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Chiral models for exciting baryons*

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Abstract. We study low lying resonances in models in which the pions linearly couple to the quark core. We derive the coupled channel equations for pion scattering, and discuss preliminary results for pion scattering in the Roper channel.

1 Introduction

In our previous work [1–3] we have presented a method to calculate the K-matrix for pion scattering and electro-production in quark models with chiral mesons. The method has several advantages over more standard methods because it allows for a clear separation of the resonant part of the amplitude from the background. We have successfully applied it to the calculation of the phase shift and electro-production amplitudes in the P33 channel.

In the present work we extend the method to cover the cases where it is necessary to include two or more channels. This allows us to attack perhaps the most intriguing among the low lying resonances – the Roper resonance. In this contribution we develop the coupled channel formalism for scattering and present some preliminary results.

2 K matrix in chiral quark models

We consider quark models in which p-wave pions couple linearly to the three-quark core. Assuming a pseudo-scalar quark-pion interaction, the part of the Hamiltonian referring to pions can be written as

$$H_\pi = \int dk \sum_{mt} \left\{ \omega_k \, a^\dagger_{mt}(k) a_{mt}(k) + \left[V_{mt}(k) a_{mt}(k) + V^\dagger_{mt}(k) \, a^\dagger_{mt}(k) \right] \right\} , \quad (1)$$

where $a^\dagger_{mt}(k)$ is the creation operator for a p-wave pion with the third components of spin m and isospin t, and

$$V_{mt}(k) = -v(k) \sum_{i=1}^{3} \sigma^i_m \tau^i_t \quad (2)$$

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is the general form of the pion source, with $\nu(k)$ depending on the model.

In the basis with good total angular momentum J and isospin T, in which the K and T matrices are diagonal, it is possible to express the K matrix in the form[3][1]

$$K^{JT}(k,k_0) = -\pi\sqrt{\frac{\omega_k}{k}}\langle\Psi^P_{JT}(W)||V(k)||\Phi_N\rangle\,. \tag{3}$$

The corresponding principal-value state[4] obeys

$$|\Psi^P_{JT}(W)\rangle = \sqrt{\frac{\omega_0}{k_0}}\left\{\left[a^\dagger(k_0)|\Phi_N\rangle\right]^{JT} - \frac{\mathcal{P}}{H-W}\,[V(k_0)|\Phi_N\rangle]^{JT}\right\}, \tag{4}$$

where $[\;]^{JT}$ denotes coupling to good J and T, and k_0 and ω_0 the pion mementum and energy.

We assume that the operator V, acting on the ground state Φ_N, does not only flip the quark spin and isospin but also excites quarks to higher spatial states. As an example let us mention the state with the flipped spins (the bare delta) which plays a crucial role in the formation of the $\Delta(1232)$ resonance, and the excitation of one quark to the 2*s* state which is believed to be the main mechanism in the formation of the Roper resonance.

The general form (4) therefore suggests the following ansatz in which the states with excited quark core, Φ_B, are separated from the state corresponding to pion scattering on the nucleon. Neglecting the two-pion states we can write

$$|\Psi_{JT}(W)\rangle = \sqrt{\frac{\omega_0}{k_0}}\left\{\left[a^\dagger(k_0)|\Phi_N\rangle\right]^{JT} + \int dk\,\frac{\chi(k,k_0)}{\omega_k-\omega_0}\left[a^\dagger(k)|\Phi_N\rangle\right]^{JT} + \sum_B c_B(W)|\Phi_B\rangle\right\}. \tag{5}$$

The pion amplitude is related to the K matrix by

$$\chi(k_0,k_0) = \frac{k_0}{\pi\omega_0}\,K(k_0,k_0)\,. \tag{6}$$

3 Coupled channels

We have shown [3] that the above ansatz successfully describes scattering as well as electro-production of pions in the P33 channel at lower energies. At higher energies, the two pion decay channel becomes important and cannot be neglected. In most cases, the two pion decay proceeds through an intermediate resonance; in the P11 channel as well as in the P33 channel this is the $\Delta(1232)$ which accounts for 30 %–40 % of the width in the region of the Roper resonance and even 40 %–70 % in the region of the $\Delta(1600)$ resonance.

[1] In the static approximation, k_0 is uniquely related to the energy $W = E_N + \omega_0$, so one can use either k_0 or W to label the states; for the on-shell K matrix we write $K(k_0,k_0) = K(W)$.

In the simplest extension of the model we therefore include an additional channel representing the pion scattering on the $\Delta(1232)$. The corresponding principle-value state takes the form:

$$|\Psi^{\Delta}_{JT}(W,E)\rangle = \sqrt{\frac{\omega_0}{k_0}}\Bigg\{\left[a^\dagger(k_E)|\Psi_\Delta(E)\rangle\right]^{JT} + \int dk\,\frac{\chi_\Delta(k,k_E)}{\omega_k-\omega_E}\left[a^\dagger(k)|\Psi_\Delta(E)\rangle\right]^{JT} + \sum_B c_B(W,E)|\Phi_B\rangle\Bigg\}. \quad (7)$$

The key point in the above ansatz is that the energy of the delta state, E, is not fixed (e.g. to 1232 MeV) but is varied from the threshold value $E_N + m_\pi$ to the maximum allowed value $W - m_\pi$. (Obviously, this channel opens at the two-pion threshold, i.e at $W = E_N + 2m_\pi$.) For simplicity we work in the static limit in which the pion energy and momentum can be written as $\omega_E = W - E$, $k_E = \left(\omega_E^2 - m_\pi^2\right)^{1/2}$. The delta state in (7) is given by (5) except that it is now normalized to $\delta(E - E')$ rather than to $(1 + K_\Delta(E)^2)\delta(E - E')$:

$$|\Psi_\Delta(E)\rangle = \frac{1}{\sqrt{1+K_\Delta(E)^2}}\sqrt{\frac{\omega_0}{k_0}}\Bigg\{\left[a^\dagger(k_0)|\Phi_N\rangle\right]^{\frac{3}{2}\frac{3}{2}} + \int dk\,\frac{\chi(k,k_0)}{\omega_k-\omega_0}\left[a^\dagger(k)|\Phi_N(k)\rangle\right]^{\frac{3}{2}\frac{3}{2}} + c_\Delta(E)|\Phi_\Delta\rangle\Bigg\}. \quad (8)$$

with $\omega_0 = E - E_N$ and $k_0 = \left(\omega_0^2 - m_\pi^2\right)^{1/2}$.

By a straightforward extension of the formula (3) we can now write down the K-matrix, which has two discrete indexes and one continuous index E, as

$$K_{NN}(W) = -\pi\sqrt{\frac{\omega_0}{k_0}}\,\langle\Phi_N||V^\dagger(k_0)||\Psi_{JT}(W)\rangle\,, \quad (9)$$

$$K_{N\Delta}(W,E) = -\pi\sqrt{\frac{\omega_E}{k_E}}\,\langle\Psi_\Delta(E)||V^\dagger(k_E)||\Psi_{JT}(W)\rangle\,, \quad (10)$$

$$K_{\Delta N}(W,E) = -\pi\sqrt{\frac{\omega_0}{k_0}}\,\langle\Phi_N||V^\dagger(k_0)||\Psi^{\Delta}_{JT}(W,E)\rangle\,, \quad (11)$$

$$K_{\Delta\Delta}(W,E,E') = -\pi\sqrt{\frac{\omega_E}{k_E}}\,\langle\Psi_\Delta(E')||V^\dagger(k_{E'})||\Psi^{\Delta}_{JT}(W,E)\rangle\,. \quad (12)$$

The full and the partial widths are related to the T matrix, and the phase shift and the inelasticity to the S matrix. The T matrix is obtained from $T = -K/(1 - iK)$

or $T = -K + iKT$ which yields the following set of integral equations:

$$T_{NN}(W) = -K_{NN}(W) + i\left[K_{NN}(W)T_{NN}(W) + \int_{E_N+m_\pi}^{W-m\pi} dE\, K_{N\Delta}(W,E)T_{\Delta N}(W,E)\right], \tag{13}$$

$$T_{N\Delta}(W,E) = -K_{N\Delta}(W,E) + i\left[K_{NN}(W)T_{N\Delta}(W,E) + \int_{E_N+m_\pi}^{W-m\pi} dE'\, K_{N\Delta}(W,E')T_{\Delta\Delta}(W,E',E)\right], \tag{14}$$

$$T_{\Delta N}(W,E) = -K_{\Delta N}(W,E) + i\left[K_{\Delta N}(W,E)T_{NN}(W,E) + \int_{E_N+m_\pi}^{W-m\pi} dE'\, K_{\Delta\Delta}(W,E,E')T_{\Delta N}(W,E')\right], \tag{15}$$

$$T_{\Delta\Delta}(W,E,E') = -K_{\Delta\Delta}(W,E,E') + i\left[K_{\Delta N}(W,E)T_{N\Delta}(W,E') + \int_{E_N+m_\pi}^{W-m\pi} dE''\, K_{\Delta\Delta}(W,E,E'')T_{\Delta\Delta}(W,E'',E')\right]. \tag{16}$$

From the first T matrix we deduce the phase shift δ and the elasticity η thought the relation

$$S = 1 - 2iT_{NN}(W) = \eta(W)e^{2i\delta(W)}\,. \tag{17}$$

4 Solution of the coupled equations in a simplified model

To get more insight in the method let us consider a simplified case in which we assume that the bare states dominate the channel states Ψ_{JT} and Ψ_{JT}^{Δ} as well as the states Φ_N and Ψ_Δ. Then, to evaluate the matrix elements of the K matrix (12) we use

$$|\Psi_{JT}(W)\rangle \approx \sqrt{\frac{\omega_0}{k_0}}\left[c_B(W)|B\rangle + \delta_{J\frac{1}{2}}\delta_{T\frac{1}{2}}c_N(W)|N\rangle\right] \tag{18}$$

$$|\Psi_{JT}^{\Delta}(W,E)\rangle \approx \sqrt{\frac{\omega_E}{k_E}}\left[c_{B'}(W,E)|B'\rangle + \delta_{J\frac{1}{2}}\delta_{T\frac{1}{2}}c_{\Delta}^{N}(W,E)|N\rangle + \delta_{J\frac{3}{2}}\delta_{T\frac{3}{2}}c_{\Delta}^{\Delta}(W,E)|\Delta\rangle\right] \tag{19}$$

where for the P11 channel $B = B' = R$ (i.e. $N^*(1440)$) while for the P33 channel we use $B = \Delta(1232)$ and $B' = \Delta(1600)$.[2] The second terms in the above expressions ensure the mutual orthogonality of the channel states and the ground state. The intermediate state appearing in Ψ_{JT}^{Δ} can be approximated as

$$|\Psi_\Delta(E)\rangle \approx \sqrt{\frac{\omega}{k}}\frac{\Psi_{JT}^{\Delta}}{\sqrt{1+K_\Delta(E)^2}}\,|\Delta\rangle \approx \frac{2\sin\delta_\Delta(E)}{\sqrt{2\pi\Gamma_\Delta}}|\Delta\rangle\,, \tag{20}$$

[2] Note that we have not included the $\Delta(1600)$ in the first channel because of the relatively small πN branching ratio.

where $\omega = E - E_N$, $k = \sqrt{\omega^2 - m_\pi^2}$, $\Gamma_\Delta = 2\pi\omega|\langle N||V||\Delta\rangle|^2/k$, $K_\Delta(E) = \frac{1}{2}\Gamma_\Delta/(E_\Delta - E)$. The coefficients c_B and $c_{B'}$ are most easily determined from

$$(H - W)|\Psi_{ST}(W)\rangle = 0 \quad \text{and} \quad (H - W)|\Psi_{ST}^\Delta(W, E)\rangle = 0\,, \tag{21}$$

which, after multiplying by $\langle B|$, yields

$$(E_B - W)c_B(W) = -\langle B||V(k_0)||N\rangle\,, \tag{22}$$

$$(E_{B'} - W)c_{B'}^\Delta(W, E) = -\frac{2\sin\delta_\Delta(E)}{\sqrt{2\pi\Gamma_\Delta}}\langle B'||V(k_E)||\Delta\rangle\,. \tag{23}$$

Here E_B and $E_{B'}$ include the self-energy. For the coefficients c_N and c_Δ we get

$$c_N^N(W) = -\langle\Phi_N||a^\dagger(k_0)||\Phi_N\rangle \approx \frac{\langle N||V(k_0)||N\rangle}{W - E_N}\,, \tag{24}$$

$$c_\Delta^N(W, E) = -\langle\Phi_N||a^\dagger(k_E)||\Psi_\Delta(E)\rangle \approx \frac{2\sin\delta_\Delta(E)}{\sqrt{2\pi\Gamma_\Delta}}\frac{\langle N||V(k_E)||\Delta\rangle}{W - E_N}\,, \tag{25}$$

and similarly for c_Δ^Δ

We immediately notice that the K matrices can be written in a separable form; in the P11 channel we find

$$K_{i,j} = \frac{a_i a_j}{E_R - W} - \frac{b_i b_j}{\omega_0}\,, \qquad i = N, \Delta;\ j = N, \Delta \tag{26}$$

with

$$a_N(W) = \sqrt{\frac{\pi\omega_0}{k_0}}\langle B||V(k_0)||N\rangle\,, \quad a_\Delta(W, E) = \sqrt{\frac{\pi\omega_E}{k_E}}\frac{2\sin\delta_\Delta(E)}{\sqrt{2\pi\Gamma_\Delta}}\langle B'||V(k_E)||\Delta\rangle\,, \tag{27}$$

$$b_N(W) = \sqrt{\frac{\pi\omega_0}{k_0}}\langle N||V(k_0)||N\rangle\,, \quad b_\Delta(W, E) = \sqrt{\frac{\pi\omega_E}{k_E}}\frac{2\sin\delta_\Delta(E)}{\sqrt{2\pi\Gamma_\Delta}}\langle N||V(k_E)||\Delta\rangle\,. \tag{28}$$

The coefficients b_N and b_Δ strongly influence the phase shift close to the threshold but in the vicinity of the resonance we can neglect them. In this in case it is possible to find the solution for the T matrices in a simple form:

$$T_{ij} = -\frac{a_i a_j}{E_R - W - i\left[a_N^2 + \int_{E_N+m_\pi}^{W-m_\pi} dE\, a_\Delta(E)^2\right]}\,. \tag{29}$$

The partial widths read

$$\Gamma_{NN}(W) = 2a_N(W)^2 = \frac{2\pi\omega_0}{k_0}\langle B||V(k_0)||N\rangle^2\,, \tag{30}$$

$$\Gamma_{N\Delta}(W) = 2\int dE\, a_\Delta(W, E)^2 \approx \frac{2\pi\omega_E}{k_E}\langle B'||V(k)||\Delta\rangle^2\,. \tag{31}$$

To get the latter expression we have assumed that $a_\Delta(W, E)$ is sufficiently strongly peaked around $E = E_\Delta$. The phase shift for $\pi N \to \pi N$ is obtained from (17):

$$\tan 2\delta(W) = \frac{\Gamma_{NN}(W)(E_R - W)}{(E_R - W)^2 - \frac{1}{4}\left(\Gamma_{NN}(W)^2 - \Gamma_{N\Delta}(W)^2\right)}\,. \tag{32}$$

The inelasticity is expressed as

$$\mathrm{Im}T^{\mathrm{in}} = -\mathrm{Im}T_{NN} - |T_{NN}|^2 = \frac{\frac{1}{4}\Gamma_{NN}(W)\Gamma_{N\Delta}(W)}{(E_R - W)^2 + \frac{1}{4}\left(\Gamma_{NN}(W) + \Gamma_{N\Delta}(W)\right)^2} . \qquad (33)$$

5 Preliminary results for the Roper in the Cloudy Bag Model

We shall illustrate the method using the simplified approach presented in the previous section by calculating scattering in the P11 channel which is dominated by the Roper resonance. Though the above expressions are general and can be applied to any model in which the pions linearly couple to the quark core we choose here the Cloudy Bag Model, primarily because of its simplicity. The Hamiltonian of the model has the form (1) and (2) with

$$v(k) = \frac{1}{2f_\pi} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \frac{\omega^0_{\mathrm{MIT}}}{\omega^0_{\mathrm{MIT}} - 1} \frac{j_1(kR)}{kR} , \qquad (34)$$

when no radial excitation of of the core takes place, and

$$v^*(k) = r_\omega v(k) , \qquad r_\omega = \frac{1}{\sqrt{3}} \left[\frac{\omega^1_{\mathrm{MIT}}(\omega^0_{\mathrm{MIT}} - 1)}{\omega^0_{\mathrm{MIT}}(\omega^1_{\mathrm{MIT}} - 1)} \right]^{1/2} , \qquad (35)$$

when one quark is excited from the 1s state to the 2s state. Here $\omega^0_{\mathrm{MIT}} = 2.04$ and $\omega^1_{\mathrm{MIT}} = 5.40$ The free parameter is the bag radius R. Though the bare values of different 3-quark configurations are in principle calculable in the model, the model lacks a mechanism that would account for large hyperfine splitting between certain states, e.g. the nucleon and the delta. For each R, we therefore adjust the splitting between the bare states such that the experimental position of the resonance is reproduced. Furthermore, using the experimental value of f_π in (34) leads to too small coupling constants irrespectively of the bag radius; in our calculation we have therefore decreased this value by 10 %.

Preliminary results for the phase shift and inelasticity in the P11 channel have been calculated using the simplified model of the previous section. We have used the parameters with which the P33 phase shift is reproduced in the region of the delta resonance, i.e. the bag radius in the range 0.8 fm $< R <$ 1.1 fm and $f_\pi = 81$ MeV.

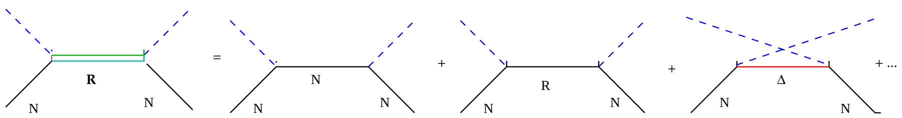

Fig. 1. Processes dominating scattering in the P11 channel: the nucleon pole term, the direct term and the crossed term with the delta intermediate state.

The experimental phase shift (Fig. 2 can be reasonable well reproduced by the the direct term only (see Fig. 1) provided we use a smaller value of the bag radius. (Larger values yield too small the resonance width.). Yet, close to the threshold the phase shift exhibits a wrong behavior; it should be negative with the strength dictated by the piNN coupling constant. The proper behavior at low

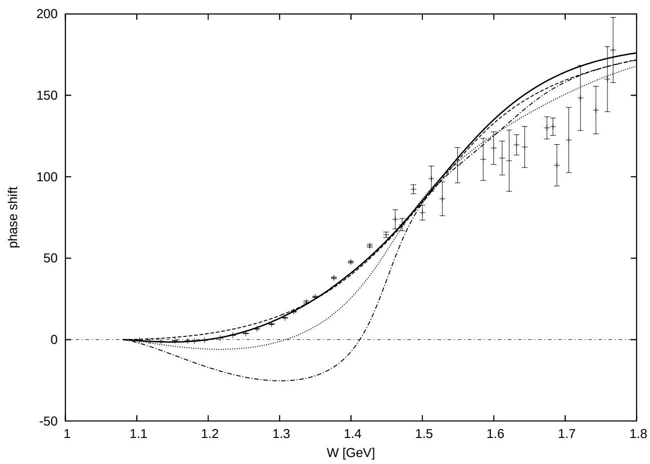

Fig. 2. Different contribution to the phase shift in the P11 channel: the direct term (dashed line), the inclusion of the nucleon pole (dotted line), the inclusion of the crossed term with the $\Delta(1232)$ (dashed and dotted line), the inclusion of the second $\Delta(1600)$ (full line). The bag radius $R = 0.83$ fm is used. The data points are from [7].

energies is established through the inclusion of the nucleon pole term. The nucleon pole term is in our model generated by requiring the orthogonality of the channel state vector to the ground state; including this term provides a consistent behavior at small energies (and in particular in the limit $\omega_0 \to 0$). However, at intermediate energies the agreement is strongly deteriorated. Since the strength of this term is fixed by the πNN coupling constant which is well reproduced in the model, additional, non-resonant, terms are needed to cure this behavior. The important contribution that increases the phase shift comes from the crossed terms, and in particular from the term with the intermediate delta states (see the last term in Fig. 1), as noted already in [6]. To the leading order it contributes the term

$$K^{\Delta}_{NN} = \pi \frac{\omega_0}{k_0} \frac{4}{9} \frac{\langle N||V||\Delta\rangle^2}{\omega_0 + E_\Delta - E_N} \tag{36}$$

to the K matrix in the πN channel. But even if we increase the model value of the $\pi N\Delta$ vertex such that the experimental width of $\Delta(1232)$ is reproduced, the phase shift at lower energies remain still too negative. Another intermediate state that has a relatively strong coupling to the πN channel is the excited delta state, $\Delta(1600)$. If we include the corresponding term in our calculation we obtain an almost perfect agreement with the experiment at lower energies; which, taking into account the crudeness of our model, should not be considered as a proof of a great predictive power of our approach but rather as an indication of the important role that other resonances may play in the formation of the Roper resonance.

Fig. 3 shows that the calculated inelasticity in the resonance region qualitatively agrees with the experimental one. It does, however, not reproduce the large inelasticity above the resonance energy which approaches the unitarity limit. From our formula (33) such an behavior can be explained by assuming that the partial widths, $\Gamma_{N\Delta}$ and Γ_{NN}, remain almost equal over a relatively large interval of energies. This could again be attributed to the interplay of different processes involving neighboring resonances not included in the present calculation.

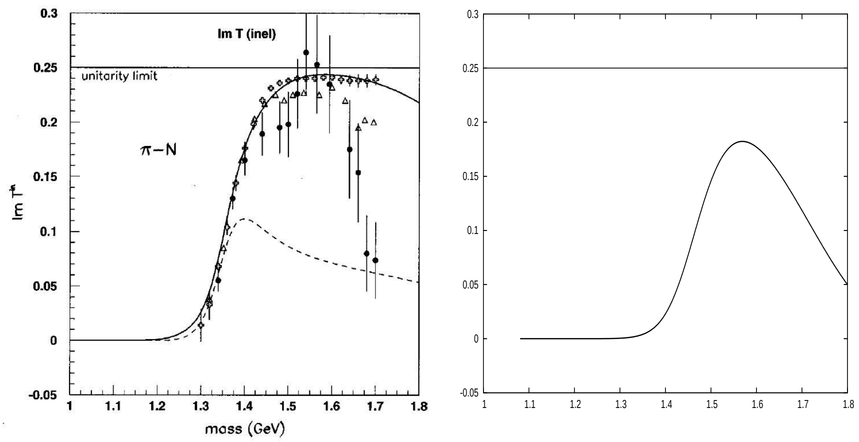

Fig. 3. The inelasticity in the P11 channel. The left figure is from [8], the right figure is our calculation with the direct terms only.

To conclude, our very preliminary calculation points out the importance of including different contributions stemming from the neighboring resonances to explain the rather peculiar properties of the Roper resonance. We believe that such conventional mechanisms have to be carefully exploited before making any conclusion about possible necessity of exotic degrees of freedom.

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