Cost Optimal Project Scheduling

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This paper presents the cost optimal project scheduling. The optimization was performed by the nonlinear programming approach, NLP. The nonlinear total project cost objective function is subjected to the rigorous system of the activity precedence relationship constraints, the activity duration constraints and the project duration constraints. The set of activity precedence relationship constraints was defined to comprise Finish-to-Start, Start-to-Start, Start-to-Finish and Finish-to-Finish precedence relationships between activities. The activity duration constraints determine relationships between minimum, maximum and possible duration of the project activities. The project duration constraints define the maximum feasible project duration. A numerical example is presented at the end of the paper in order to present the applicability of the proposed approach.

Key words: project management, scheduling, optimization, nonlinear programming, NLP

1 Introduction

Project scheduling is an important phase in the planning process of the project. While general project scheduling is performed before the submission of a tender, the executive project scheduling is performed before or during the realization of a project (Pšunder and Rebolj, 1991). Each activity within project network is characterized by its precedence relations, duration and resource requirements. In this way, execution of each project activity in normal duration requires employment of certain resources. In cases when faster execution of an activity is required, the additional resources at extra cost must be utilized. Traditional methods for cost optimal project scheduling include either Critical path method (CPM) or program evaluation and review technique (PERT) combined with trial-and-error procedure. This way, the cost optimal project schedule is achieved in the time-consuming analysis of various alternatives for start times and durations of activities.

To surmount the mentioned disadvantages, various different optimization methods have been proposed for the cost optimal project scheduling. Considering the exact mathematical programming methods, the cost optimal project scheduling has been handled mainly by different linear programming methods (e.g. Demeulemeester *et al.*, 1998; Achuthan & Hardjawidjaja, 2001; Möhring *et al.*, 2001; Vanhoucke *et al.*, 2002;). In these studies, the nonlinear cost functions were approximated with piece-wise linear functions. The nonlinear programming methods have been also proposed to solve optimal project scheduling problems with continuous nonlinear cost functions. A survey of literature and nonlinear time/cost trade-off models in this field were presented by Deckro *et al.* (1995). However, in most of the published research works the cost optimal project scheduling was performed considering only the finish-to-start precedence relationships between project activities.

This paper presents the cost optimal project scheduling performed by the nonlinear programming approach, NLP. The nonlinear total project cost objective function is subjected to the rigorous system of the activity precedence relationship constraints, the activity duration constraints and the project duration constraints. The set of activity precedence relationship constraints is defined to comprise Finish-to-Start, Start-to-Start, Start-to-Finish and Finish-to-Finish precedence relationships between activities. The activity duration constraints determine relationships between minimum, maximum and possible duration of the project activities. The project duration constraints define the maximum feasible project duration. A numerical example is presented at the end of the paper in order to present the applicability of the proposed approach.

2 NLP problem formulation

The cost optimal project scheduling was performed by the nonlinear programming approach, NLP. The general NLP optimization problem may be formulated in the following form:

$$\text{Min } z = f(x)$$

subjected to:

$$h(x) = 0 \qquad \text{(NLP)}$$

$$g(x) \le 0$$

$$x \in X = \{x \mid x \in R^n, x^{LO} \le x \le x^{UP}\}$$

where x is a vector of the continuous variables, defined within the compact set X. Functions $f(x)$, $h(x)$ and $g(x)$ are the (non)linear functions involved in the objective function z, the equality and inequality constraints, respectively. All the functions $f(x)$, $h(x)$ and $g(x)$ must be continuous and differentiable.

In the context of the project scheduling optimization problem, the continuous variables define schedule parameters such as activity durations, start times, direct costs, etc. Equality and inequality constraints and the bounds of the continuous variables represent a rigorous system of precedence relationship constraints, the activity duration constraints and the project duration constraints of the project scheduling optimization problem.

3 NLP model formulation

Considering the general NLP formulation, the NLP model formulation for project scheduling optimization problem is more specific, particularly in terms of variables and constraints. This way, the proposed NLP model formulation consists of the objective function, the precedence relationship constraints, activity duration constraints and the project duration constraints.

The optimal project scheduling may include various objectives worthy of consideration. The most frequently used objectives for the optimal project scheduling are: the minimum project completion time and the minimum total project cost. In this paper, the following total project cost objective function is defined for the optimal project scheduling:

$$CT = \sum_{i \in I} C_i\left(D_i\right) + C0 + C1 \cdot DP \tag{1}$$

where objective variable CT represents the total project cost, set I comprises the project activities i, $i \in I$, $C_i(D_i)$ denotes the activity direct cost-duration function, $C0$ is the initial project cost, $C1$ is the daily project cost and DP is the project duration. This way, the total project cost objective includes direct cost of all project activities, the initial project cost and the indirect daily expenditures. The total project cost objective function is subjected to the rigorous system of the activity precedence relationship constraints, the activity duration constraints and the project duration constraints.

Each project activity i, $i \in I$, is connected with its succeeding activities j, $j \in J$ by fulfilling at least one of the following precedence relationship constraints:

Finish-to-Start:

$$S_i + D_i + L_{i,j} \leq S_j \tag{2}$$

Start-to-Start:

$$S_i + L_{i,j} \leq S_j \tag{3}$$

Start-to-Finish:

$$S_i + L_{i,j} \leq S_j + D_j \tag{4}$$

Finish-to-Finish:

$$S_i + D_i + L_{i,j} \leq S_j + D_j \tag{5}$$

where S_i is the start time of activity, D_i is the activity duration, $L_{i,j}$ is the lag/lead time between activity i, $i \in I$, and the succeeding activity j, $j \in J$.

Duration of each activity defined within project network is constrained not to pass defined minimum and maximum possible activity duration:

$$D_i - Dmin_i \leq 0 \tag{6}$$

$$D_i - Dmax_i \leq 0 \tag{7}$$

where $Dmin_i$ and $Dmax_i$ denote minimum and maximum possible duration of activity i, $i \in I$.

The project duration DP is determined as follows:

$$DP = S_{i\omega} + D_{i\omega} - S_{i\alpha} \tag{8}$$

where $S_{i\omega}$ and $D_{i\omega}$ represent the start time and the duration of the last project activity $i\omega$, $i\omega \in I$, and $S_{i\alpha}$ denotes the start time of the first project activity $i\alpha$, $i\alpha \in I$.

Project must be completed before the predetermined date. This way, the project duration is constrained not to exceed a given maximum feasible project duration:

$$S_{i\omega} + D_{i\omega} - S_{i\alpha} \leq D_{Pmax} \tag{9}$$

where D_{Pmax} denotes the maximum feasible project duration.

It should be noted that the start times of activities S_i, the activity durations D_i and the activity direct cost-duration functions $C_i(D_i)$ are included into NLP model formulation as positive variables.

4 Modelling and solving the NLP optimization problem

The developed NLP model formulation must be transformed into suitable modelling software. The spreadsheet-oriented optimizers, such as MS Excel Solver and What'sBest, are applicable tool for formulating small- and medium-sized models with reasonable number of parameters to be filled on a spreadsheet. On the other hand, the mathematical modelling languages, such as AMPL, GAMS, LINGO and MPL may be used for large, complex, one-of-a-kind optimization problems which may require many revisions to establish an accurate model. The modelling languages are especially applicable in cases where large number of functional constraints of the same type follow the same pattern. This way, the modelling language may simultaneously formulate all the constraints of the same type by simultaneously dealing with the variables of each type. Moreover, the modelling language hastens a number of model management tasks, such as accessing the data, transforming the data into model parameters, modifying the model, and analyzing solutions from the model.

After the optimization model formulation is transformed into modelling software, the defined optimization problem may be solved by the use of a suitable solver. A general NLP class of optimization problems can be solved by several commercially available NLP solvers, such as CONOPT, KNITRO, LANCELOT, MINOS, NPSOL, etc.

5 Numerical example

In order to present the applicability of the proposed NLP approach, the paper presents an example of the cost optimal project scheduling. The considered example project consists of 14 non-splittable activities. The precedence relationships and the lag times between succeeding activities are presented in Table 1. The initial project cost of 5500,00 € and the daily project cost of 2000,00 € are defined in the input data. The minimum durations, the maximum durations and the direct cost-duration functions of example project activities are given in Table 2. Maximum project duration is set to be 17 working days.

The objective of the optimization is to find a project schedule with optimal activity start times and durations so as to minimize total project cost, subjected to the activity precedence relationship constraints, the activity duration constraints and the project duration constraints.

The proposed optimization NLP model formulation was applied. A high-level language GAMS (General Algebraic Modelling System) (Brooke *et al.*, 1988) was used for modelling and for data inputs/outputs. CONOPT

Table 1: Precedence relationships and the lag times between project activities

Activity	Succeeding activity	Precedence relationship	Lag time (day)
1	2	Finish-to-Start	0
2	3	Start-to-Start	2
	4	Start-to-Start	2
3	5	Finish-to-Finish	3
4	6	Start-to-Finish	2
	7	Finish-to-Finish	4
5	8	Finish-to-Start	0
6	9	Start-to-Start	1
7	13	Finish-to-Start	0
8	10	Finish-to-Finish	4
	11	Finish-to-Finish	2
9	13	Finish-to-Start	2
10	12	Finish-to-Start	0
11	14	Finish-to-Start	0
12	14	Finish-to-Finish	1
13	14	Finish-to-Start	0

Table 2: Minimum duration, maximum duration and direct cost-duration functions of the project activities

Activity	Minimum duration (day)	Maximum duration (day)	Direct cost-duration function (€)
1	1	2	$5500-80D_1-40D_1^2$
2	1	3	$4300-65D_2-30D_2^2$
3	4	7	$5300-75D_3-35D_3^2$
4	1	4	$3400-50D_4-25D_4^2$
5	1	5	$4400-65D_5-25D_5^2$
6	2	6	$4600-70D_6-30D_6^2$
7	2	5	$4400-65D_7-25D_7^2$
8	1	3	$3100-40D_8-20D_8^2$
9	1	5	$4400-65D_9-25D_9^2$
10	2	6	$5100-75D_{10}-30D_{10}^2$
11	7	9	$6700-100D_{11}-50D_{11}^2$
12	1	2	$2600-40D_{12}-15D_{12}^2$
13	4	8	$6700-90D_{13}-40D_{13}^2$
14	2	5	$3800-55D_{14}-20D_{14}^2$

D_1 to D_{14} denote the durations of the project activities labelled 1 to 14 measured in days.

(Generalized reduced-gradient method) (Drud, 1994) was used for the optimization.

Since the NLP denotes the continuous optimization technique, the optimization of the project schedule was performed in two successive steps. In the first step, the ordinary NLP optimization was performed to calculate the optimal continuous variables (e.g. start times, durations, etc.) inside their upper and lower bounds. In the second step, the calculation was repeated/checked for the fixed and rounded variables (from in the first stage obtained continuous values to their nearest upper discrete values). In this paper, a day was selected to be the discrete time unit for the example project schedule.

The minimum total project obtained by the NLP optimization cost was found to be 88555,00 €. The gained optimal results include the optimal start times, durations and direct costs of the project activities. Table 3 summarizes the optimum results for the example project. The optimal project schedule is presented in Figure 1.

The obtained cost optimal solution was determined by the activity precedence relationship constraints, the minimum activity duration constraints (activities 1, 3, 4, 8, 12 and 14, see Table 2 and Fig. 1.), the maximum activity duration constraints (activities 2, 5, 6, 7, 9, 10, and 11, see Table 2 and Fig. 1.) and the project duration constraints. Since the calculated optimal duration of the 13th activity was equal to 7 days, the duration constraints on this activity were not decisive for the optimum solution in this case, see Table 2 and Fig. 1.

Table 3: Optimal results

Activity	Start time (day)	Duration (day)	Direct cost (€)
1	1	1	5380,00
2	2	3	3835,00
3	4	4	4440,00
4	4	1	3325,00
5	6	5	3450,00
6	1	6	3100,00
7	4	5	3450,00
8	11	1	3040,00
9	2	5	3450,00
10	10	6	3570,00
11	5	9	1750,00
12	16	1	2545,00
13	9	7	4110,00
14	16	2	3610,00
		Direct project cost (€):	49055,00
		Indirect project cost (€):	39500,00
		Total project cost (€):	88555,00

Project scheduling with CPM and PERT methods considers only time dimension of the project activities. Accordingly, the minimization of the total project cost with traditionally used CPM and PERT methods must be performed in a time-consuming trial-and-error procedure of analysing different project schedule alternatives by varying the start times and the durations of the project activities. Moreover, doubt always exists as to whether or not the obtained project schedule is optimal.

On the other hand, the presented example shows that the total cost optimization of the project schedule performed by the NLP approach is carried out in a single uniform calculating process, where the start times and durations of project activities are considered simultaneously in order to obtain the minimum total project cost. The obtained maximum values for durations of the project activities 2, 5, 6, 7, 9, 10, and 11 demonstrate that the cost optimization of project schedules not necessarily minimize the project duration. In this way, the additional feature of the total project cost optimization represents the advantage of the proposed NLP approach to project scheduling over the traditionally used CPM and PERT methods.

6 Conclusion

This paper presents the cost optimal project scheduling performed by the nonlinear programming approach, NLP. The nonlinear continuous total project cost objective function was subjected to the rigorous system of the activity precedence relationship constraints, the activity duration constraints and the project duration constraints. The set of activity precedence relationship constraints was defined to comprise Finish-to-Start, Start-to-Start, Start-to-Finish, and Finish-to-Finish precedence relationships between activities. The activity duration constraints were included to determine relationships between minimum, maximum and possible duration of the project activities. The project

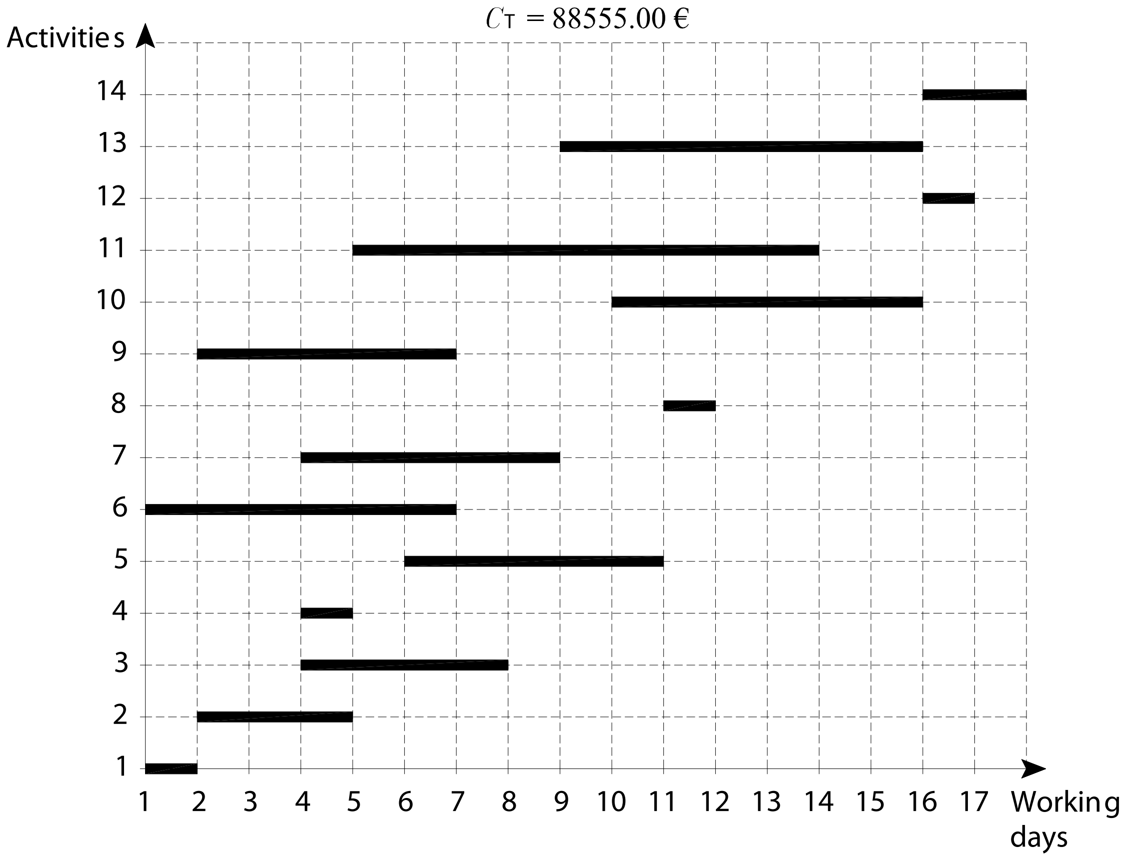

Figure 1: Optimal project schedule

duration constraints were defined to constrain the maximum feasible project duration.

The NLP model formulation was developed and applied for the cost optimal project scheduling. The input data within proposed NLP optimization model include: the project network with determined preceding and succeeding activities, the precedence relationships and the lag/lead times between activities, the minimum and the maximum durations of the activities, the initial project cost, the daily project cost, and the direct cost-duration functions of the activities. Linear or nonlinear functions may be used for direct cost-duration relationships of the project activities. For specified input data, the proposed NLP optimization model yields the minimum total project cost. The gained optimal results also include the optimal start times, durations and direct costs of the project activities. On account of the additional feature of the total project cost optimization, the proposed NLP approach to the project scheduling surpasses traditionally used CPM and PERT methods. Furthermore, the contribution provided an alternative tool which enables the cost optimal project scheduling to be carried out fast and in a single uniform calculating process. Since the proposed optimization approach enables an insight into the interdependence between the project duration and the total project cost, the decision-maker can more effectively estimate the effect of the project deadline on a total project cost before the submission of a tender. Finally, the project schedule can also be updated and compressed during the project execution in such a way to cause the minimum additional costs or to make maximum additional cost savings.

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Appendix

Notations

C_i	activity direct cost (variable)
CT	total project cost (objective variable)
$C0$	initial project cost (constant)
$C1$	daily project cost (constant)
D_i	activity duration (variable)
$Dmax_i$	maximum duration of activity (constant)
$Dmin_i$	minimum duration of activity (constant)
DP	project duration (variable)
$DPmax$	maximum project duration (constant)
I	set of project activities
J	set of succeeding project activities
$L_{i,j}$	lag/lead time between the preceding and the succeeding activity (constant)
$\boldsymbol{R}^n$	n-dimensional space of real numbers
S_i	start time of activity (variable)
$\boldsymbol{X}$	compact set
$\boldsymbol{x}$	vector of continuous variables
z	objective function

Subscripts

i	project activity (index)
j	succeeding project activity (index)

Superscripts

LO	lower bound
UP	upper bound

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Stroškovno optimalno planiranje projektov

V članku je predstavljeno stroškovno optimalno planiranje projektov. Optimiranje je izvedeno s pristopom nelinearnega programiranja, NLP. Nelinearna namenska funkcija celotnih stroškov projekta je podvržena rigoroznemu sistemu pogojnih neenačb časovnih odvisnosti med aktivnostmi, trajanja aktivnosti in trajanja projekta. Množica pogojnih neenačb časovnih odvisnosti med aktivnostmi vključuje časovne povezave konec-začetek, začetek-začetek, začetek-konec in konec-konec. Pogojne neenačbe trajanja aktivnosti določajo odnose med minimalnim, maksimalnim in možnim časom trajanja projektnih aktivnosti. Pogojne (ne)enačbe trajanja projekta omejijo maksimalni možni čas trajanja projekta. Za predstavitev uporabnosti predlaganega pristopa je na koncu članka prikazan računski primer.

Ključne besede: projektni management, planiranje, optimizacija, nelinearno programiranje, NLP