DESIGNING INDUCTORS FOR A FREQUENCY DEPENDENT Q-FACTOR

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Key words: plating, higher harmonics, matching network, nickel plating, nonlinear load, oscillations, plated coil, Q-factor, resonance, RF applications, RF plasma, signal distortion

Abstract: Power dissipation in inductors for radio frequency (RF) applications is a function of frequency because of the skin effect. If a coil is plated with a ferromagnetic material, the dissipation as a function of frequency can be tailored to yield a Q-factor that decreases with frequency much faster than that of an unplated coil. This effect is useful to prevent oscillations at higher harmonics of a frequency of interest. One such application is RF power systems for plasma processing, in which RF plasma presents a time-variable, nonlinear load where nonlinearities introduce harmonic frequencies.

Zasnova tuljave z izrazito frekvenčno odvisnostjo upornosti

Kjučne besede: harmonske frekvence, impedančna prilagoditev, kvaliteta nihajnega kroga, kvaliteta tuljave, načrtovanje RF vezij, nelinearno breme, nelinearno ojačenje, nelinearna vezja, niklanje, oscilacije, resonanca, RF plazma

Izvleček: Upornost tuljave, in z njo povezana intenzivnost prehajanja elektromagnetne v toplotno energijo, je funkcija frekvence zaradi kožnega pojava. Če tuljavo galvaniziramo s feromagnetnim nanosom ustrezne debeline, močno povečamo odvisnost upornosti tuljave od frekvence toka, oziroma z ustreznim feromagnetnim nanosom povzročimo izrazito frekvenčno odvisnost kvalitete tuljave. Pojav je uporaben za preprečevanje oscilacij pri višjih harmonskih frekvencah osnovnega signala, kar predstavlja problem pri, na primer, dovajanju RF energije v plazemske procese.

1. Introduction

Power dissipation in high frequency inductors takes place mostly within the skin depth δ, which is a function of frequency. Magnitudes of the electric field $\vec{E}$, magnetic field $\vec{B}$, and current $\vec{j}$ decrease exponentially with the penetration into the conductor. Skin depth is defined as the depth at which fields and current decrease to *1/e* of their values at the surface. For example, skin depths for copper at frequencies of 1, 10, and 100 MHz are 0.06, 0.02, and 0.003 mm /9/. This is the primary reason to build inductors for RF applications mostly from tubes instead of from solid conductors. Benefits of tube usage are savings in weight, material, and possibility of intra-coil cooling with air circulation. High frequency and high power inductors are used in RF power equipment such as generators, impedance matching networks /1, 3/, and power delivery coils, i.e., antennas /10/. Low power RF inductors are usually made from solid material, i.e., from solid wire since low power implies small dimensions where usage of tubes in inductor manufacturing becomes unrealistic and nonprofitable.

Inductors are classified by inductance and by quality, that is, by Q-factor. This is a measure of a relationship between stored energy and rate of energy dissipation in a component, thus indicating the components' efficiency.

In circuits, the Q-factor is a measure of the quality of a resonance. A high Q-factor is important for the efficiency of RF transmitters and for the sensitivity of RF receivers. A low Q-factor at the higher harmonics of the operating frequency is important to prevent propagation and amplification of harmonics, and, in the worst case, to prevent resonance of a resonant circuit at harmonic frequencies. Such unwanted resonance does happen in plasma processes such as RF plasma enhanced sputtering, etching, coating, and cleaning, where RF induced plasma behaves as a time-variable nonlinear load.

Unwanted resonance at higher harmonics is detrimental to the operation of circuits by causing sustained and transient oscillations that cause noise, signal distortion and the possibility of damage to circuit elements. In plasma processing, such resonance affects quality of the process.

The Q-factor is defined as the ratio between the total energy of a system with periodic action and the energy lost in one period /6/. When the resonance is reasonably narrow (Q ≤ about 3), the Q-factor can be approximated by the resonant frequency divided by the bandwidth:

$$Q = \omega_0 / \Delta\omega . \qquad (1)$$

Bandwidth is defined as the difference between frequencies where circuit response corresponds to 0.7 (or, -3 *dB*) of a maximum response. The Q-factor for a simple resonant circuit (an ideal capacitor coupled to a non-ideal inductor) equals

$$Q = \frac{2\pi E}{|\Delta E|} = \frac{\omega_0 L}{R} = 1/R_L \sqrt{L/C}. \qquad (2)$$

The present paper shows that by plating power inductors one can make a coil's resistance a controlled function of frequency within a certain range. An obvious application is the reduction of the coil's Q-factor at higher harmonics of the operating frequency. We derive here the general design formulae for such an application.

The independent variables are the frequency (f), and variables in Figure 1: the tube's inner radius (a), the wall thickness (w), the plating thickness (p), the tube's length (dL), the conductances (σ_w, σ_p), the permeabilities (μ_w, μ_p) and the skin depths (δ_w, δ_p) of the wall and plating metals. The power dissipation in such a coil is

$$P = RI^2, \qquad (3)$$

where I is the RMS current and R is the effective resistance at the given frequency.

It is usually convenient to express a physical property that is a function of geometry and materials (the resistance R in this case) as the product of a reference property i.e., R_0 and of a dimensionless factor referred to as a "form factor" i.e., F, which characterizes the geometry and materials. Taking as a reference R_0, the resistance of the unplated tube, we may thus write

$$R = R_0 F. \qquad (4)$$

Our objectives are to determine the reference resistance R_0 and the form factor F as functions of all listed variables.

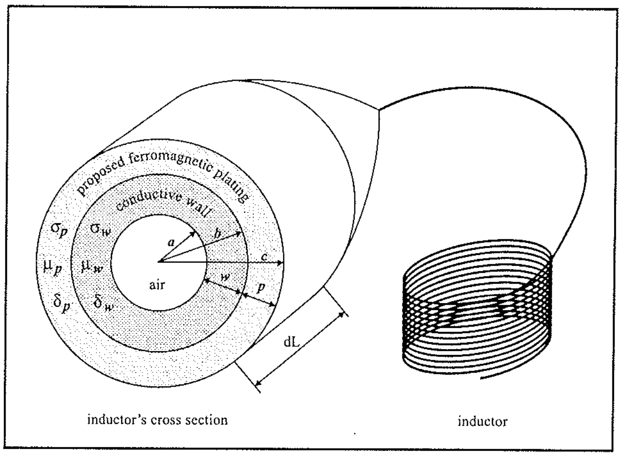

Figure 1 Proposed structure of an inductor for RF power applications.

2. Derivation of the reference resistance R_0, and of the form factor F

The total effective resistance is computed from the total power dissipation as

$$R = P/I^2. \qquad (5)$$

The power dissipation is obtained by integration over the cross section. To this end, we need the current densities

$$\left.\begin{aligned} \vec{j}_p(r) &= \sigma_p \vec{E}_p(r) \text{ for } b \le r \le c \\ \vec{j}_w(r) &= \sigma_w \vec{E}_w(r) \text{ for } a \le r \le b \end{aligned}\right\}, \qquad (6)$$

where r denotes the radial distance from the centerline of the tubing, b the outer radius of the tube ($b = a + w$) and c that of the plating, ($c = b + p$). The current densities are determined by longitudinal electric fields, which are, respectively

$$\left.\begin{aligned} E_p(r) &= E_0 \exp\left(-(c-r)/\delta_p\right) \\ E_w(r) &= E_0 \exp\left(-p/\delta_p\right)\exp\left(-(b-r)/\delta_w\right) \end{aligned}\right\}, \qquad (7)$$

where E_0 is the field at the outer surface and the skin depths δ_p and δ_w are given by the general expression

$$\delta = \sqrt{\frac{1}{\sigma\mu\pi f}}. \qquad (8)$$

The total current,

$$I = I_p + I_w, \qquad (9)$$

is proportional to the field E_0,

$$\left.\begin{aligned} I_p &= K_p E_0 \\ I_w &= K_w E_0 \end{aligned}\right\} \qquad (10)$$

where the coefficients K_p and K_w characterize the tubing. They depend only on the two metals and on the geometry:

$$\begin{aligned} K_p &= 2\pi\sigma_p \exp\left(-c/\delta_p\right)\int_b^c \exp\left(r/\delta_p\right) r\,dr \\ &= 2\pi\sigma_p\delta_p\left[c - \delta_p - \left(b - \delta_p\right)\exp\left(-p/\delta_p\right)\right], \end{aligned} \qquad (11)$$

$$\begin{aligned} K_w &= 2\pi\sigma_w \exp\left(-p/\delta_p\right)\exp\left(-b/\delta_w\right)\int_a^b \exp\left(r/\delta_w\right) r\,dr \\ &= 2\pi\sigma_w\delta_w \exp\left(-p/\delta_p\right)\left[\left(b-\delta_w\right) - \left(a-\delta_w\right)\exp\left(-w/\delta_w\right)\right]. \end{aligned} \qquad (12)$$

The field variable E_0 may now be expressed as a function of the total current I,

$$E_0 = \frac{I}{K_p + K_w}. \qquad (13)$$

The power dissipated in a length dL of plated tubing is

$$P = P_w + P_p = \int_b^c dP_p + \int_a^b dP_w, \qquad (14)$$

where

$$dP_p = \frac{\rho_p dL}{2\pi r dr}\left[2\pi r dr j_p\right]^2 = 2\pi\rho_p dL j_p^2 r dr, \qquad (15)$$

$$dP_w = \frac{\rho_w dL}{2\pi r dr}\left[2\pi r dr j_w\right]^2 = 2\pi\rho_w dL j_w^2 r dr. \qquad (16)$$

Integration yields

$$\begin{aligned} P_p &= 2\pi\rho_p dL\int_b^{b+p} j_p^2 r dr \\ &= 2\pi\rho_p\sigma_p^2 E_0^2 dL \exp\left(-2c/\delta_p\right)\int_b^c \exp\left(2r/\delta_p\right) r dr \\ &= I^2\frac{\pi\sigma_p\delta_p dL}{\left(K_p+K_w\right)^2}\left(\left(c-\frac{1}{2}\delta_p\right)-\left(b-\frac{\delta_p}{2}\right)\exp\left(-2p/\delta_p\right)\right) \end{aligned} \qquad (17)$$

$$\begin{aligned} P_w &= 2\pi\rho_w dL\int_a^b j_w^2 r dr \\ &= I^2\frac{2\pi\rho_w\sigma_w^2 dL}{\left(K_p+K_w\right)^2}\exp\left(-2p/\delta_p\right)\exp\left(-2b/\delta_w\right)\int_a^b \exp\left(2r/\delta_w\right) r dr \\ &= I^2\frac{2\pi\rho_w\sigma_w^2 dL}{\left(K_p+K_w\right)^2}\exp\left(-2p/\delta_p\right)\left(\left(b-\frac{\delta_w}{2}\right)-\left(a-\frac{\delta_w}{2}\right)\exp\left(-2w/\delta_w\right)\right) \end{aligned} \qquad (18)$$

By relation (5), the effective resistance is

$$\begin{aligned} R &= \frac{\pi\sigma_p\delta_p dL}{\left(K_p+K_w\right)^2}\left(b+p-\frac{\delta_p}{2}-\left(b-\frac{\delta_p}{2}\right)\exp\left(-2p/\delta_p\right)\right) \\ &+ \frac{\pi\sigma_w\delta_w dL}{\left(K_p+K_w\right)^2}\exp\left(\frac{-2p}{\delta_p}\right)\left(b-\frac{\delta_w}{2}-\left(a-\frac{\delta_w}{2}\right)\exp\left(\frac{-2w}{\delta_w}\right)\right), \end{aligned} \qquad (19)$$

where

$$K_p = 2\pi\sigma_p\delta_p\left(b+p-\delta_p-\left(b-\delta_p\right)\exp\left(\frac{-p}{\delta_p}\right)\right) \qquad (20)$$

$$K_w = 2\pi\sigma_w\delta_w\exp\left(\frac{-p}{\delta_w}\right)\left(b-\delta_w-(a-\delta_w)\exp\left(\frac{-w}{\delta_w}\right)\right). \qquad (21)$$

The reference resistance corresponds to the special case of no plating, i.e., $p = 0$,

$$R_0 = \frac{\pi\sigma_w\delta_w dL}{K_{w0}^2}\left(b-\frac{\delta_w}{2}-\left(a-\frac{\delta_w}{2}\right)\exp\left(\frac{-2w}{\delta_w}\right)\right), \qquad (22)$$

where

$$K_{w0} = 2\pi\sigma_w\delta_w\left(b-\delta_w-(a-\delta_w)\exp\left(\frac{-w}{\delta_w}\right)\right). \qquad (23)$$

The reference resistance of a tube is a function of eight variables (three geometric variables, four material constants and frequency):

$$R_0 = f\left(dL, a, w; \sigma_w, \sigma_p, \delta_w, \delta_p; f\right),$$

likewise with the form factor, with the modification that the form factor does not depend on the length dL, but rather on the plating thickness p,

$$F = F\left(a, w, p; \sigma_w, \sigma_p, \delta_w, \delta_p, f\right).$$

3. Impact of plating thickness and frequency on the form factor F

Explicitly writing the full functions for R_0 and F is impractical on the printed page, but also unnecessary for practical calculations by computer. To get insight into the form factor, we consider two examples: a copper tube (a = 5 mm, w = 1 mm) plated with nickel, (a ferromagnetic material - assumed linear), and a nickel tube of the same cross-section plated with copper. The material properties are /9/:

$$\begin{aligned} \sigma_{cu} &= 5.8\times10^7 \text{ S/m}, \\ \sigma_{ni} &= 1.1\times10^7 \text{ S/m}, \\ \mu_{cu} &= 4\pi\times10^{-7} \text{ H/m}, \\ \mu_{ni} &= 250\times4\pi\times10^{-7} \text{ H/m}, \\ \delta_{cu} &= \frac{0.0670}{\sqrt{f}} \text{ mm}, \\ \delta_{ni} &= \frac{0.0088}{\sqrt{f}} \text{ mm}. \end{aligned}$$

For the purpose of this calculation, we designed and coded a simulation tool in C++ Builder. Other tools, i.e., simulation packages for mathematics could alternatively have been utilized for the simulation.

Displaying the form factor as a function of plating thickness (in the range of 0 to 0.07 mm) and frequency (in the range of 0 to 50 MHz), we obtain the graph in Figure 2.

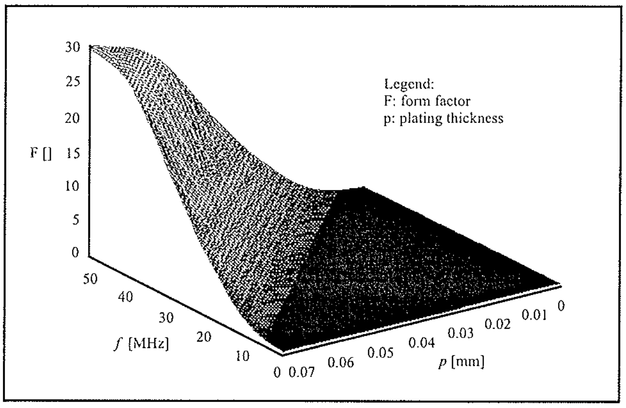

Figure 2 Form factor F for nickel plating on copper tubing.

For p = 0 μm, we have F = 1 at all frequencies, which is exactly true by definition, since, in this case, $R = R_0$. We also have F = 1 for direct current, since the thin plating of nickel affects the DC resistance of the copper tube negligibly. The greatest effect is F = 30, achieved at 50 MHz and 70 μm of plating.

Interchanging the metals and leaving the geometry unchanged results in the graph in Figure 3.

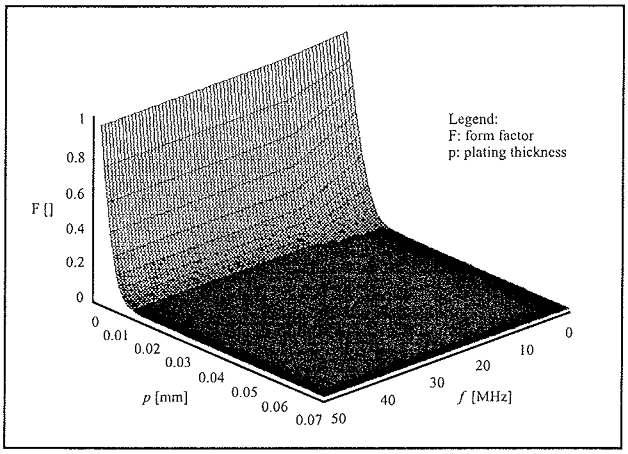

Figure 3 Form factor F for copper plating on nickel tubing.

DC resistance gradually decreases as the plating thickness of the better conductor increases. The same is true at all frequencies, but the effect is more pronounced at higher frequencies. This is because less current flows in the more resistive nickel tubing as the skin depth limits it to the copper plating.

4. Form factor F and the quality of resonance

A low form factor *F*, i.e., low resistance of an inductor is preferred to minimize losses and to reach high quality for resonant circuits at a resonant frequency. A high form factor, i.e., high resistance of an inductor is preferred to obtain high losses at frequencies where resonance should not take place. The authors predict that the frequency-dependent form factor of an inductor will be recognized as important when:

- a power inductor delivers power to a nonlinear load, such as RF plasma where nonlinearities /8/ introduce harmonic frequencies,
- the load capacitance varies, as it is the case with RF plasma at the onset of plasma excitation /2, 4, 5/, and when
- the RF signal on the output of RF power stages, at the input of the load impedance matching networks, is less than a perfect sine (power amplifiers do trade fidelity for efficiency, i.e., efficient class AB, B, C, and E /7/ RF amplifiers introduce higher harmonics).

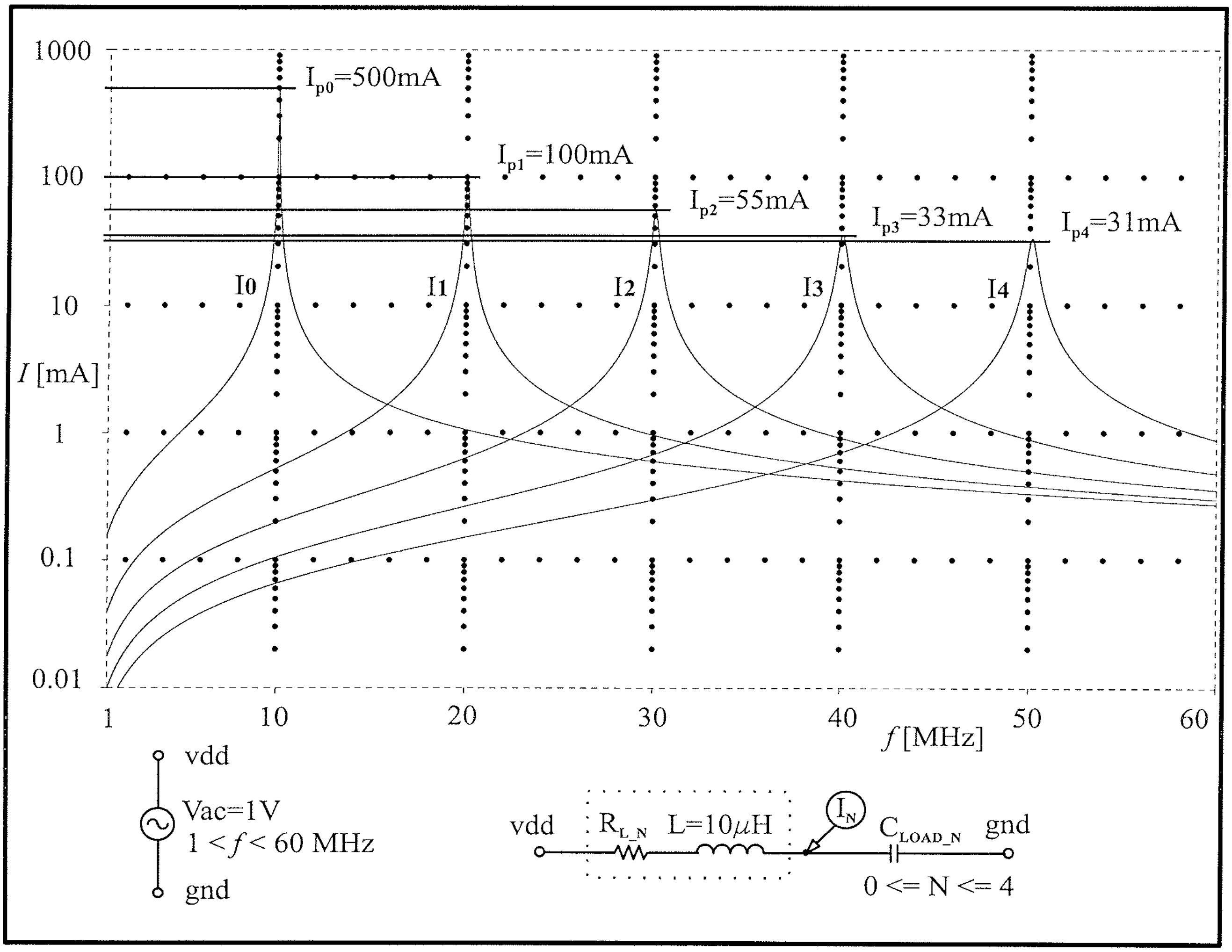

Figure 4 Frequency responses of five series resonance circuits with different Q-factors.

Figure 4 shows frequency responses of five series resonance (SR) circuits, built from ideal capacitors, and from inductors with L = 10 mH and with different Q-factors. Form factors are read from Figure 2 for 70 mm nickel plating at 10, 20, 30, 40, and 50 MHz, and R_0 is one Ohm. Corresponding inductor resistances at 10 MHz and at the first four harmonics are 2, 9.5, 18, 28, and 30 Ohm.

$$\mathrm{SR0}: f_r = 10\ \mathrm{MHz},\ Q\ \mathrm{B}320,\ I_{peak} = 500\ \mathrm{mA},$$
$$(R_L = 2\ \mathrm{Ohm},\quad C_{LOAD} = 25.33\ \mathrm{pF},)$$

$$\mathrm{SR1}: f_r = 20\ \mathrm{MHz},\ Q\ \mathrm{B}130,\ I_{peak} = 100\ \mathrm{mA},$$
$$(R_L = 9.5\ \mathrm{Ohm},\ C_{LOAD} = 6.33\ \mathrm{pF}\ ,)$$

$$\mathrm{SR2}: f_r = 30\ \mathrm{MHz},\ Q\ \mathrm{B}100,\ I_{peak} = 55\ \mathrm{mA},$$
$$(R_L = 18\ \mathrm{Ohm},\quad C_{LOAD} = 2.81\ \mathrm{pF}\ ,)$$

$$\mathrm{SR3}: f_r = 40\ \mathrm{MHz},\ Q\ \mathrm{B}100,\ I_{peak} = 33\ \mathrm{mA},$$
$$(R_L = 28\ \mathrm{Ohm},\quad C_{LOAD} = 1.58\ \mathrm{pF}\ ,)$$

$$\mathrm{SR4}: f_r = 50\ \mathrm{MHz},\ Q\ \mathrm{B}100,\ I_{peak} = 31\ \mathrm{mA},$$
$$(R_L = 30\ \mathrm{Ohm},\quad C_{LOAD} = 1.01\ \mathrm{pF}\ .)$$

The resonant current ratios of a 10 MHz signal and its first four harmonics are 1 / 0.20 / 0.11 / 0.07 / 0.06. Were the Q-factor of an inductor not frequency-dependent, then all five resonant currents would be equal. That being the case, a resonant circuit would be more sensitive to propagation of harmonic frequencies (introduced by nonlinear loads and to some extent by nonlinear gain stages) and to resonance at harmonic frequencies (as load capacitance changes, as it is the case with the capacitance of RF plasma). In RF plasma processing, impedance matching networks address changes in load impedance. However, the frequency-dependent Q-factor of inductors reacts to load changes instantly since it is implemented in the inductors' material structure, which has no built-in memory[1] and therefore reacts instantly.

5. Conclusion

Power dissipation in high frequency inductors takes place mostly within the skin depth, which is a function of frequency. By plating high current inductors used in RF power equipment, one makes the coil's resistance a controlled function of frequency within a certain range. An obvious application is the reduction of the Q-factor at higher harmonics of the operating frequency. This effect needs to get recognition in applications with time-variable and/or nonlinear components that introduce higher harmonics.

Acqnowledgments

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1 *In matching networks, motors adjust geometries of variable capacitors to adjust impedances. Matching process takes more or less time, depending on the difference between the new and the last setting.*