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THE ROBUST ALTERNATIVE TO STANDARD VALIDITY APPROACH

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ABSTRACT

The usual approach to criterion-related validity is developed under the canonical correlation model and is based on the maximization of the correlation of test results and the chosen criteria. The standard measures of validity are canonical correlation in the case of several test results and criteria, multiple correlation in the case of several test results and one criterion, and bivariate correlation in the case of one test and one criterion. In kinesiology, as well as some other disciplines, standard measures of validity are not always appropriate, being sensitive of the value of degrees of freedom. Therefore, the measures of validity based on the maximization of covariance of test results and chosen criteria proposed by Momirović et al. (1983), including robust canonical correlation analysis, robust regression analysis, robust discriminant analysis and redundancy analysis, may be more appropriate. The example in favour of this method of validation is presented.

Keywords: validity, robust methods, quasi canonical analysis

ROBUSTNA ALTERNATIVA STANDARDNEMU POSTOPKU TESTA VELJAVNOSTI

IZVLEČEK

Pristop določanja kriterijske veljavnosti v modelu kanonične korelacijske analize temelji na maksimizaciji korelacije rezultatov testa in zbranih meril. Standardna mera veljavnosti v primeru več testov z več kriteriji je kanonična korelacija, v primeru enega kriterija je mera veljavnosti multipla korelacija, bivariatna korelacija pa je mera ve-

ljavnosti v primeru enega prediktorja in enega kriterija. Kot v drugih vedah, tudi v kineziologiji standardna mera veljavnosti zaradi občutljivosti števila stopenj prostosti ni vedno primerna. Zato so mere veljavnosti, zasnovane na maksimizaciji kovarianc med rezultati testa in kriteriji, ki jih je predlagal Momirović s sodelavci (1983) in vključujejo robustne kvazikanonične analize kovariance, robustno regresijsko analizo, robustno deskriminantno analizo in analizo prepokrivanja mogoče primernejše. Predstavljen je primer v potrditev opisane metodologije.

Ključne besede: veljavnost, robustne metode, kvazi kanonična analiza

INTRODUCTION

The usual approach to establish any criterion-related validity is developed under a correlation model and is based on the maximization of the correlation of test results and chosen criteria (Gliner & Morgan, 2000). The standard measures of validity are canonical correlations in the case of several test results and criteria, multiple correlation in the case of several test results and one criterion, and bivariate correlation in the case of one test and one criterion. As a measure of reliability, the first canonical correlation is well defined as being the maximal correlation between linear combinations of two sets of variables on the given set of data. The properties of canonical correlation are that it is sensitive to the regularity of correlation matrices, outliers, and the difference between the number of entities and the number of variables. Statistically significant canonical correlation can be obtained if only two variables (one variable from each set) have substantial product-moment correlation. Multiple correlation is the special case of canonical correlation and it is sensitive to the same condition related instabilities when there is a relatively small number of entities in relation to the number of variables. The promoters of canonical correlation analysis as a validity technique are in favour to this approach (Galton, 1954; Mekota & Blahus, 1984), but there are some contra statements which points out its inadequacy (Cohen & Cohen 1983, 2010).

In kinesiology, as well as some other disciplines, standard measures of validity are not always appropriate. Most often, the problem lies in the small number of participants in the research sample. In some cases, the population is so small that it is impossible to cumulate even the modest number of subjects in the study. For example, try to estimate the predictive validity of the Slovenian translation of an anxiety test in prediction of the success of Formula One drivers, or curlers while they sweep the rock down the ice, or competitors in pole vaulting.

Momirović, Dobrić & Karaman (1983) proposed the method of the analysis of the relationship of two sets of quantitative data based on the maximization of covariances of not necessarily orthogonal linear combinations of two sets of variables. The method is not dependent on the regularity of correlation matrices, and it not so sensitive to outliers. The results are not sensitive to high correlations between a single pair of variables. It is robust to the number of degrees of freedom and could be applied when the research sample is small (Knežević & Momirović, 1996). The same applies to robust linear regression analysis based on the maximization of covariance proposed by Štalec & Momirović (1983). Robust methods proved to be much more convenient in different analyses of the relationship between two sets of data. Here is the proposition to use robust methods based on the covariance maximization in defining criterion-related validity in kinesiology.

ALGORITHM

Based on the development in the standard canonical correlation approach (Hotelling, 1933) and Tucker's interbattery factor analysis (Tucker, 1958) Momirović et al. (1983) proposed the method of analysis of the relationship of two sets of quantitative data based on the maximization of covariances of not necessarily orthogonal linear combinations of two sets of variables. To facilitate the comparison of both of the methods, a procedure for simultaneous analyses was developed (Bosnar, Prot & Momirović, 1984; Knežević & Momirović, 1996). A synopsys of the algorithm which simultaneously applies canonical correlation and quasi canonical analysis of covariance is presented (Prot, 2008). SPSS matrix macro program QCCR (Momirović, 1996), an implementation of the simultaneous canonical correlation and quasi canonical covariance analysis procedures is used and the main results presented.

We have a set of entities, E, measured with two set variables V_1 and V_2 and as a result we have two matrices $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in standard normal form (i.e. columns of $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are centered to 0 normalised to 1, and divided by $n^{-1/2}$), and unit summation vectors $\mathbf{e}_1$ and $\mathbf{e}_2$.

$$\mathbf{Z}_1 = E \otimes V_1 \qquad \begin{array}{l} | \; \mathbf{Z}_1 = (z_1)_{iv} \,; \\ | \; i = 1, \ldots n \,;\; v = 1, \ldots, m_1 \,; \\ | \; \mathbf{Z}_1^t \mathbf{e}_1 = \mathbf{0} \,; \\ | \; \mathrm{diag}\,(\mathbf{Z}_1^t \mathbf{Z}_1) = \mathbf{I} \end{array}$$

$$\mathbf{Z}_2 = E \otimes V_2 \qquad \begin{array}{l} | \; \mathbf{Z}_2 = (z_2)_{iw} \,; \\ | \; i = 1, \ldots n \,;\; w = 1, \ldots, m_2 \,; \\ | \; \mathbf{Z}_2^t \mathbf{e}_2 = 0 \,, \\ | \; \mathrm{diag}\,(\mathbf{Z}_2^t \mathbf{Z}_2) = \mathbf{I} \end{array}$$

The description of the set of entities *E* on two sets of variables V_1 i V_2, one of them is set of variables to be validated with respect to another one as a set of criterion variables, in standard normal form. So, then: $\mathbf{R}_{11} = \mathbf{Z}^t_1\ \mathbf{Z}_1$; $\mathbf{R}_{22} = \mathbf{Z}^t_2\ \mathbf{Z}_2$; and $\mathbf{R}_{12} = \mathbf{Z}^t_1\ \mathbf{Z}_2$, are: matrix of correlations within the first set; matrix of correlations within the second set and matrix of correlations between the first and second set, respectively.

Canonical correlation analysis (Hotelling, 1933)

$$\mathbf{Z}_1\ \mathbf{x}_{1p} = \mathbf{k}_{1p} \quad | \quad r_p = \mathbf{k}_{1p}{}^t\ \mathbf{k}_{1p} = \max$$
$$\mathbf{Z}_2\ \mathbf{x}_{2p} = \mathbf{k}_{2p} \quad | \quad \mathbf{k}_{1p}{}^t\ \mathbf{k}_{1p} = \mathbf{k}_{2p}{}^t\ \mathbf{k}_{2p} = d_{pq}$$
$$| \quad \mathbf{k}_{1p}{}^t\ k_{2q} = 0;\ p^1\ q$$

where p, q = 1,...,m; m = min(m_1,m_2), a δ_{pq} Kronecker symbol.

The well known solution (see Anderson, 1984) defined as an extremum of:

$$f(\mathbf{x}_{1p}, \mathbf{x}_{2p}, l_{1p}, l_{2p}) = \mathbf{x}_{1p}{}^t\ \mathbf{R}_{12}\ \mathbf{x}_{2p} - 1/2\ l_{1p}\ (\mathbf{x}_{1p}{}^t\ \mathbf{R}_{11}\ \mathbf{x}_{1p} - 1) - 1/2\ l_{2p}\ (\mathbf{x}_{2p}{}^t\ \mathbf{R}_{22}\ \mathbf{x}_{2p} - 1)$$

where l_{1p} and l_{2p} are Lagreange multipliers.

As a result, we have two sets of canonical variates represented with matrices $\mathbf{K}_1$ and $\mathbf{K}_2$ in standard normal form for significant canonical roots. Correlation between $\mathbf{K}_1$ and $\mathbf{K}_2$ are canonical validites.

Quasi canonical covariance analysis (Momirović et al., 1983)

Let $\mathbf{Z}_1$ (n, m_1) and $\mathbf{Z}_2$ (n, m_2) be two centred data matrices, obtained as a description of set *E* of n entities over two sets *V*1 and *V*2 of quantitative, elliptically distributed variables. Maximization of covariances between linear composites of variables belonging to the sets *V*1 and *V*2 is defined, under some constraint, as:

$$\mathbf{Z}_1\ \mathbf{x}_p = \mathbf{l}_{1p} \quad | \quad r_p = \mathbf{l}_p{}^t\ \mathbf{l}_{2p} \rightarrow \max$$
$$\mathbf{Z}_2\ \mathbf{y}_p = \mathbf{l}_{2p} \quad | \quad r_p > r_{p+1},\ p = 1,\dots, r-1;$$
$$| \quad r = \min(m1, m2)$$
$$| \quad \mathbf{x}_p{}^t\ \mathbf{x}_p = \mathbf{y}_p{}^t\ \mathbf{y}_p = d_p$$

and can be reduced to the singular value decomposition of matrix $\mathbf{C}_{12}\ \mathbf{Z}_1{}^t\ \mathbf{Z}_2\ n^{-1}$, defined as extremum of

$$f(\mathbf{y}_{1p}, \mathbf{y}_{2p}, h_{1p}, h_{2p}) = \mathbf{y}_{1p}{}^t\ \mathbf{R}_{12}\ \mathbf{y}_{2p} - 1/2\ h_{1p}\ (\mathbf{y}_{1p}{}^t\ \mathbf{y}_{1p} - 1) - 1/2\ h_{2p}\ (\mathbf{y}_{2p}{}^t\ \mathbf{y}_{2p} - 1)$$

where h_{1p} and h_{2p} are Lagrange multipliers, and maximisations of covariance as pointed out by Tucker (1958). As a result, we have two set of canonical variates represented by matrices $\mathbf{L}_1$ and $\mathbf{L}_2$ in standard normal form for important spectral values.

Correlations between $\mathbf{K}_1$ and $\mathbf{K}_2$, and between $\mathbf{L}_1$ and $\mathbf{L}_2$ are canonical and quasi canonical validites, respectively, which is the main interest of this paper.

The example

The comparison of standard measures of validity and measures based on the maximization of the covariance appropriate linear combination of test results and the chosen criteria was performed on the data in an attempt to define the ability of tactical thinking in team sports. Lanc (1967) proposed the measure of tactical thinking ability in team sports composed of four problem tasks from football, handball, basketball and volleyball. The convergent validation of the concept was done by four intelligence tests by Reuchlin & Valin (1953), measuring spatial, perceptual, verbal and numerical ability. The results, as well as conclusions, were ambiguous and Lanc (1967) lost interest in researching tactical thinking in team sports. The data obtained on the sample of 90 students of physical education were re-analyzed by biorthogonal canonical correlation analysis (Hotelling, 1936) and by canonical analysis of covariance (Momirović et al., 1983).

The correlations of variables are in Table 1, and the comparisons of the results of the canonical correlation analysis and canonical covariance analysis are in Tables 2–4. The correlation of variables of two sets are low, starting from zero values to highest $r=0.292$. Hotelling's canonical correlation analysis show an insignificant first canonical correlation ($\rho=0.387$), however, an algorithm by Momirović et al. (1983) provided a smaller but statistically significant correlation ($\rho=0.291$, $p=0.005$). The sample of 90 subjects was too small to prove the relationships by maximization the correlation of linear composites of two sets of variables, but large enough for obtaining significant correlation when using the algorithm based on maximization the covariance of linear composites of two sets of variables.

Table 1: The correlations of intelligence tests results (spatial, verbal, perceptive and numerical) and results in four problem tasks of tactical thinking (football, handball, basketball and volleyball).

	Spatial	Verbal	Percep-tual	Numeri-cal	Football	Handball	Basket-ball	Volley-ball
Spatial	1							
Verbal	.168	1						
Perceptual	.462	.135	1					
Numerical	.337	.277	.522	1				
Football	.115	.083	-.057	.109	1			
Handball	.292	.063	.162	.126	.181	1		
Basketball	.227	.031	.186	.038	.195	.502	1	
Volleyball	.193	.003	.031	.204	-.007	.437	.351	1

Table 2: The comparison of the results of canonical correlation analysis and canonical covariance analysis: ρ denotes correlation coefficient, ρ^2 denotes determination coefficient, F-value is the value of F test, df 1 and df 2 are degrees of freedom 1 and 2, respectively, and p_F and p_x are the probabilities of statistical significance of F-test and ρ^2 test, respectively; for both analysis n=90.

Analysis	**ρ**	**ρ^2**	**F-value**	**df 1**	**df 2**	**p_F**
Canonical covariance	.291	.085	8.157	1	88	.005
			Wilk's λ	**χ^2 test**	**df**	**p_x**
Canonical correlation	.387	.150	.760	23.169	16	.109

There is no doubt that canonical covariance analysis is an appropriate method for samples qualified as small. As proposed by Hošek et al. (1984) it seems reasonable to perform both canonical correlation and canonical covariance analysis whenever investigating the relations of two sets of variables using samples with a modest number of entities. This example shows that only taking into consideration the results of Hotelling's canonical correlation analysis could lead to misinterpretation.

Table 3: The comparison of the results of canonical correlation analysis and canonical covariance analysis: X denotes weights, F denotes structure of the first factor and C denotes structure of the first cross-factor; index cc denotes the results of canonical correlation analysis and qc denotes the results of canonical covariance analysis.

Tactical thinking in team sports	Xcc	Xqc	Fcc	Fqc	Ccc	Cqc
Football	.718	.242	.537	.334	.208	.097
Handball	.007	.672	.217	.856	.084	.270
Basketball	.499	.514	.134	.772	.052	.206
Volleyball	.998	.475	.662	.687	.257	.191

Table 4: The comparison of the results of canonical correlation analysis and canonical covariance analysis: X denotes weights, F denotes structure of the first factor and C denotes structure of the first cross-factor; index cc denotes the results of canonical correlation analysis and qc denotes the results of canonical covariance analysis.

Intelligence tests	Xcc	Xqc	Fcc	Fqc	Ccc	Cqc
Spatial	.067	.806	.295	.864	.114	.313
Verbal	.032	.149	.104	.340	.040	.058
Perceptual	.187	.384	.298	.749	.115	.149
Numerical	.121	.425	.586	.704	.227	.165

Program

Data analysis have been realised by the general macro program QCCR (Momirović, 1997) and for the simultaneous application of canonical correlation analysis and canonical analysis of covariances, more often called as quasi canonical covariance analysis. The 767 lines of code macro program QCCR is realized in SPSS Macro language. Canonical correlation analysis is implemented according to Hotelling (1935, 1936) and Cooley and Lohnes (1971), where the significance of canonical correlations is tested according to the procedure proposed by Bartlett (1941). Canonical analysis of covariances is implemented according to the method proposed by Momirović et al. (1983). The comparison of results of those two methods is realised by the procedure proposed by Bosnar, Prot & Momirović (1984). This program is a revision of the program QCCR written by Momirović, Dobrić, Bosnar & Prot (1984) in SS language Zakrajšek, Momirović & Štalec (1974).

REFERENCES

Bosnar, K., Prot, F. & Momirović, K. (1984). Neke relacije između kanoničke i kvazikanoničke korelacijske analize. In K. Momirović, J. Štalec, F. Prot, K. Bosnar, N. Viskić-Štalec, L. Pavičić, & V. Dobrić, Kompjuterski programi za klasifikaciju, selekciju, programiranje i kontrolu treninga (pp. 5–22). Zagreb: Fakultet za fizičku kulturu.

Cohen, J., & Cohen, P. (1983, 2003). Appendix 4. Set correlation as a General Multivariate Data-analytic Method. Applied Multiple Regression/Correlation Analysis for the Behavioral Sciences (2nd edition) (pp. 487–518). Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Gliner, J. A., & Morgan, G. A. (2000). Research Methods in Applied Settings: An Integrated Approach to Design and Analysis. Mahawah: Lawrence Erlbaum Associates.

Hotelling, H. (1936). Relations between two sets of variantes. Biometrika, 28, 321–377.

Hošek, A., Bosnar, K., & Prot, F. (1984). Comparison of the results of quasicanonical and canonical correlation analysis in various experimental situations. In V. Lužar, & M. Cvitaš (eds.), Proceedings of International Symposium 'Computer at the University' (pp. 610.1–610.7). Zagreb: University Computing Centre.

Knežević, G., & Momirović, K. (1996). Algorithm and program for analysis of relations between canonical correlation analysis and covariance canonical analysis. [Algoritam i program za analizu relacija kanoničke korelacijske analize i kanoničke analize kovarijansi in Serbian]. In P. Kostić (ed.) Merenje u psihologiji, 2, 57–73. Beograd: Institut za kriminološka i sociološka istraživanja.

Lanc, M. (1967). Neke relacije između testova kognitivnih funkcija i taktičkih sposobnosti u sportskim igrama (magistarski rad), Zagreb: Visoka škola za fizičku kulturu.

Mekota, K., & Blahuš, P. (1983). Motoricke testy v telesne vychove. Praha: Statni pedagogicke nakladatelstvo.

Momirović, K., Dobrić, V., & Karaman, Ž. (1983). Canonical covariance analysis. In: Proceedings of 5th International Symposium 'Computer at the University' (pp. 463–473). Cavtat.

Reuchlin, M., & Valin, E. (1953). Tests collectifs du Centre de Recherche BCR. Bulletin de l'Institut National d'Orientation Professionelle, 9, 1–152.

Štalec, J., & Momirović, K. (1983). Some properties of a very simple model for robust regression analysis. Proceedings of International Symposium "Computer at the University" (pp. 453–461). Cavtat.

Tucker, L. R. (1958). An Inter-Battery Method of Factor Analysis, Psychometrika, 23(2), 111–136.