-inventory problem. They added an approximation of routing costs, which was only dependent to routing-assignment decisions, to the model. Ahmadi Javid and Azad [4] proposed a model that had great advantage compared to Shen and Qi's model. They calculated routing costs precisely by considering stochastic demand with normal distribution. In 2008, Chanchan et al. [5] considered the location-routing-inventory problem for closed loop reverse logistics. Li et al. [6] proposed a model to optimize the reverse logistics system in the location-routing-inventory problem for gathering urban wastes. Among other researches in this field, we can refer to Jiang and Ma (2009) [7] and Yang et al. (2010) [8] that developed solution methods based on meta heuristics. Sajjadi and Cheraghi (2011) [9] surveyed a three-level, multiple-product network with constrained capacity warehouse. Nekooghadirli et al. (2014) [10] considered the problem in multiple-periods and multiple-products situation with two objectives, including minimization of total costs and the minimization of maximum average product delivery-time. Recently, Chen et al. [11] surveyed the problem considering fuzzy demands. Other papers in the literature have a limited view to the inventory problem and have considered it as part of a larger problem. Among those, we can refer to Ambrosino and M. Grazia Scutellà (2005) [12], Ahmadi Javid and Seddighi (2012) [13], Seyedhosseini et al. (2014) [14], Guerrero et al. (2013) [15], and Zhang et al. (2014) [16].

As stated before, we use the METRIC approach to model the problem. The METRIC was first proposed by Sherbrooke [17] in 1968 to optimize inventory levels of spare parts at warehouses of US Air Force. He considered that the repair times of the parts are stochastic and time between two arrival demands to repair or replace follows exponential distribution. He used queue theory to calculate performance criteria in order to find the optimal level of inventories. Muckstadt [18] generalized Sherbrooke's METRIC model and considered a two-level hierarchical structure for the products. He assumed when a part needs to be repaired, one or more of its sub-parts (module) needs to be repaired. So the model should state the inventory level of both parts and sub-parts. Slay [19] developed a METRIC model and named it VARI-METRIC. He assumed the mean of items under repair equals to its variance and used negative binomial distribution to tackle it. Grave [20] used this distribution for two exact and approximate methods in a multiple-level problem. Sherbrooke [21] considered a model similar to Muckstadt model, but used a different approximation approach. The numerical results showed that this method makes considerable improvement in METRIC accuracy. Many researchers later developed this model as a reference model and utilized other improvements for it. For example, Wang et al. [22] developed a two-level repairable inventory system by considering stochastic lead time for replenishment. Rustenburg et al. [23] developed an exact model for multiple-level problem and Wong et al. [24] developed an analytical model to determine inventory levels of spare parts in a repairable inventory system with multiple factories, multiple distributors, and multiple products.

Andersson and Melchiors [25] considered a two-level model with a central depot and multiple retailers in which customers' demands were stochastic and followed Poisson distribution. They considered fixed lead-times, continuous review and (S-1, S) inventory policy in the model. They compared the proposed model with METRIC approach, and then solved the model for 13 problems.

In this paper, we propose a location-routing-inventory model in which the (S-1, S) inventory policy is considered for retailers and distributors. In addition, we consider that demands of retailers are stochastic and follow Poisson distribution. The lead-times of retailers are also stochastic and functions of shortage at distributors according to METRIC approach relations. Since the proposed model is NP-hard, an efficient hybrid method was developed to solve it. The hybrid method consists of a simulated annealing method to solve the location, assignment and routing problem and a genetic algorithm to solve the inventory problem. It is able to find the solution in reasonable time.

The remainder of the paper is organized as follows: In sections 2 and 3, problem and model description is given, respectively. Then the mathematical model is formulated in section 4. Mathematical results are brought in section 5 and finally section 6 is assigned to conclusion remarks.

2. Problem description

The problem assumptions are as follows:

- A three-level supply chain including a manufacturer, multiple distributor, and multiple retailers
- Demands process of retailers are stochastic and follow Poisson distribution. Therefore, demands of distributors are also stochastic and follow Poisson distribution.

- The manufacturer has no limit in manufacturing the product
- The locations of the manufacturer and retailers are known and the objective is to determine the optimal location of distributors
- Inventory is held in warehouses of retailers and distributors.
- The retailers and distributors follow the (S-1, S) inventory policy.
- The lead-time to replenish retailers by distributors is function of shortage at distributors. Therefore, the lead-time is stochastic.

The main objectives of the problem are as follows:

- Determine the optimal number and location of distributors
- Assign the retailers to the distributors
- Determine the optimal product delivery routes from distributors to retailers
- Determine optimal inventory levels at retailers and distributors such that the total costs of supply chain design – including holding and shortage costs, routing costs, establishment and location costs, ordering and purchasing costs – are minimized.

3. Model description

The symbols used in this model are as follows.

Indices:

I – Index of retailers
K – Index of distributors
M – Index of retailers and distributors ($M = I \cup K$)
V – Index of vehicles

Parameters:

f_k – Fixed activation cost of distributor k
h_{0k} – Unit inventory holding cost at distributor k
h_i – Unit inventory holding cost at retailer i
π_i – Unit inventory shortage cost at retailer i
λ_i – Demand rate of retailer i
C_{0k} – Unit purchase cost of distributor k from the manufacturer
A_{0k} – Ordering cost of distributor k
C_{ik} – Unit purchase cost of retailer i from distributor k
A_{ik} – Unit ordering cost of retailer i from distributor k
τ_{0k} – Delivery time from manufacturer to distributor k
τ_{ik} – Delivery time from distributor k to retailer i
T_{gh} – Travel cost from node g to node h
Cap – Capacity of vehicle
$D(n)$ – Stochastic demand during n periods
λ_{0k} – The received demand rate of distributor k
W_{0k} – Random variable of delay in the warehouse of distributor k due to lack of inventory
$\bar{\tau}_i$ – Lead-time plus waiting time of retailer i

Decision variables:

z_k – 1 if distributor k is activated, 0 otherwise
y_{ik} – 1 if retailer i is assigned to distributor k, 0 otherwise
X_{ghv} – 1 if the node h is serviced immediately after node g using vehicle v, 0 otherwise
B_{kghiv} – 1 if vehicle v of distributor k passes nodes g and h to service retailer i, 0 otherwise
N_{gv} – The addition variable for retailer g to eliminate sub-tours of vehicle v
I_{0k}^{+} – The average inventory of distributor k
I_{0k}^{-} – The average shortage of distributor k

I_i^+ – The average inventory of retailer i
I_i^- – The average shortage of retailer i
S_{0k} – Stock order-up-to position at distributor k
S_i: – Stock order-up-to position at retailer i

As stated earlier, the distributors and retailers hold inventories to serve the demand and both follow the (S-1, S) inventory policy. In this policy, the inventory level decreases as soon as a demand is received and served. By decreasing each unit of inventory, an order is placed to substitute the reduced product. The demand is stochastic and follows Poisson distribution with rate of λ_i. Each retailer is assigned to a distributor. Therefore, the received demand of each distributor would be sum of demands of retailers that has been assigned to it as shown in the following relation:

$$\lambda_{0k} = \sum_{i\epsilon I} \lambda_i y_{ik} \qquad \forall k\epsilon K \tag{1}$$

Product delivery time from manufacturer to distributor k is shown by $\tau_{0\mathrm{k}}$ and product delivery time from distributor k to retailer i is shown by τ_{ik}. In addition, the retailer may confront with a waiting time at distributor to serve retailer's need. This waiting time is occurred when the distributor faces shortage and the retailers need to wait until distributor's replenishment. This time can be found based on the Little's law as follows:

$$W_{0k} = \frac{I_{0k}^-}{\lambda_{0k}} \qquad \forall k\epsilon K \tag{2}$$

Hence, the total retailer's waiting time to replenishment would be:

$$\bar{\tau}_i = \sum_{k\epsilon \mathrm{K}} (\tau_{ik} + W_{0k}) y_{ik} \qquad \forall i\epsilon I \tag{3}$$

In order to find average inventory and average shortage at distributors using the METRIC approach, the following procedure should be accomplished. Let i_{0k}^+ be the stochastic inventory level at distributor k in steady state. The probability of inventory level at j ($j \geq 0$) would be:

$$p(i_{0k}^+ = j) = p(S_{0k} - D(\tau_{0k}) = j) = p(D(\tau_{0k}) = S_{0k} - j) \qquad \forall k\epsilon K \tag{4}$$

Since demand process was supposed stochastic process with Poisson distribution, we have:

$$p(D(\tau_{0k}) = S_{0k} - j) = \frac{e^{-\lambda_{0k}\tau_{0k}}(\lambda_{0k}\tau_{0k})^{S_{0k}-j}}{(S_{0k}-j)!} \qquad \forall k\epsilon K \tag{5}$$

Hence, the average inventory at distributor k can be found using the below expected value:

$$I_{0k}^+ = \sum_{j=1}^{S_{0k}} j \left[\frac{e^{-(\lambda_{0k}\tau_{0k})}(\lambda_{0k}\tau_{0k})^{S_{0k}-j}}{(S_{0k}-j)!}\right] \qquad \forall k\epsilon K \tag{6}$$

In addition, average shortage at distributor k can be found using the below equation:

$$I_{0k}^- = \sum_{j=-\infty}^{-1} -j \left[\frac{e^{-(\lambda_{0k}\tau_{0k})}(\lambda_{0k}\tau_0)^{S_{0k}-j}}{(S_{0k}-j)!}\right] \qquad \forall k\epsilon K \tag{7}$$

As finding the average shortage at distributor k using the above equation (7) is difficult, let define $E(I_{0k})$ as follows:

$$E(I_{0k}) = I_{0k}^+ - I_{0k}^- \qquad \forall k\epsilon K \tag{8}$$

And so,

$$I_{0k}^- = I_{0k}^+ - E(\mathrm{I}_{0k}) \qquad \forall k\epsilon K \tag{9}$$

On the other hand:

$$E(I_{0k}) = S_{0k} - \lambda_{0k}\tau_{0k} \qquad \forall k\epsilon K \tag{10}$$

Hence, the average shortage equals:

$$I_{0k}^{-}=I_{0k}^{+}-(S_{0k}-\lambda_{0k}\tau_{0k}) \qquad \forall k\epsilon K \qquad (11)$$

Similarly, the average inventory and average shortage at retailers can be computed but with the difference that $\bar{\tau}_i$ (as introduced in Eq. 3) was used instead of τ_i. Hence, the average inventory and average shortage at retailer i would be:

$$I_i^{+}=\sum_{j=1}^{S_i} j\left[\frac{e^{-(\lambda_i\bar{\tau}_i)}(\lambda_i\bar{\tau}_i)^{S_j-j}}{(S_j-j)!}\right] \qquad \forall i\epsilon I \qquad (12)$$

$$I_i^{-}=I_i^{+}-(S_i-\lambda_i\bar{\tau}_i) \qquad \forall i\epsilon I \qquad (13)$$

4. Mathematical description

Based on assumptions, notations and descriptions in previous sections, the developed location-routing-inventory optimization model is proposed as follows:

$$\begin{aligned} Min\ z = &\sum_{k\epsilon K} f_k z_k+\sum_{i\epsilon I}(h_i I_i^{+}+\pi_i I_i^{-})+\sum_{k\epsilon K} h_{0k} I_{0k}^{+} \\ &+\sum_{k\epsilon K}\lambda_{0k}(C_{0k}+A_{0k})z_k+\sum_{k\epsilon K}\sum_{i\epsilon I}\lambda_i(C_{ik}+A_{ik})y_{ik}+\sum_{g\epsilon M}\sum_{h\epsilon M}\sum_{v\epsilon V} T_{gh}X_{ghv} \end{aligned} \qquad (14)$$

Subject to:

$$\sum_{k\epsilon K} y_{ik}=1 \qquad \forall i\epsilon I \qquad (15)$$

$$y_{ik}\le z_k \qquad \forall k\epsilon K\ ,\forall i\epsilon I \qquad (16)$$

$$\lambda_{0k}=\sum_{i\epsilon I}\lambda_i y_{ik} \qquad \forall k\epsilon K \qquad (17)$$

$$I_{0k}^{+}=\sum_{j=1}^{S_{0k}} j\left[\frac{e^{-(\lambda_{0k}\tau_{0k})}(\lambda_{0k}\tau_0)^{S_{0k}-j}}{(S_{0k}-j)!}\right] \qquad \forall k\epsilon K \qquad (18)$$

$$I_{0k}^{-}=I_{0k}^{+}-(S_{0k}-\lambda_{0k}\tau_{0k}) \qquad \forall k\epsilon K \qquad (19)$$

$$W_{0k}=\frac{I_{0k}^{-}}{\lambda_{0k}} \qquad \forall k\epsilon K \qquad (20)$$

$$\tau_{ik}=\sum_{v\epsilon V}\sum_{h\epsilon M}\sum_{g\epsilon M} T_{gh}B_{kghiv} \qquad \forall k\epsilon K\ \forall i\epsilon I \qquad (21)$$

$$y_{ik}X_{giv}\le B_{kgiiv} \qquad \forall k\epsilon K\ \forall i\epsilon I,\forall g\epsilon M\ \forall v\epsilon V \qquad (22)$$

$$X_{ghv}B_{khliv}\le B_{kghiv} \qquad \forall k\epsilon K\ \forall i\epsilon I\ \ \forall g\ h\ l\epsilon M\ \forall v\epsilon V \qquad (23)$$

$$B_{kghiv}\le X_{ghv} \qquad \forall k\epsilon K\ \forall i\epsilon I,\forall g\ h\epsilon M\ \forall v\epsilon V \qquad (24)$$

$$\sum_{v\epsilon V} B_{kghiv}\le z_k \qquad \forall k\epsilon K\ \forall i\epsilon I,\forall g\ h\epsilon M \qquad (25)$$

$$\bar{\tau}_i=\sum_{k\epsilon \mathrm{K}}(\tau_{ik}+W_{0k})y_{ik} \qquad \forall i\epsilon I \qquad (26)$$

$$I_i^{+}=\sum_{j=1}^{S_i} j\left[\frac{e^{-(\lambda_i\bar{\tau}_i)}(\lambda_i\bar{\tau}_i)^{S_j-j}}{(S_j-j)!}\right] \qquad \forall i\epsilon I \qquad (27)$$

$$I_i^{-}=I_i^{+}-(S_i-\lambda_i\bar{\tau}_i) \qquad \forall i\epsilon I \qquad (28)$$

$$\sum_{g\epsilon M}\sum_{i\epsilon I}\lambda_i X_{giv} \leq Cap \qquad \forall v\epsilon V \qquad (29)$$

$$\sum_{g\epsilon M}\sum_{v\epsilon V} X_{giv} = 1 \qquad \forall i\epsilon I \qquad (30)$$

$$\sum_{k\epsilon K}\sum_{i\epsilon I} X_{kiv} \leq 1 \qquad \forall v\epsilon V \qquad (31)$$

$$\sum_{g\epsilon M} X_{ghv} - \sum_{g\epsilon M} X_{hgv} = 0 \qquad \forall h\epsilon M\ \forall v\epsilon V \qquad (32)$$

$$\sum_{h\epsilon M} X_{khv} - \sum_{h\epsilon M} X_{hiv} - y_{ik} \leq 1 \qquad \forall k\epsilon K\ \forall i\epsilon I\ \forall v\epsilon V \qquad (33)$$

$$N_{gv} - N_{iv} + |I|X_{giv} \leq |I| - 1 \qquad \forall i\epsilon I\ g\epsilon M\ \forall v\epsilon V \qquad (34)$$

$$z_k \epsilon \{0\ 1\} \qquad \forall k\epsilon K \qquad (35)$$

$$y_{ik} \epsilon \{0\ 1\} \qquad \forall k\epsilon K\ , \forall i\epsilon I \qquad (36)$$

$$X_{ghv} \epsilon \{0\ 1\} \qquad \forall g\ h\epsilon M\ , \forall v\epsilon V \qquad (37)$$

$$B_{kghiv} \epsilon \{0\ 1\} \qquad \forall k\epsilon K\ \forall i\epsilon I\ , \forall g\ h\epsilon M\ \forall v\epsilon V \qquad (38)$$

$$N_{gv} \epsilon \{0\ 1\} \qquad \forall g\ \epsilon M\ , \forall v\epsilon V \qquad (39)$$

$$S_i, I_i^+, I_i^- \geq 0 \qquad \forall i\epsilon I\ \text{(integer)} \qquad (40)$$

$$S_{0k}, I_{0k}^+, I_{0k}^- \geq 0 \qquad \forall k\epsilon K\ \text{(integer)} \qquad (41)$$

The first term of objective function shows fixed activation costs of distributors. The second and third represent holding and lack of inventory costs through retailers and distributors, respectively. The fourth and fifth terms correspond to selling and ordering costs of retailers and distributors. The last term expresses routing cost with lowest cost-to-serve from distributors to retailers. Constraints set (Eq. 15) ensures that each retailer is assigned to only one distributor. Constraints set (Eq. 16) prevents assigning the retailers to inactive distributors. Constraints sets (Eqs. 17 to 20) and (Eqs. 26 to 28) were explained earlier.

Constraints set (Eq. 21) determines delivery time between retailer i and distributor k based on their routing travel times. Constraints set (Eq. 22) explains that if retailer i is assigned to distributor k with a route from node g to i then the route g-i is the way that transfers the distributor k to retailer i (i.e. B_{kgiiv} = 1). Constraints set (Eq. 23) has the same concept of constraints set (Eq. 22). It determines that with existing the route between nodes g and h (i.e. X_{ghv} = 1), and the route between h and i underlaid in the pass of distributor k to retailer i (i.e. B_{khliv} = 1), the route g-h should be laid in the pass of distributor k to retailer i (i.e. $B_{kghiv} = 1$). Constraints set (Eq. 24) explains that when a route can be laid in the way of distributor k to retailer i that it is an active route. Constraints set (Eq. 25) explains that when a route can be defined from a distributor that the distributor is active. It is worth mentioning that constraints sets (Eqs. 22 to 25) are supplementary to constraints set (Eq. 21) to control B_{kghiv}. Constraints set (Eq. 29) explains capacity restriction of vehicles. it should be noted that all vehicles are considered the same capacity. Constraints set (Eq. 30) determines that each retailer is assigned to only one route of vehicle. Constraints sets (Eq. 31) explains that several vehicles could not serve in one route. Constraints set (Eq. 32) ensures arriving and leaving of the vehicles to retailer or distributor nodes. Meanwhile, these constraints set makes the routes to be closed. Constraints set (Eq. 33) explains that a route is activated between a distributor and retailer if the retailer has assigned to distributor. Constraints set (Eq. 34) ensures eliminating sub tours and determines that each distributor should be in the beginning and each retailer should be in ending of each route. Finally, constraints sets (Eqs. 35 to 41) are standard constraints that describe the nature of the variables considered in the model.

5. Solution method

Because of stochastically nature of the model, solving the model with exact methods is not easy specially in medium and large-scale problems. It should be noted that some constraints contains variables (i.e. S_i) on their upper limit of summations. Although the model can be decomposed into sub-models and some optimizations solvers (i.e. GAMS, Lingo) can solve such models; but this makes the exact solution approach very hard and even impossible in some larger cases.

In such circumstances, Meta-heuristics are used instead of exact methods successfully. There are many Meta-heuristics that can be used in solving process but specifically genetic algorithm and simulated annealing are widely used in similar aspects. Since solving the model with only one Meta-heuristic is very complicated a hybrid approach is developed by composing these two well-known Meta-heuristics. So, a hybrid genetic-simulated annealing algorithm is used in this study. To this end, the model is decomposed into a location-routing model, which is solved using simulated annealing and METRIC inventory model, which is solved using genetic algorithm.

The location-routing solutions is developed based on previous studies [26] as a vector of positive, negative and zero numbers which respectively represent retailers, distributors and routes. Fig. 1 illustrates a sample of this vector representation and related routing. In fact, each route, containing a distributor and its covering retailer, is started and ended with zeros. Distributors without retailers are not located. Algorithm makes the neighborhoods by changing the vector to reach the optimal solution.

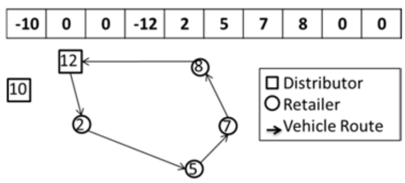

Fig. 1 Sample of location-routing solution in SA

In genetic algorithm, the METRIC inventory model is optimized after identifying the active distributors and related retailers in previous algorithm. The chromosomes represent the stock level of active distributors and retailers respectively. For example if two distributors and six retailers are activated, then a string with length 8 is developed such as [4-6, 8, 9, 11, 28, 35]. Then crossover and mutation operators are used to generate new and really feasible offspring solutions to reach the best near-optimal solutions of the model. Specifically for crossover, single point and two point methods and for mutation, bit inversion method was used. The schematic representation of hybrid algorithm is shown in Fig. 2.

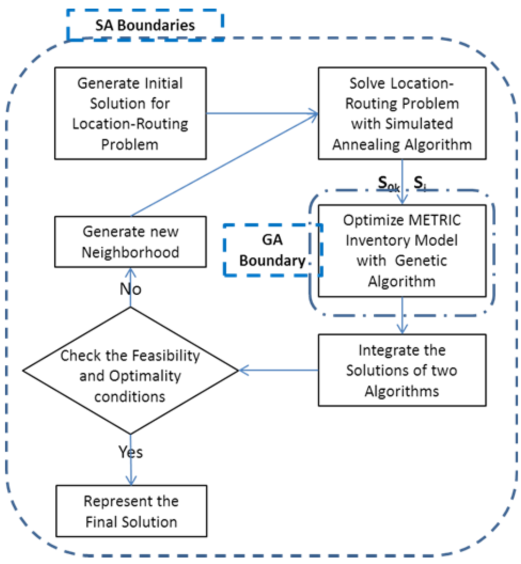

Fig. 2 Schematic representation of developed hybrid SA-GA algorithm

6. Results and discussion

Location inventory and location routing inventory models are categorized in p-median problems and accounted as NP-hard problems. Hence, solving the models with deterministic methods would be very hard; specially in large scale sizes. For this reason, as mentioned in previous section, a hybrid genetic and simulated annealing algorithm is proposed as solution method. We used Lingo 9 and MATLAB R2013b to solve the sample problems of this study in PC with Intel (R) Core i5 CPU 2.40Ghz and 4.00 GB RAM specification.

Parameter tuning for genetic and simulated annealing algorithms is conducted based on information brought in Tables 1 and 2.

Table 1 Simulated annealing parameters

$T_{initial}$	T_{min}	R	$I_{SA\text{-}main}$	$I_{SA\text{-}inner}$
700	0.01	0.9	20	5

Table 2 Genetic algorithm parameters

Pop size	$P_{crossover}$	$P_{mutaion}$	I_{GA}
20	0.8	0.2	400

In Table 1, $T_{initial}$ and T_{min} represent initial and final temperatures, R is acceptance threshold of bad neighborhood solution and $I_{SA\text{-}inner}$ and $I_{SA\text{-}main}$ depict the number of iterations in a fixed certain temperature and in all temperatures, respectively. In Table 2 *Pop size* is population size, $P_{crossover}$ and $P_{mutation}$ are crossover and mutation rates of the algorithm respectively and I_{GA} represents number of iterations in genetic algorithm. The parameters are adjusted based on Taguchi method; in which the parameters are categorized with three high, medium and low levels and then the method identifies the best values of the parameters.

In addition, the parameters of the model are determined as shown in Table 3. As observed, the parameters are generated randomly with uniform distribution in their specified ranges.

The sample problems were solved with regards to 0 and the results are depicted in both deterministic and meta-heuristic methods. Sample problems were designed in three short, medium and large scale sizes. The deviations of meta-heuristic from deterministic method in objective function are brought in the last column of Table 4.

As shown in Fig. 3, the results illustrate that meta-heuristic method decreases the solving time of large scale problems. The curve of deterministic method has been stopped after problem No 5 since after that, the time is exponentially increased.

Table 3 Location-routing-inventory parameters

Pameters	Distribution	Value
f_k	Uniform	U[3000, 5500]
h_i	Uniform	U[3, 6]
π_i	Uniform	U[7, 10]
h_{0k}	Uniform	U[3, 6]
C_{0k}	Uniform	U[20, 40]
A_{0k}	Uniform	U[20, 40]
λ_i	Uniform	U[1, 40]
C_{ik}	Uniform	U[25, 45]
A_{ik}	Uniform	U[20, 40]
T_{gh}	Uniform	U[0, 5]

Table 4 Results of location routing inventory model

Scale	No #	Number of distributions	Number of retailers	Deterministic Cost	Deterministic Time	Meta heuristic Cost	Meta heuristic Time	Error, %
Short	1	1	3	12482.80	3	12483.44	37.66	0.005
	2	2	4	12146.63	184	12257.18	42.57	0.91
	3	3	6	15004.29	1335	15899.94	91.35	5.96
	4	4	10	26988.35	5874	27683.46	300.79	2.57
Medium	5	6	15	44331.11	41715	46388.44	731.68	4.64
	6	10	30	*	*	125433.30	3624.24	-
	7	14	45	*	*	182542.83	2305.59	-
	8	20	60	*	*	239439.74	5702.48	-
Large	9	25	70	*	*	270975.04	2236.25	-
	10	30	90	*	*	301140.58	5986.73	-
	11	35	120	*	*	392606.95	1818.88	-
	12	40	150	*	*	456251.43	4911.29	-

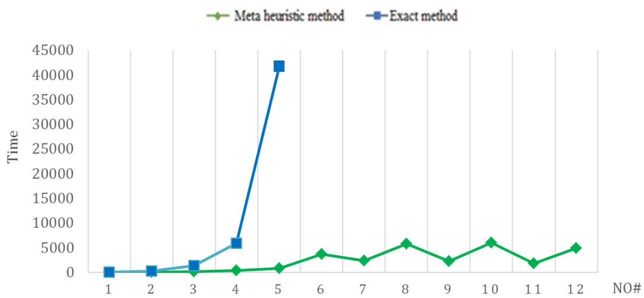

Fig. 3 Comparison of deterministic and meta-heuristic methods

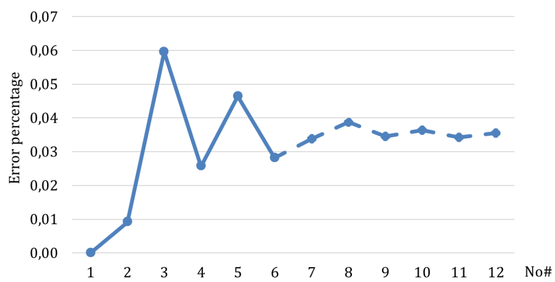

Fig. 4 Error percentage of Meta heuristic and deterministic methods

In addition, the difference error between Meta heuristic and deterministic methods are brought in the Fig. 4. The dotted lines demonstrate the predicted values with moving average (n = 5). As shown, the developed meta-heuristic has trivial difference with deterministic method in cost objective values and additionally has shorter running time than deterministic method in medium and large scale problems.

Finally, sensitivity analyses were performed for all problems. As known, the stock position (S) is one of the most important variables in METRIC approach. So, sensitivity analyses were performed on changing effects of the parameters on this variable. Generally, three parameters were used in the analyses including holding cost, shortage cost, and demand rate.

Here as an instance, the analyses were illustrated for problem No 5. This problem is a medium size with 6 distributors and 15 retailers. Figs. 5 and 6 illustrate the results.

Figs. 5(a) and 5(b) represent changing costs of retailer and their effects on optimal stock level of retailer. Fig. 5(c) represent the same for distributor. As mentioned previously, shortage of distributor is inducted as delays for retailers and so no direct analysis was brought for distributor's shortage cost. As known, with increasing the holding cost the stock level of inventory should be decreased accordingly This behavior can be clearly observed in Fig. 5(a). In contrast, by increasing shortage cost, the stock level would be increased; as shown in Fig. 5(b). Since the model tries to deal less with lack of inventory and so increases the stock level. Fig. 5(c) has the same behavior of Fig. 5(a) for distributors.

Figs 6(a) and 6(b) represent the changing of optimal stock level in different demand rates in both retailer and distributor levels. As shown in Fig. 6(a), by increasing the demand rate, the model tries to increase the stock level to prevent the shortfalls. Meanwhile, increasing on retailer's demand would cause as well as increasing on distributor's demand. It is therefore logical that with increasing this parameter, the stock level of distributor is also increased.

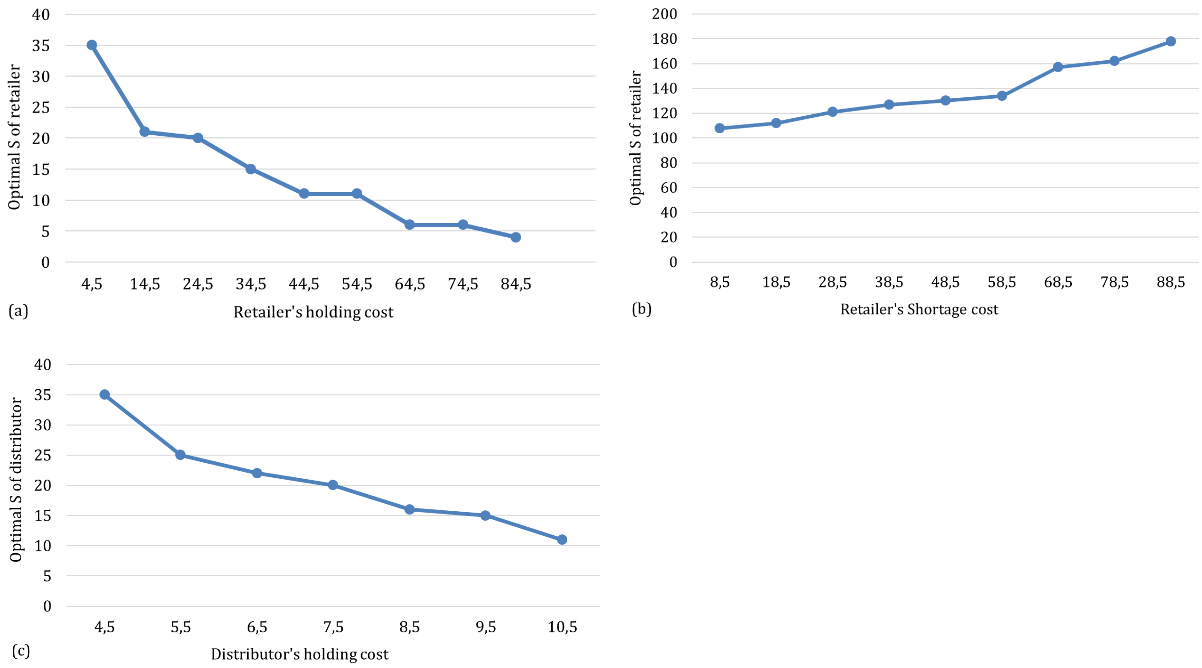

Fig. 5 Sensitivity analysis of optimal S based on inventory costs

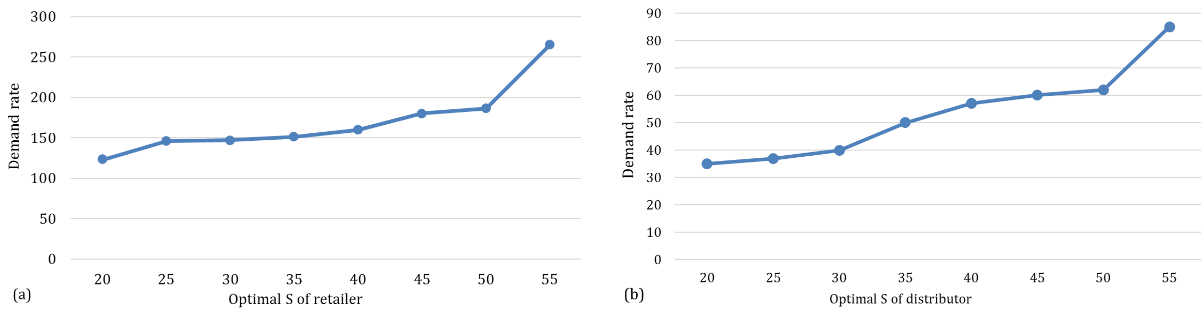

Fig. 6 Sensitivity analysis of S based on demand rate

7. Conclusion

Generally, a supply chain involves different level of decision-making. Strategic and long term decisions are at macro level; mid-term and tactical decisions are at the next level and short-term and operational decisions are at the lowest level. Nowadays organizations, for remaining in competitive market, are being forced to continuously improve their performance and in this viewpoint, one of the most important improving factors, would be supply chain management decisions. In supply chain, location and allocation are of strategic decisions, inventory control and management are of tactical decision and transportation is of operational decisions. According to these points, in this study an integrated supply chain model is investigated that includes simultaneous optimization of all decisions in location, inventory control, transportation and routing. The model was developed in stochastic conditions based on METRIC approach and (S-1, S) policy. The model was investigated with stochastic conditions to meet better to real-world situations. To solve the model, a hybrid genetic and simulated annealing algorithm was developed and used.

Experimental efforts were performed with 12 sample problems in different (short, medium, large) scales. The problems were solved and sensitivity analyses were performed on optimal stock level as an important variable of the model.

As recommendation for future studies can be mentioned to using other inventory policies such as (*R*, *Q*) in the model, developing multi-objective and multi-level models and using the vehicles with different capacities. Meanwhile the model can be developed with other objective functions such as minimization of transportation cost, maximization of customer satisfaction and minimization of the risk.

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