BLED WORKSHOPS
IN PHYSICS
VOL. 15, NO. 2

Proceedings to the 17th Workshop
What Comes Beyond ... *(p. 163)*
Bled, Slovenia, July 20-28, 2014

10 The *Spin-charge-family* Theory Explains Why the Scalar Higgs Carries the Weak Charge $\pm\frac{1}{2}$ and the Hyper Charge $\mp\frac{1}{2}$

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Abstract. One Weyl representation of $SO(13+1)$ contains [1–7], if analysed with respect to the charge and the spin groups of the *standard model*, left handed weak ($SU(2)_{I}$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons, and right handed weakless and $SU(2)_{II}$ charged quarks and leptons (neutrinos and electrons). In the *spin-charge-family* theory [1–12] spinors carry also the family quantum numbers, explaining the origin of families and correspondingly the masses of fermions and weak bosons and the origin of the scalar Higgs and Yukawa couplings. I am demonstrating in this paper that all the fields appearing in the simple starting action of the *spin-charge-family* theory in $d = (13+1)$ with the scalar index with respect to $d = (3+1)$ and determining masses of quarks and leptons (and correspondingly also of the weak boson fields) carry the weak and the hyper charge required by the *standard model* for the scalar Higgs.

Povzetek. Ena Weylova upodobitev $SO(13+1)$ vsebuje [1–7], če jo analiziramo glede na grupe nabojev in spinov *standardnega modela*, levoročne kvarke z barvnim tripletnim nabojem in brezbarvne leptone s šibkim nabojem ($SU(2)_{I}$), ki nimajo naboja $SU(2)_{II}$ ter desnoročne barvne triplete kvarkov in brezbarvnih leptonov, ki ne nosijo šibkega naboja, nosijo pa naboj $SU(2)_{II}$. V teoriji *spinov-nabojev-družin* [1–12] nosijo spinorji tudi kvantna števila družin, kar pojasni izvor družin in tudi mase fermionov in šibkih bozonov ter izvor Higgsovega skalarja in Yukawinih sklopitev. V tem prispevku pokažem, da nosijo vsa polja s skalarnim indeksom glede na $d = (3+1)$ $s = (7,8)$, ki nastopajo v enostavni začetni akciji teorije *spinov-nabojev-družin* v $d = (13+1)$ in določajo mase kvarkov in leptonov, s tem pa tudi mase šibkih bozonov, šibki in hiper naboj tak, kot ju zahteva *standardni model* za skalarno Higgsovo polje. Teorija tako ponudi razlago za izmerjene lastnosti Higgsovega skalarja ter Yukawinih sklopitev.

10.1 Introduction

The *standard model* assumed and the LHC confirmed the existence of the Higgs scalar - the only so far observed bosons with the charge in the fundamental representation.

I am demonstrating in this paper that the *spin-charge-family* theory explains the appearance of the scalar fields with the charges of the Higgs scalar fields. There are, namely, in this theory, in its simple starting action in $d = (13+1)$, the fields with the scalar index with respect to $d = (3+1)$, which have the properties of the higgs and explain masses of quarks and leptons together with the Yukawa couplings, and correspondingly also the masses of the weak vector boson fields.

Let me add that all the scalars, that is all the gauge fields of this theory with the space index ≥ 5, have in the starting action the corresponding charges with respect to the scalar index in the fundamental representations: They are either doublets with respect to the two $SU(2)$ groups (the weak $SU(2)_I$ and the second $SU(2)_{II}$, which correspondingly result in the properties of the Higgs scalars with respect to the weak and the hyper charge) or they are colour triplets (the properties of these triplets are discussed in a separate paper [13]). All these scalar fields carry the additional charges (the charges not originating in the space index - the family charges, for example) in adjoint representations.

The referee of this paper stated that it is not at all remarkable that there are the scalar fields which are doublets with respect to the weak charge after starting with so many independent fields.

It is, of course, true that a large enough orthogonal group can contain any desired subgroups. *But this is not what the spin-charge-family theory proposes*: It starts with an (very simple) action for spinors and the corresponding gauge fields, manifesting very limited properties, and it is not at all self-evident that some of these fields manifest the desired properties in the low energy regime while all the other spinors and vector and scalar gauge fields - unobserved in the low energy regime - get high masses through the interaction with only one scalar condensate, what is happening in the *spin-charge-family* theory.

On the contrary, it is an extremely encouraging fact that one scalar condensate makes all the vector and the scalar gauge fields appearing in the *spin-charge-family* theory, except those which are observable at the low energy regime (the gravity, the colour vector gauge field, the weak and the hyper charge vector gauge fields, and the eight families of quarks and leptons, decoupled into two times four families), very massive with respect to the weak scale and correspondingly unobservable in the low energy regime. Several scalar gauge fields, however, which when gaining nonzero vacuum expectation values (changing in this case also their masses) cause the electroweak break, have the weak charge equal to $\pm\frac{1}{2}$ and the hyper charge correspondingly $\mp\frac{1}{2}$, as the scalar Higgs in the *standard model*, while they have all the other quantum numbers in the adjoint representations. All the rest of the scalar fields are colour triplets with respect to the scalar space index.

Those who are proposing unifying theories, must offer for the chosen groups and the chosen representations of these groups also the Lagrange densities, designed for those groups and representations, what calls for the theory beyond those effective actions. I am proposing a simple starting action, out of which - after the breaks of symmetries triggered by boundary conditions in a complicated many body problem - manifests in the low energy regime the observable phenomena.

Let me make in this introduction make a short overview of the *spin-charge-family* theory [1,2,7,6,3–5,8–12], pointing out that this theory is offering the explanation for the assumptions of the *standard model*: For the properties of quarks and leptons (right handed neutrinos are in this theory the regular members of a family) and antiquarks and antileptons, and for the existence of the gauge vector fields [1] of the charges. It is offering also the explanation for the existence of the families of quarks and leptons and correspondingly for the scalar gauge fields, which are responsible for masses of quarks and leptons and of weak gauge fields and for Yukawa couplings.

There are, namely, (only) two kinds [3–5,7,16–18] of the Clifford algebra objects (connected by the left and the right multiplication of any Clifford object): the Dirac γ^a's and the second $\tilde{\gamma}^a$'s, respectively. These two Clifford algebra objects (Eq. (9.35)) anticommute, forming the equivalent representations with respect to each other. If using the Dirac γ^a's in $d = (13+1)$ to describe in $d = (3+1)$ the spin and all the charges, then $\tilde{\gamma}^a$'s describe families.

All predictions of the *spin-charge-family* theory in the low energy regime (after the break of the starting symmetry) follow from the simple starting action (Eq.(10.1)) in $d = (13+1)$ for spinors carrying two kinds of a spin (no charges) and for the vielbeins and the two kinds of spin connection fields, with which spinors interact.

Let us first tell that one Weyl representation of $SO(13,1)$ contains [1,2,7,6,14], if analysed with respect to the subgroups $SO(3,1)\times$ $SU(2)_I\times$ $SU(2)_{II}$ $\times$ $SU(3)$ $\times U(1)$, all the family members, required by the *standard model*, with the right handed neutrinos in addition: It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), and right handed weakless and $SU(2)_{II}$ charged quarks and leptons, as well as right handed weak charged and $SU(2)_{II}$ chargeless colour antitriplet antiquarks and (anti)colourless antileptons, and left handed weakless and $SU(2)_{II}$ charged antiquarks and antileptons. The antifermions are reachable from the fermions by the application of the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$, presented in ref. [14].

The theory accordingly explains how and why is the weak charge connected with the handedness determined by the spin degrees of freedom in $d = (3+1)$. One Weyl (one family) representation of spinors of the group $SO(13,1)$ is presented in table 9.3. Each state is written as a product of nilpotents and projectors defined in the "technique" [4,16,18,17], short version of which can be found in appendix 9.9. Quantum numbers of each of the family members, all are presented in table 9.3 together with the quantum numbers, are defined in Eqs. (10.8, 10.9, 10.10).

The symmetry of both kinds of groups, $SO(13,1)$ and $\widetilde{SO}(13,1)$ (are assumed to) break simultaneously, influencing family members and families of spinors, as well as the gauge fields. After the break of symmetries from the manifold $M^{(13+1)}$ to $M^{(7+1)}\times M^{(6)}$, which makes all the families, except the $2^{\frac{7+1}{2}-1}$ ones determined by the group $\widetilde{SO}(7,1)$, massive [2], carries each family member the family

[1] In this sense the *spin-charge-family* is the Kaluza-Klein like theory [15].

[2] In this paper the break of symmetries in the way that only $2^{\frac{(7+1)}{2}-1}$ families stay massless, while all the others get high masses of the order above the unifying scale, is just assumed. This assumption, however, is supported by several works on the toy model with the same starting action (Eq. (10.1)) but with $d = (5+1)$, ref. [20,23], while the preliminary work on this more complex case is in progress.

quantum numbers, belonging in $\widetilde{SO}(7,1)$ to two times $\widetilde{SU}(2) \times \widetilde{SU}(2)$ groups, originating in $\widetilde{SO}(3,1)$ and in $\widetilde{SO}(4)$, respectively (where $\widetilde{SO}(n)$ represent the subgroups the generators of which are expressed by $\tilde{\gamma}^a$). The families correspondingly decouple to two times four families.

The generators of the corresponding subgroups of the family group $\widetilde{SO}(7,1)$, are defined in Eqs. (10.11, 10.12). To each family member of each family the antimember corresponds, accessible from the member by the discrete symmetry operator $\mathcal{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$, which does not depend on $\tilde{\gamma}^a$'s, as explained in the ref. [14,25].

Let us add that since all the charges, with the family charge included, emerge from the spins, correspondingly all the charges are quantized.

Quarks and leptons have the "spinor" quantum number (τ^4, originating in $SO(6)$, Eq. (10.10)) $\frac{1}{6}$ and $-\frac{1}{2}$, respectively [3], with the sum of both equal to $3 \times \frac{1}{6} + (-\frac{1}{2}) = 0$.

The *spin-charge-family* theory therefore predicts that there are two decoupled groups of four families: The fourth [1,7,6,9] to the already observed three families of quarks and leptons should (sooner or later) be measured at the LHC [11], while the lowest of the upper four families constitute the dark matter [10].

Let me *summarize* this first part of the introduction with the *statement*: The *spin-charge-family theory is offering the explanation for the assumptions of the standard model*, having correspondingly a chance to be the right step beyond the *standard model*.

This paper presents in section 10.2 that the properties of the scalar field, the weak and the hyper charge of the scalar Higgs, which are in the *standard model* just assumed to properly "dress" the right handed members by the weak and the hyper charge, *appear in the spin-charge-family* naturally, offering the explanation for the appearance of the scalar fields, observed so far as the scalar Higgs.

In the subsection of this introductory section the simple starting action of the *spin-charge-family* theory is presented, as well as all the assumptions made to achieve that the theory manifests at low energies the observed phenomena.

In section 10.3 the resume and conclusions are presented. In the first appendix 9.9 a short review of the technique, used in this paper to manifest properties of the spinor states, as well as the expressions for the two kinds of spin connection fields, in terms of vielbeins and the spinor sources, are added.

In section 10.3 the resume and conclusions are presented. In appendix a short review of the technique, used in this paper to manifest properties of the spinor states, as well as the expressions for the two kinds of spin connection fields, in terms of vielbeins and the spinor sources, are added.

10.1.1 The action of the *spin-charge-family* theory and the assumptions

Let me present the *assumptions* on which the theory is built, starting with the (simple) action in $d = (13+1)$:

[3] In the Pati-Salam model [21] this "spinor" quantum number is named $\frac{B-L}{2}$ quantum number and is twice the "spinor" quantum number, for quarks equal to $\frac{1}{3}$ and for leptons to -1.

i. In the simple action [7,1] fermions ψ carry in $d = (13+1)$ as the *internal degrees of freedom only two kinds of spins, no charges*, and *interact correspondingly with only the two kinds of the spin connection gauge fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and the vielbeins, $f^{\alpha}{}_{a}$.*

$$
\begin{aligned}
S\, &= \int \, d^dx \; E\; \mathcal{L}_f + \\
&\int \, d^dx \; E\; (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
\mathcal{L}_f &= \frac{1}{2}\, (\bar{\psi}\, \gamma^a p_{0a} \psi) + h.c., \\
p_{0a} &= f^{\alpha}{}_{a} p_{0\alpha} + \frac{1}{2E}\, \{p_{\alpha}, E f^{\alpha}{}_{a}\}_{-}, \\
p_{0\alpha} &= p_{\alpha} - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \\
R &= \frac{1}{2}\, \{f^{\alpha[a} f^{\beta b]}\; (\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\, \omega^{c}{}_{b\beta})\} + h.c.\,, \\
\tilde{R} &= \frac{1}{2}\, f^{\alpha[a} f^{\beta b]}\; (\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha} \tilde{\omega}^{c}{}_{b\beta}) + h.c.\,.
\end{aligned} \tag{10.1}
$$

Here [4] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. S^{ab} and $\tilde{S}^{ab}$ are generators of the groups $SO(13,1)$ and $\widetilde{SO}(13,1)$, respectively, expressible by γ^a and $\tilde{\gamma}^a$.

ii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (which manifests as $SU(3)\times U(1)$), affecting both internal degrees of freedom - the one represented by γ^a and the one represented by $\tilde{\gamma}^a$, leading to $2^{((7+1)/2-1)}$ massless families, all the rest families get heavy masses [5]. Both internal degrees of freedom, the ordinary $SO(13+1)$ one (where γ^a determine spins and charges of spinors) and the $\widetilde{SO}(13+1)$ (where $\tilde{\gamma}^a$ determine family quantum numbers), break simultaneously with the manifolds.

iii. There are additional breaks of symmetry: The manifold $M^{(7+1)}$ breaks further into $M^{(3+1)}\times M^{(4)}$.

iv. There is a scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale above the unification scale, to all the vector and scalar gauge fields, which interact with the condensate.

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[4] $f^{\alpha}{}_{a}$ are inverted vielbeins to $e^{a}{}_{\alpha}$ with the properties $e^{a}{}_{\alpha} f^{\alpha}{}_{b} = \delta^{a}{}_{b},\; e^{a}{}_{\alpha} f^{\beta}{}_{a} = \delta^{\beta}_{\alpha}$, $E = \det(e^{a}{}_{\alpha})$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[5] A toy model [23,24,14] was studied in $d = (5+1)$ with the same action as in Eq.'(10.1). For a particular choice of vielbeins and for a class of spin connection fields the manifold M^{5+1} breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families stay massless and mass protected. Equivalent assumption, although not yet proved that it really works, is made also in the case that $M^{(13+1)}$ breaks first into $M^{(7+1)}\times M^{(6)}$. The study is in progress.

v. There are nonzero vacuum expectation values of the scalar fields with the scalar indices $(7, 8)$, which cause the electroweak break and bring masses to the fermions and weak gauge bosons, conserving the electromagnetic and colour charge.

Comments on the assumptions:

i.: This starting action enables to represent the *standard model* as the effective low energy manifestation of the *spin-charge-family* theory, explaining all the *standard model* assumptions, with the families included. There are (before the electroweak break all massless) observable gauge fields: gravity, colour ($SU(3)$, from $SO(6)$) octet vector gauge fields, weak ($SU(2)_I$ from $SO(4)$) triplet vector gauge field and "hyper" ($U(1)$ from $SO(6)$) singlet vector gauge fields. All are superposition of $f^{\alpha}{}_{c}\, \omega_{ab\alpha}$. And there are (before the electroweak break all massless) observable (eight rather than observed three) families of quarks and leptons. (There are indeed two decoupled groups of four families, in the fundamental representations of twice $\widetilde{SU}(2)\times \widetilde{SU}(2)$ groups, the subgroups of $\widetilde{SO}(3,1)\times \widetilde{SO}(4)$. There are correspondingly the scalar fields with the weak and the hyper charge of the scalar Higgs and with either two kinds of the family quantum numbers in the adjoint representations - they are two times two triplets, emerging from the superposition of $f^{\sigma}{}_{s}\tilde{\omega}_{ab\sigma}$ with $s \in (7, 8)$, in accordance with twice $\widetilde{SU}(2)\times \widetilde{SU}(2)$ groups, the subgroups of $\widetilde{SO}(3,1)\times \widetilde{SO}(4)$ - or with the quantum numbers (Q, Q', Y') emerging from the superposition of $f^{\sigma}{}_{s}\tilde{\omega}_{ab\sigma}$. Both determine the Yukawa couplings.) The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge field and the scalar fields with the space index $s \in (5, 6)$ and $t \in (9, 10, 11, 12)$, as well as the auxiliary vector gauge fields expressible with vielbeins, which are the superposition of $f^{\mu}{}_{m}\tilde{\omega}_{ab\mu}$. They all remain either auxiliary or become massive after the appearance of the condensate.

ii., iii.: The assumed breaks explain why the weak and the hyper charge are connected with the handedness of spinors, manifesting the observed properties of the family members - the quarks and the leptons, left and right handed - and of the observed vector gauge fields. Since the left handed members are weak charged while the right handed are weak chargeless, the family members stay massless and mass protected up to the electroweak break. Antiparticles are accessible from particles by the $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$, as explained in refs. [14,25]. This discrete symmetry operator does not contain $\tilde{\gamma}^a$'s degrees of freedom. To each family member there corresponds the antimember, with the same family quantum number.

iv.: It is the condensate of two right handed neutrinos with the quantum numbers of the upper four families, which makes all the scalar gauge fields (with the index (5, 6, 7, 8), as well as those with the index $(9, \ldots, 14)$) and the vector gauge fields, manifesting nonzero τ^4, τ^{23}, Q ,Y, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{Q}$,$\tilde{Y}$,$\tilde{N}^3_R$ (Eqs. (10.8, 10.9, 10.10, 10.11, 10.12, 10.13)) massive [13].

v.: At the electroweak break the scalar fields with the space index $s = (7, 8)$, originating in $\tilde{\omega}_{abs}$, as well as some superposition of $\omega_{s's''s}$, those which conserve the electromagnetic charge, get nonzero vacuum expectation values, what changes also their masses. They determine mass matrices of twice the four families, as well as the masses of the weak bosons. All the rest scalar fields keep masses of the condensate scale and are correspondingly (so far) unobservable in the low energy regime [6]. The fourth family to the observed three ones will (sooner or later) be observed at the LHC. Its properties are under the consideration [11], while the stable of the upper four families is the candidate for the dark matter.

The above assumptions enable that the starting action (Eq. (10.1)) manifests effectively in $d = (3+1)$ in the low energy regime by the *standard model* required degrees of freedom of fermions and bosons [1,2,7,6,3–5,8–12], that is the quarks and the leptons, left and right handed, the families of quarks and leptons and all the known gauge fields, with (several, explaining the Yukawa couplings) scalar fields included.

To see this let us rewrite formally the action for the Weyl spinor of (Eq.(10.1)) as follows

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m(p_m - \sum_{A,i} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
&\quad \{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\
&\quad \{\sum_{t=5,6,9,\dots,14} \bar{\psi}\gamma^t p_{0t}\, \psi\}, \\
p_{0s} &= p_s - \frac{1}{2}S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}, \\
p_{0t} &= p_t - \frac{1}{2}S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt},
\end{aligned} \tag{10.2}
$$

where $m \in (0,1,2,3)$, $s \in 7,8$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within $(0,1,2,3)$ and $(5,6,7,8)$, $t \in (5,6,9,\dots,13,14)$.

The first line of Eq. (10.2) determines the kinematics and dynamics of spinor fields in $d = (3+1)$, coupled to the vector gauge fields. The generators τ^{Ai} of the charge groups are expressible in terms of S^{ab} through the complex coefficients $c^{Ai}{}_{ab}$, as presented in Eqs. (10.9, 10.10, 10.13)

$$
\tau^{Ai} = \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}, \tag{10.3}
$$

and the commutation relations

$$
\{\tau^{Ai}, \tau^{Bj}\}_- = i\delta^{AB} f^{Aijk} \tau^{Ak}. \tag{10.4}
$$

The corresponding vector gauge fields A^{Ai}_m are expressible with the spin connection fields ω_{stm}, with (s,t) either $\in (5,6,7,8)$ or $\in (9,10,\dots,13,14)$, in agreement with the assumptions **ii.** and **iii.**. Before the electroweak break the vector gauge fields appearing in the first line of Eq. (10.2) are all massless: $\vec{A}^3_m$ carries the colour charge $SU(3)$ (originating in $SO(6)$), $\vec{A}^1_m$ carries the weak charge $SU(2)_I$ ($SU(2)_I$ and $SU(2)_{II}$ are the subgroups of $SO(4)$) and $A^Y_m = \sin\vartheta_2 A^{23}_m + \cos\vartheta_2 A^4_m$ (Y is defined in Eq. (10.13), τ^4 in Eq. (10.10), the corresponding $U(1)$ group originates in $SO(6)$), A^4_m is defined in Eq. (10.15), if the scalar space index s is replaced by

[6] Correspondingly $d = (13+1)$ manifests in $d = (3+1)$ spins and charges as if one would start with $d = (9+1)$ instead of with $d = (13+1)$, since the plane $(5,6)$ and the plane in which the vector τ^4 lies, are unobservable at low energies.

the space vector index m, A^{23}_m is the third component of the second $SU(2)_{II}$ field $\vec{A}^2_m$. The corresponding charges $(\vec{\tau}^3, \vec{\tau^1}, Y)$ are the conserved charges.

Before the appearance of the condensate of the two right handed neutrinos with the quantum numbers of the upper four families (properties of the condensate are presented in table 10.1) at the scale far about the electroweak scale, all the three components of the field $\vec{A}^2_m$ are massless. The condensate gives the mass of the order of the scale of the appearance of the condensate to $A^{Y'}_m = \cos\vartheta_2 A^{23}_m - \sin\vartheta_2 A^4_m$, and to all the scalar gauge fields, presented in the second and the third line of Eq. (10.2), leading to $A^{Ai}_s, s \in (5, 6, \ldots, 13, 14)$ and $\tilde{A}^{Ai}_t$, $t \in (5, 6, \ldots, 13, 14)$.

Vector gauge fields $A^Y_m, \vec{A}^1_m$ and $\vec{A}^3_m$ do not couple to the condensate (table 10.1).

In Eqs. (10.15, 10.14) the expressions for the scalars with the scalar index $(7, 8)$ in terms of both kinds of the spin connection fields are presented. These scalar fields (the second line in Eq. (10.2)) determine after the electroweak break the mass matrices of the two decoupled groups of four families. Getting nonzero vacuum expectation values they cause the electroweak break, changing also their own masses. These scalar fields determine also the masses of the gauge bosons.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\vert\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_1 \vert\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_2)$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\vert\nu^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\vert e^{VIII}_{1R} >_1 \vert e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 10.1. The condensate of the two right handed neutrinos ν_R, with the $VIII^{th}$ family quantum numbers, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The right handed neutrino has $Q = 0 = Y$. The triplet carries $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in table 9.4, taken from the ref. [13].

Among the vector gauge fields $\vec{A}^3_m$ and $\vec{\tilde{A}}^3_m$ and the corresponding vielbeins only one of these three vector gauge fields is the propagating one, while the rest two are the auxiliary fields as one can learn from Eqs. (9.55, 9.56) of the second appendix section 9.10, if taking into account that there is no spinor (fermion) sources with the corresponding quantum numbers. Equivalently, also only one of the three vector gauge fields $\vec{A}^1_m$, $\vec{\tilde{A}}^1_m$ and the corresponding vielbein field is the propagating field, the other two are the auxiliary fields, as well as only one of the three vector gauge fields $\vec{A}^{N_L}_m$, $\vec{\tilde{A}}^{N_L}_m$ and the corresponding vielbein field is the propagating field, while $\vec{A}^{N_R}_m$ is massive due to the interaction with the condensate of the two right handed neutrinos through quantum numbers $\vec{\tilde{N}}_R$, presented in Eqs. (10.8, 10.11).

Let me *summarize* this subsection: The starting action (Eq.(10.1)) of the *spin-charge-family* theory manifests under the assumptions **i.-v.** in the low energy regime properties of the *standard model*, explaining the *standard model* assumptions: Before the electroweak break all the scalar gauge fields and the vector gauge fields -

except the colour, the weak and the hyper vector fields (and the gravity), which stay massless - are massive, due to the interaction with the scalar condensate of the two right handed neutrinos with the family quantum numbers of the upper four families. There are also the two decoupled massless groups of four families.

At the electroweak break the scalar gauge fields, carrying the scalar space index and keeping the electromagnetic charge conserved and changing their own masses, bring masses to all the fermions and all the gauge fields, except to the gavity, electromagnetic and the colour ones.

Let me comment that in the presence of the spinor fields (as it is the condensate, for example) all three gauge fields - the vielbeins and the two kinds of the spin connection fields - are in general the propagating fields. If there are no spinors present, only one of the three fields is the propagating field, the other two are expressible with the propagating one (as it is well known). In the second appendix 9.10 the expressions for the spin connection fields of both kinds in terms of the vielbeins and the spinor sources are presented, taken from the ref. [20].

The assumed breaks should occur spontaneously, determined by the starting action and the boundary conditions. To prove that this really can happen is a very difficult (many body) problem. Although several studies made so far, for either a toy model in $d = (5+1)$ or for the $d = (13+1)$ case, support these assumptions, yet several additional studies are needed to justify the assumptions and to clarify further the properties of the scalar and vector gauge fields and of the spinor families, appearing in the starting action. Also the comparison with all the other works made on the unifying theories are needed to see to which extend predictions of this theory coincide with the other theories in the literature, in which sense and what one can learn out of them.

The *standard model* **subgroups of the $SO(13+1)$ and of the $\widetilde{SO}(13+1)$ group and the corresponding gauge fields** To calculate quantum numbers of one Weyl representation presented in table 9.3 in terms of the generators of the *standard model* groups τ^{Ai} $(= \sum_{a,b} c^{Ai}{}_{ab}\; S^{ab})$ one must look for the coefficients $c^{Ai}{}_{ab}$ (Eq. (10.4)). The generators τ^{Ai} are the generators of the charge groups. Similarly one expresses also the spin and the family degrees of freedom.

The same coefficients $c^{Ai}{}_{ab}$ determine operators which apply on spinors and on vectors. The difference among the three kinds of operators - vectors and two kinds of spinors - lies in S^{ab}.

While S^{ab} for spins of spinors is equal to

$$S^{ab} = \frac{i}{4}(\gamma^a \gamma^b - \gamma^b \gamma^a)\,, \tag{10.5}$$

and $\tilde{S}^{ab}$ for families of spinors is equal to

$$\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)\,, \tag{10.6}$$

one must take, when $\mathcal{S}^{ab}$ apply on the spin connections ω_{bde} $(= f^{\alpha}{}_{e}\; \omega_{bd\alpha})$ and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ $(= f^{\alpha}{}_{e}\; \tilde{\omega}_{\tilde{b}\tilde{d}\alpha})$, on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, the operator

$$(\mathcal{S}^{ab})^{c}{}_{e} A^{d\dots e\dots g} = i(\eta^{ac}\delta^{b}_{e} - \eta^{bc}\delta^{a}_{e})\, A^{d\dots e\dots g}\,. \tag{10.7}$$

This means that the space index (e) of ω_{bde} transforms according to the requirement of Eq. (10.7), and so do b, d and $\tilde{b}$, $\tilde{d}$. I used the notation $\tilde{b}$, $\tilde{d}$ to point out that S^{ab} and $\tilde{S}^{ab}$ ($= \tilde{S}^{\tilde{a}\tilde{b}}$) are generators of two independent groups.

One finds [1,7,6,3–5,8,12] for the generators of the spin and the charge groups, which are the subgroups of $SO(13,1)$, the expressions:

$$\vec{N}_{\pm}(=\vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03})\,, \tag{10.8}$$

where the generators $\vec{N}_{\pm}$ determine representations of the two $SU(2)$ subgroups of $SO(3,1)$, generators $\vec{\tau}^{1}$ and $\vec{\tau}^{2}$,

$$\begin{aligned} \vec{\tau}^{1} &:= \frac{1}{2}(S^{58} - S^{67},\, S^{57} + S^{68},\, S^{56} - S^{78})\,, \\ \vec{\tau}^{2} &:= \frac{1}{2}(S^{58} + S^{67},\, S^{57} - S^{68},\, S^{56} + S^{78}), \end{aligned} \tag{10.9}$$

determine representations of the $SU(2)_{I}\times\ SU(2)_{II}$ invariant subgroups of the group $SO(4)$, which is further the subgroup of $SO(7,1)$ ($SO(4), SO(3,1)$ are subgroups of $SO(7,1)$), and the generators $\vec{\tau}^{3}$, τ^{4} and $\tilde{\tau}^{4}$

$$\begin{aligned} \vec{\tau}^{3} :=&\ \frac{1}{2}\{S^{9\,12} - S^{10\,11}\,, S^{9\,11} + S^{10\,12},\, S^{9\,10} - S^{11\,12}, \\ &S^{9\,14} - S^{10\,13},\, S^{9\,13} + S^{10\,14}\,, S^{11\,14} - S^{12\,13}\,, \\ &S^{11\,13} + S^{12\,14},\, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\ \tau^{4} :=&\ -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})\,, \\ \tilde{\tau}^{4} :=&\ -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14})\,, \end{aligned} \tag{10.10}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$, and of $\tilde{\tau}^{4}$ originating in $\widetilde{SO}(6)$.

One correspondingly finds the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03})\,, \tag{10.11}$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\begin{aligned} \vec{\tilde{\tau}}^{1} &:= \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67},\, \tilde{S}^{57} + \tilde{S}^{68},\, \tilde{S}^{56} - \tilde{S}^{78})\,, \\ \vec{\tilde{\tau}}^{2} &:= \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67},\, \tilde{S}^{57} - \tilde{S}^{68},\, \tilde{S}^{56} + \tilde{S}^{78})\,, \end{aligned} \tag{10.12}$$

determine representations of $\widetilde{SU}(2)_{I}\times\ \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both, $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$ are the subgroups of $\widetilde{SO}(7,1)$.

One further finds

$$Y = \tau^4 + \tau^{23}, \quad Y' = -\tau^4 \tan^2 \vartheta_2 + \tau^{23}, \quad Q = \tau^{13} + Y, \quad Q' = -Y \tan^2 \vartheta_1 + \tau^{13},$$
$$\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}, \quad \tilde{Y}' = -\tilde{\tau}^4 \tan^2 \tilde{\vartheta}_2 + \tilde{\tau}^{23}, \quad \tilde{Q} = \tilde{Y} + \tilde{\tau}^{13}, \quad \tilde{Q}' = -\tilde{Y} \tan^2 \tilde{\vartheta}_1 + \tilde{\tau}^{13}. \tag{10.13}$$

The scalar fields, responsible [1,2,7] - after getting in the electroweak break nonzero vacuum expectation values - for masses of the family members and of the weak bosons, are presented in the second line of Eq. (10.2). These scalar fields are included in the covariant derivatives as $-\frac{1}{2}\, S^{s's"} \omega_{s's"s} - \frac{1}{2} \tilde{S}^{\tilde{a}\tilde{b}} \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s \in (7,8)$, $(\tilde{a}, \tilde{b})$, $\in (\tilde{0}, \tilde{1}, \ldots, \tilde{8})$, where $\tilde{a}, \tilde{b}$ is again used to point out that (a, b) belong in this case to the "tilde" space.

One finds, by taking into account Eqs. (10.11, 10.12) and Eq. (10.13), for the choice of the $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ scalar gauge fields, contributing to the electroweak break, the expressions

$$\begin{aligned}
-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s} &= -(\vec{\tilde{\tau}}^{\tilde{1}}\, \vec{\tilde{A}}^{\tilde{1}}_{s} + \vec{\tilde{N}}_{\tilde{L}}\, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} + \vec{\tilde{\tau}}^{\tilde{2}}\, \vec{\tilde{A}}^{\tilde{2}}_{s} + \vec{\tilde{N}}_{\tilde{R}}\, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s}),\\
\vec{\tilde{A}}^{\tilde{1}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s},\, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s},\, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{1}s},\, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{2}s},\, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\, \tilde{\omega}_{\tilde{0}\tilde{3}s}),\\
\vec{\tilde{A}}^{\tilde{2}}_{s} &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s},\, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s},\, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{s} &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{1}s},\, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{2}s},\, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\, \tilde{\omega}_{\tilde{0}\tilde{3}s}),\\
&(s \in (7,8)).
\end{aligned} \tag{10.14}$$

Among ω_{abs}, which contribute to the mass matrices of quarks and leptons, one finds when using Eqs. (10.9, 10.10, 10.13), the expressions

$$\begin{aligned}
-\frac{1}{2}S^{s's"}\, \omega_{s's"s} &= -(g^{23}\, \tau^{23}\, A^{23}_{s} + g^{13}\, \tau^{13}\, A^{13}_{s} + g^{4}\, \tau^{4}\, A^{4}_{s}),\\
g^{13}\, \tau^{13}\, A^{13}_{s} + g^{23}\, \tau^{23}\, A^{23}_{s} + g^{4}\, \tau^{4}\, A^{4}_{s} &= g^{Q}\, Q A^{Q}_{s} + g^{Q'}\, Q' A^{Q'}_{s} + g^{Y'}\, Y'\, A^{Y'}_{s},\\
A^{4}_{s} &= -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s}),\\
A^{13}_{s} &= (\omega_{56s} - \omega_{78s}), \quad A^{23}_{s} = (\omega_{56s} + \omega_{78s}),\\
A^{Q}_{s} &= \sin\vartheta_1\, A^{13}_{s} + \cos\vartheta_1\, A^{Y}_{s},\\
A^{Q'}_{s} &= \cos\vartheta_1\, A^{13}_{s} - \sin\vartheta_1\, A^{Y}_{s},\\
A^{Y'}_{s} &= \cos\vartheta_2\, A^{23}_{s} - \sin\vartheta_2\, A^{4}_{s},\\
&(s \in (7,8)).
\end{aligned} \tag{10.15}$$

Scalar fields from Eq. (10.14) couple to the family quantum numbers, while those from Eq. (10.15) distinguish among family members. In Eq. (10.15) the coupling constants were explicitly written to see the analogy with the gauge fields in the *standard model*.

Expressions for the vector gauge fields in terms of the spin connection fields and the vielbeins, which correspond to the colour charge ($\vec{A}^{3}_{m}$), the $SU(2)_{II}$ charge ($\vec{A}^{2}_{m}$), the weak charge ($SU(2)_{I}$) ($\vec{A}^{1}_{m}$) and the $U(1)$ charge originating in $SO(6)$

($\vec{A}^4_m$), can be found by taking into account Eqs. (10.9, 10.10). Equivalently one finds the vector gauge fields in the "tilde" sector. One really can use just the expressions from Eqs. (10.15, 10.14), if replacing the scalar index s with the vector index m.

Let me *summarize* this subsubsection: The expressions for the operators τ^{Ai} are presented, either in terms of S^{ab} (Eq. (10.5)) or (in this case we name them $\tilde{\tau}^{Ai}$) in terms of $\tilde{S}^{ab}$ (Eq. (10.6)), valid also in terms of $\mathcal{S}^{ab}$ (Eq. (10.7)), affecting correspondingly spinors spin and charges quantum numbers, spinors families quantum numbers and scalar or vector gauge fields, respectively. Also the expressions for those scalar gauge fields, which contribute to the electroweak break by getting nonzero vacuum expectation values, in terms of the corresponding spin connection fields are presented (Eqs.(10.15, 10.14)). When the scalar index s is replaced by the vector index m, the expressions for the vector gauge fields in terms of spin connection fields follow.

10.2 Scalar fields contributing to the electroweak break are weak charge doublets

In this main part of the paper is demonstrated that all the scalar gauge fields with the scalar index $s \in (7,8)$, which get nonzero vacuum expectation values causing the electroweak break, carry the weak and the hyper charge as does the scalar Higgs of the *standard model*.

All the scalars (the gauge fields with the scalar index with respect to $d = (3+1)$) of the action (Eq. 10.1) contribute charges in the fundamental representations: The scalars with the space indices $s \in (7,8)$ and $s \in (5,6)$ are, with respect to this scalar space degree of freedom, before the appearance of the condensate (table 10.1), the weak ($SU(2)_I$) and the second $SU(2)_{II}$ *doublets*. After the appearance of the condensate only the weak and the hyper charge Y remain the conserved charges, so that it is the third component of τ^{23}, which determines the hyper charge ($Y = \tau^{23} + \tau^4$, Eq. (10.13)) of these scalar fields, since τ^4 applied on the scalar index of these scalar fields gives zero, according to Eqs. (10.9, 10.10, 10.7).

The scalars with the space indices $s \in (9, 10, \ldots, 13, 14)$ are, again with respect to this scalar space degree of freedom, colour triplets [13]. There are no additional scalar indices and therefore no additional corresponding scalars with respect to the scalar indices in this theory.

The scalars, however, carry additional quantum numbers Ai, the states of which belong to the adjoint representations with respect to either $\tilde{\tau}^{Ai}$ or τ^{Ai}. While, to reproduce the low energy phenomena, the scalar fields of all the family quantum numbers are allowed, only those τ^{Ai} are acceptable, which conserve after the electroweak break the electromagnetic charge. The scalar fields with nonzero vacuum expectation values carrying nonzero weak charge also due to $\vec{\tau}^1$ would cause nonconservation of the electromagnetic charge (see the assumption **v.** and the corresponding comments in subsection 10.1.1).

The colour triplet scalars contribute to transition from antileptons into quarks and antiquarks into quarks and back, unless the scalar condensate of the two right handed neutrions, presented in table 10.1, breaks matter-antimatter symmetry [13]. This condensate breaks also the $SU(2)_{II}$ symmetry, leaving massless (besides gravity) only the colour, weak and the hyper charge vector gauge fields. Also all the scalar fields get masses through the interaction with the condensate.

When at the electroweak break the scalar fields with the scalar indices $s \in (7, 8)$ originating either in $\tilde{\omega}_{abs}$ or in those superposition of $\omega_{s's''s}$ which conserve the electromagnetic charge (Eq. (10.16)) get nonzero vacuum expectation values, changing also their own masses, they bring masses to all the massless fermions (spinors), breaking their mass protection, and to weak bosons.

Let us recognize, by taking into account Eq. (9.44) and table 9.3, that $\gamma^0 \gamma^s$ appearing in $\{\sum_{s=7,8} \bar{\psi}\gamma^s p_{0s} \psi\}$ in the second line of Eq. (10.2), transform for either $s = 7$ or $s = 8$ the right handed u-quark (u_R^{c1}), weak chargeless, with the hyper charge $Y = \frac{2}{3}$ from the first line of table 9.3 to the left handed weak charged u-quark (u_L^{c1}) with the hyper charge $\frac{1}{6}$ from the seventh line of the same table, or that $\gamma^0 \gamma^s$ transform the right handed ν-lepton (ν_R), weak chargeless, with the hyper charge $Y = 0$ from the 25^{th} line of the same table 9.3 to the left handed weak charged ν-lepton (ν_L) with the hyper charge $-\frac{1}{2}$ from the 31^{st} line of the same table.

Now is the time to prove that the scalar fields with the scalar index $s \in (7, 8)$ from the second line of Eq. (10.2) and with quantum numbers of Eq. (10.16) really carry the weak and the hyper charge as required by the *standard model*. I introduce in Eq. (10.16) common notation A_s^{Ai} for all these scalar fields, independently of whether they origin in ω_{abs} - in this case they do not carry the additional weak or hyper charge due to $\vec{\tau}^A$ - or $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ fields.

$$
\begin{aligned}
A_s^{Ai} &\supset (\, A_s^{Q}\,, A_s^{Y}\,, A_s^{Y'}\,, \vec{\tilde{A}}_s^{\tilde{1}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{L}}}\,, \vec{\tilde{A}}_s^{\tilde{2}}\,, \vec{\tilde{A}}_s^{\tilde{N}_{\tilde{R}}}\,)\,,\\
\tau^{Ai} &\supset (Q,\, Y,\, Y' = -\tan^2\vartheta_2 \tau^4 + \tau^{23},\, \vec{\tilde{\tau}}^1,\, \vec{\tilde{N}}_L,\, \vec{\tilde{\tau}}^2,\, \vec{\tilde{N}}_R)\,. \qquad (10.16)
\end{aligned}
$$

These scalars, the gauge scalar fields of the generators τ^{Ai} and $\tilde{\tau}^{Ai}$ (Eqs. (10.11, 10.12, 10.9, 10.10)), are expressible in terms of the spin connection fields (Eqs. (10.14, 10.15)).

One expects that the solutions with nonzero momentum lead to higher masses of the fermion fields in $d = (3 + 1)$ [23,24]. We shall correspondingly pay no attention to the momentum p_s , $s \in (4, 8)$, when having in mind the lowest energy solutions, manifesting at low energies.

Scalars, which do not get nonzero vacuum expectation values, keep masses on the condensate scale.

Let me now, by taking into account Eqs. (10.7, 10.9), calculate properties of all scalar fields A_s^{Ai} of Eq. (10.13).

To do this let us first recognize

$$
\begin{aligned}
&\tau^{1\boxed{\pm}} = \frac{1}{2}[(S^{58} - S^{67}) \;\boxed{\mp}\; i\,(S^{57} + S^{68})]\,, \tau^{13} = \frac{1}{2}(S^{56} - S^{78})\,,\\
&Y = \tau^{23} + \tau^4\,, \quad Q = Y + \tau^{13}\,,
\end{aligned}
$$

and rewrite the scalar fields A_s^{Ai}, which determine masses of fermions and weak bosons in Eq. (10.2), appearing in the second line of Eq. (10.2), as follows (the momentum p_s is left out)

$$\sum_{s=(7,8),Ai} \bar{\psi}\gamma^s(-\tau^{Ai} A^{Ai}_s) =$$
$$-\psi^\dagger\gamma^0\{ \overset{78}{(+)}\ \tau^{Ai}(A^{Ai}_7 - i A^{Ai}_8) + \overset{78}{(-)}\ (\tau^{Ai}(A^{Ai}_s + i A^{Ai}_8)\psi\},$$
$$\overset{78}{(\pm)} = \frac{1}{2}(\gamma^7 \pm i\gamma^8), \tag{10.17}$$

with the summation over Ai performed, since A^{Ai}_s represent the scalar fields (A^Q_s, A^Y_s, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$ and $\vec{\tilde{A}}^{N_L}_s$).

Application of the operators Y and τ^{13} on the fields ($A^{Ai}_7 \mp i A^{Ai}_8$), leads after using Eq. (10.7) for $\mathcal{S}^{ab}$ and expressions for τ^{13} and Y (Eq. (10.17)), to

$$\tau^{13}(A^{Ai}_7 \mp i A^{Ai}_8) = \pm\frac{1}{2}(A^{Ai}_7 \mp i A^{Ai}_8),$$
$$Y(A^{Ai}_7 \mp i A^{Ai}_8) = \mp\frac{1}{2}(A^{Ai}_7 \mp i A^{Ai}_8),$$
$$Q(A^{Ai}_7 \mp i A^{Ai}_8) = 0. \tag{10.18}$$

Since Y and τ^{13} give zero, if applied on the upper indices (Q, Y, Y') of (A^Q_s, A^Y_s and $A^{Y'}_s$), as one can read from Eq. (10.15), and since Y and τ^{13} commute with the family quantum numbers, one sees that the scalar fields A^{Ai}_s (A^Q_s, A^Y_s, $A^{Y'}_s$, $\tilde{A}^{\tilde{4}}_s$, $\tilde{A}^{\tilde{Q}}_s$, $\vec{\tilde{A}}^{\tilde{1}}_s$, $\vec{\tilde{A}}^{\tilde{2}}_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$), rewritten as follows, $A^{Ai}_{\pm} = (A^{Ai}_7 \mp i A^{Ai}_8)$, are eigen states of τ^{13} and Y having the quantum numbers of the *standard model* Higgs' scalars.

Let us make the notation

$$A^{Ai}_{\underset{(\pm)}{78}} = (A^{Ai}_7 \mp i A^{Ai}_8), \tag{10.19}$$

and let us calculate what does the operator $\tau^{1\boxplus}$ (Eq. (10.17)) make if applied on $A^{Ai}_{\underset{(\pm)}{78}}$. Taking into account Eqs. (10.7, 10.9) one finds that

$$\tau^{1\boxplus} A^{Ai}_{\underset{(\pm)}{78}} = (A^{Ai}_5 \mp i A^{Ai}_6) =: A^{Ai}_{\underset{(\pm)}{56}},$$
$$\tau^{1\boxminus} A^{Ai}_{\underset{(\pm)}{78}} = 0. \tag{10.20}$$

The scalar fields $A^{Ai}_{\underset{(\pm)}{56}}$ are all massive fields with the masses of the condensate scale (table 10.1), while the scalar fields $A^{Ai}_{\underset{(\pm)}{78}}$ change masses at the electroweak break.

Using Eqs. (9.46, 9.44, 9.54) one finds that $\gamma^0\ \overset{78}{(-)} A^{Ai}_{\underset{(-)}{78}}$ transforms the right handed u^{c1}_R quark from the first line of table 9.3 into the leftt handed u^{c1}_L quark from the seventh line of the same table, which can be also interpreted in the *standard model* way, namely, that $A^{Ai}_{\underset{(-)}{78}}$ "dress" u^{c1}_R giving it the weak and the hyper charge of the left handed u^{c1}_L quark, while γ^0 changes handedness. Equivalently happens to ν_R from the 25^{th} line, which the action of $\gamma^0 \stackrel{78}{(-)} A^{Ai}_{\stackrel{78}{(-)}}$ on it transforms into the ν_L from the 31^{th} line, which again can be interpreted in the *standard model* way: With the action of γ^0 and the "dressing" of $A^{Ai}_{\stackrel{78}{(-)}}$ on ν_R, transforming it into ν_L.

The action of $\gamma^0 \stackrel{78}{(+)} A^{Ai}_{\stackrel{78}{(+)}}$ transform d^{c1}_R from the third line of the same table into d^{c1}_L from the fifth line of this table, or e_R from the 27^{th} line into the e_L from the 29^{th} line. One can use in this two cases, knowing the properties of the scalar fields (Eq. (10.18)), again the *standard model* interpretation, in which the scalar fields $A^{Ai}_{\stackrel{78}{(+)}}$ take care of the weak and the hyper charges of the right handed members d^{c1}_R and e_R by "dressing" them with the appropriate weak and the hyper charges, while γ^0 changes handedness. In the *standard model* there is the scalar Higgs and the Yukawa couplings, which take care of fermion and also of the weak boson properties.

In the *spin-charge-family* theory there are several scalar fields, which determine the mass matrices of the two groups of four families.

When the scalar fields ($A^{Q}_{\stackrel{78}{(\pm)}}, A^{Y}_{\stackrel{78}{(\pm)}}, A^{Y'}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{1}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{2}}_{\stackrel{78}{(\pm)}}, \vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_{\stackrel{78}{(pm)}}$) from Eq. (10.16) get nonzero vacuum expectation values, they determine mass matrices of family members - of quarks and leptons - of the lower (carrying the family quantum numbers ($\vec{\tilde{\tau}}^1, \vec{\tilde{N}}_L$)) and the upper (carrying the family quantum numbers ($\vec{\tilde{\tau}}^2, \vec{\tilde{N}}_R$)) four families, since they carry the weak and the hyper charge (Eqs. (10.9, 10.10)) which breaks the mass protection mechanism of quarks and leptons.

We clearly see that all the scalars $A^{Ai}_{\stackrel{78}{(\pm)}}$ have the following properties:

$$(\tau^{13}, Y)\, A^{Ai}_{\stackrel{78}{(\pm)}} = \pm\,(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\stackrel{78}{(\pm)}}\,. \tag{10.21}$$

The scalars $A^{Ai}_{\stackrel{78}{(-)}}$ obviously bring the right quantum numbers to the right handed partners (u_R, ν_R), and the scalars $A^{Ai}_{\stackrel{78}{(+)}}$ give the right quantum numbers to (d_R, e_R).

The scalar fields $A^{Ai}_{\stackrel{78}{(\pm)}}$ are in the *spin-charge-family* theory *triplets with respect to the family quantum numbers* ($\vec{\tilde{N}}_R$, $\vec{\tilde{N}}_L$, $\vec{\tilde{\tau}}^2$, $\vec{\tilde{\tau}}^1$; Eqs. (10.11, 10.12)) or singlets as the gauge fields of $Q = \tau^{13} + Y$, $Y = \tau^{23} + \tau^4$ and $Y' = -\tan^2\vartheta_2 \tau^4 + \tau^{23}$.

One can prove this by applying $\tilde{\tau}^{23}$, $\tilde{\tau}^{13}$, $\tilde{N}^3_R$ and $\tilde{N}^3_L$ on their eigen states. Let us do this for $\tilde{A}^{N_L i}_{\stackrel{78}{(\pm)}}$ and for $A^{Q}_{\stackrel{78}{(\pm)}}$, taking into account Eqs. (10.11), and recognizing that $\tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}} = \tilde{A}^{N_L 1}_{\stackrel{78}{(\pm)}} \boxplus i\, \tilde{A}^{N_L 2}_{\stackrel{78}{(\pm)}}$ (Eq. (10.7)).

$$\begin{aligned}
&\tilde{N}^3_L\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \boxplus\, \tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}}\,, \quad \tilde{N}^3_L\, \tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = 0\,,\\
&\tilde{A}^{\tilde{N}_L \boxplus}_{\stackrel{78}{(\pm)}} = \{(\tilde{\omega}_{\tilde{2}\tilde{3}\stackrel{78}{(\pm)}} + i\,\tilde{\omega}_{\tilde{0}\tilde{1}\stackrel{78}{(\pm)}}) \boxplus i\,(\tilde{\omega}_{\tilde{3}\tilde{1}\stackrel{78}{(\pm)}} + i\,\tilde{\omega}_{\tilde{0}\tilde{2}\stackrel{78}{(\pm)}})\}\,,\\
&\tilde{A}^{\tilde{N}_L 3}_{\stackrel{78}{(\pm)}} = (\tilde{\omega}_{\tilde{1}\tilde{2}\stackrel{78}{(\pm)}} + i\,\tilde{\omega}_{\tilde{0}\tilde{3}\stackrel{78}{(\pm)}})\\
&Q\, A^{Q}_{\stackrel{78}{(\pm)}} = 0\,, A^{Q}_{\stackrel{78}{(\pm)}} = \omega_{56\stackrel{78}{(\pm)}} - (\omega_{9\,10\stackrel{78}{(\pm)}} + \omega_{11\,12\stackrel{78}{(\pm)}} + \omega_{13\,14\stackrel{78}{(\pm)}})\,,
\end{aligned} \tag{10.22}$$

with $Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, and with τ^4 defined in Eq. (10.10)).

To masses of the lower four families only the scalar fields, which are the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute. (To masses of the upper four families only the gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute.) The three scalar fields $A^{Q}_{\stackrel{78}{(\pm)}}$, $A^{Y}_{\stackrel{78}{(\pm)}}$ and $A^{4}_{\stackrel{78}{(\pm)}}$ "see" the family members quantum numbers and contribute correspondingly to all the families.

The scalar fields, with the weak and the hyper charge in the fundamental representations (Eq. (10.21)) and the family charges in the adjoint representations, transform any family member of the lower four families into the same family member belonging to one of the lower four families (while those with the family charges of the upper four families transform any family member into the same family member belonging to one of the upper four families).

In loop corrections all the scalar and vector gauge fields which couple to fermions contribute.

The mass matrix of any family member, belonging to any of the two groups of the four families, manifests - due to the $\widetilde{SU}(2)_{(L,R)} \times \widetilde{SU}(2)_{(I,II)}$ (either (L, I) or (R, II)) structure of the scalar fields, which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$ - the symmetry presented in Eq. (10.23) [7].

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha}\,. \tag{10.23}$$

Let us *summarize* this section: It is proven that all the scalar fields with the scalar index $s \in (7, 8)$, which gain nonzero vacuum expectation values and keep the electromagnetic charge conserved, carry the weak and the hyper charge quantum numbers as required by the *standard model* for the scalar Higgs (Eq. (10.21)):

[7] Since the upper four families interact with the condensate of the two right handed neutrinos, which carry the family quantum numbers of the upper four families, the symmetry of the mass matrix presented in Eq. (10.23) is the symmetry of the upper four families.

$(\tau^{13}, Y)\, A^{Ai}_{\stackrel{78}{(\pm)}} = \pm(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\stackrel{78}{(\pm)}}$. These are the only scalar fields in this theory with the quantum numbers of Higgs' field.

These scalar fields carry additional quantum numbers: The family quantum numbers. The nonzero vacuum expectation values of the scalars with the space index $s = \in (7, 8)$ determine on the tree level the mass matrices of the two groups of four families. While the scalars with the family quantum numbers $(\vec{\tilde{1}}, \vec{\tilde{N}}_L)$ contribute to mass matrices of the lower four families, contribute those with the family quantum numbers $(\vec{\tilde{2}}, \vec{\tilde{N}}_R)$ to masses of the upper four families and those with the family members quantum numbers (Q, Y, Y') to any of these two groups of four families. In loop corrections in all orders the mass matrices of the two groups of four families follow.

All the other scalar fields: A^{Ai}_s, $s \in (5, 6)$ and $A^{Ai}_{tt'}$, $(t, t') \in (9, \ldots, 14)$ have masses of the order of the condensate scale and contribute to matter-antimatter asymmetry.

10.3 Conclusions

The *spin-charge-family* [1,2,7,6,3–5,8,12,9,10] theory, a kind of the Kaluza-Klein theories [15] with the families introduced by the second kind of gamma operators ($\tilde{\gamma}^a$ in addition to the Dirac γ^a), is offering the explanation for the properties of quarks and leptons (right handed neutrinos are in this theory regular members of each family) and antiquarks and antileptons, for the appearance of the gauge vector fields and of the scalar Higgs and Yukawa couplings. All these are in the *standard model* just assumed.

The theory offers the explanation why are the weak and hyper charges of fermions connected with their handedness (table 9.3) and where do the scalar fields originate (Eqs. (10.14, 10.15)).

It also explains why do the scalar fields carry the weak and the hyper charges as assumed by the *standard model* (Eq. (10.18)): $(\tau^{13}, Y)\, A^{Ai}_{\stackrel{78}{(\pm)}} = \pm(\frac{1}{2}, -\frac{1}{2})\, A^{Ai}_{\stackrel{78}{(\pm)}}$, where τ^{13} denotes the third component of the weak charge, Y the hyper charge, Ai denotes (Q, Y, Y') (originating in the first kind γ^a of the Clifford algebra objects) and all the family quantum numbers (originating in the second kind of the Clifford algebra objects $\tilde{\gamma}^a$). While γ^a, through S^{ab}, determine all the spin and the charges of families, determine $\tilde{\gamma}^a$, through $\tilde{S}^{ab}$, the family quantum numbers.

The *spin-charge-family* therefore, starting with the simple action (Eq.(10.1)) in $d = (13 + 1)$ for spinors (carrying two kinds of gamma operators) and interacting with the gravity only (with the vielbeins and the two kinds of the spin connection fields), differs essentially from the unifying theories of Pati and Salam [21], Georgi and Glashow [27] and other $SO(10)$ and $SU(n)$ theories [28], although all these unifying theories are answering some of the open questions of the *standard model* and accordingly have many things in common - among themselves and with the *spin-charge-family* theory.

The *spin-charge-family* theory predicts two decoupled groups of four families [7,6,9,10]: The fourth of the lower group of families will be measured at the LHC [11] and the lowest of the upper four families constitute the dark matter [10].

It also predicts that there will be several scalar fields observed sooner or later at the LHC.

Besides the scalar fields with the space index $s \in (7,8)$, which by getting non zero vacuum expectation values cause the electroweak break and take care of massless of fermions and the weak bosons, all the other scalar fields get, through the interaction with the scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, masses of the condensate scale. There are also only weak, hyper and the colour vector gauge bosons which stay massless up to the condensate scale, since they do not interact with the condensate. The scalar fields with the scalar space index $s =\in (9,\ldots,14)$ are colour triplets with respect to the scalar space index and cause, after interacting with the condensate, matter-antimatter asymmetry [13].

All the scalar fields are in the fundamental representations (Eq. ([13])) with respect to the space index. They resemble the supersymmetry particles, although they are not, since they do not meet all the requirements for the bosonic partners of fermions.

Starting with few assumptions, presented in the introduction 10.1 (**i.- iv.**), I show that the *spin-charge-family* theory is not only offering the explanation for the so far measured phenomena, with the origin of the dark matter and the scalar fields included, but offers also the predictions for new families (the fourth to the observed three families will be measured at the LHC, the fifth - the lowest of the upper four families - forming baryons [10] explains the appearance of the dark matter) and new scalar fields (there are two triplets and three singlets: A^{Q}_{s}, A^{Y}_{s}, $A^{Y'}_{s}$, $\vec{\tilde{A}}^{\tilde{1}}_{s}$, $\vec{\tilde{A}}^{\tilde{N}_L}_{s}$, Eqs. (10.14, 10.15, 10.16), which determine properties of the four lower families - the Higgs and the Yukawa couplings of the *standard model* [2,1]). The theory might be able also to answer questions about the (ordinary, mainly made out of the first family) matter/antimatter asymmetry, which is discussed in a separate paper [13]. The quantum numbers of the condensate, responsible for breaking $\mathcal{CP}$ symmetry, are presented in this paper (table 10.1). The same condensate makes massive scalar and vector gauge fields which would otherwise be as massless observed at low energies.

Although the *spin-charge-family* theory starts in $d = (13+1)$ dimensional space with the spin connection fields of two kinds (having the origin in γ^a and in $\tilde{\gamma}^a$) and with the vielbeins - all these look like having a very large number of degrees of freedom - it leads under the assumption that there is a condensate of two right handed neutrinos carrying the quantum numbers of the upper four families and that there are scalar fields, which obtain nonzero vacuum expectation values causing the electroweak break, naturally (what means that all unobserved fields of both origins get masses without additional requirements) at the low energy regime to the observed fermion and vector gauge boson fields.

This paper presents, by explaining that in this theory there are the scalar fields, which carry the quantum numbers of the scalar Higgs scalars and correspondingly offering the explanation for the appearance of the scalar Higgs and the Yukawa couplings, a further step towards understanding the properties of quarks and leptons and in particular of those scalar fields (section 10.2), which determine mass matrices of quarks and leptons.

It stays to be solved, why and how does the condensate of the two right handed neutrinos with the family quantum numbers of the upper four families appear and why do scalars, with the weak and the hyper charge required by the *standard model*, gain non zero vacuum expectation values.

Acknowledgments

Author acknowledges funding of the Slovenian Research Agency.

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