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The Roper resonance – a genuine three quark or a dynamically generated resonance?

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Abstract. We investigate two mechanisms for the formation of the Roper resonance: the excitation of a valence quark to the 2s state versus the dynamically generation of a quasi-bound meson-nucleon state. We use a coupled channel approach including the πN, πΔ and σN channels, fixing the pion-baryon vertices in the underlying quark model and using a phenomenological form for the s-wave sigma-baryon interaction. The Lippmann-Schwinger equation for the K matrix with a separable kernel is solved to all orders which results in the emergence of a quasi-bound state at around 1.4 GeV. Analysing the poles in the complex energy plane using the Laurent-Pietarinen expansion we conclude that the mass of the resonance is determined by the dynamically generated state, but an admixture of the $(1s)^2(2s)^1$ component is crucial to reproduce the experimental width and the modulus of the resonance pole.

This work has been done in collaboration with Simon Širca from Ljubljana, Hedim Osmanović from Tuzla and Alfred Švarc from Zagreb.

The recent results of lattice QCD simulation in the P11 partial wave by the Graz-Ljubljana group [1] including besides 3q interpolating fields also operators for πN in relative p-wave and σN in s-wave, has revived the interest in the nature of the Roper resonance. Their calculation and a similar calculation by the Adelaide group [2] show no evidence for a dominant 3q configuration below 1.65 GeV and 2.0 GeV, respectively, that could be interpreted as a three-quark Roper state, and therefore support the dynamical origin of the Roper resonance.

In our work [3] we study the interplay of the dynamically generated state and the three-quark resonant state in a simplified model incorporating the πN, πΔ and σN channels. The choice of the channels as well as of the parameters of the model is based on our previous calculations of the scattering and the meson photo- and electro-production amplitudes for several partial waves in which all relevant channels as well as most of the nucleon and Δ resonances in the intermediate energy regime have been included [5–9]. The bare octet-meson–baryon vertices are calculated in the Cloudy Bag Model while the parameters of the σ-baryon interaction are left free: apart of its strength, the Breit-Wigner mass and the width of the σ are varied. We have been able to consistently reproduce the results in the S and P partial waves; only the D waves typically require an increase in the strength of the meson-quark couplings compared to those predicted by the underlying quark model. The results presented here are obtained with the

σ mass and width both equal to 600 MeV, and only the σNN coupling is varied. Very similar results have been obtained for the mass and width of 500 MeV.

The central quantity in our approach is the half-on-shell K matrix[1] that consists of the resonant (pole) terms and the background (non-pole) term $\mathcal{D}$:

$$\chi_{\alpha\gamma}(k, k_\gamma) = \frac{V_{\gamma N}(k_\gamma)\, V_{\alpha N}(k)}{m_N - W} + \frac{V_{\gamma R}(k_\gamma)\, V_{\alpha R}(k)}{m_R - W} + \mathcal{D}_{\alpha\gamma}(k, k_\gamma)\,. \tag{1}$$

Indices α, β, γ ... denote the three channels, the first term corresponds to the nucleon pole, the second term is optional and generates an explicit resonance with the K-matrix pole at $W = m_R$. The Lippmann-Schwinger equation (LSE) for the K matrix splits into the equation for the dressed $N \to \alpha$ vertex,

$$\mathcal{V}_{\alpha N}(k) = V^{(0)}_{\alpha N}(k) + \sum_\beta \int dk' \, \frac{\mathcal{K}_{\alpha\beta}(k, k')\mathcal{V}_{\beta N}(k')}{\omega_\beta(k') + E_\beta(k') - W}\,, \tag{2}$$

and the equation for the background,

$$\mathcal{D}_{\alpha\delta}(k, k_\delta) = \mathcal{K}_{\alpha\delta}(k, k_\delta) + \sum_\beta \int dk' \, \frac{\mathcal{K}_{\alpha\beta}(k, k')\mathcal{D}_{\beta\delta}(k', k_\delta)}{\omega_\beta(k') + E_\beta(k') - W}\,. \tag{3}$$

If the resonant state is included, an equation analogous to (2) holds for the $R \to \alpha$ vertex. Let us note that the splitting of the K matrix is similar to the splitting used in approaches computing directly the T matrix, but is not equivalent. In the K-matrix approach the T matrix is obtained by solving the Heitler equation, $T = K + iKT$, which necessarily mixes the pole and the non-pole terms.

Our approximation consists of assuming a separable form for the kernel $\mathcal{K}_{\alpha\beta}$:

$$\mathcal{K}_{\alpha\beta}(k, k') = \sum_i \varphi^\alpha_{\beta i}(k)\, \xi^\beta_{\alpha i}(k')\,, \tag{4}$$

$$\varphi^\alpha_{\beta i}(k) = \frac{m_i}{E_\beta}\,(\omega_\beta + \varepsilon^\beta_{i\alpha})\frac{V^\alpha_{i\beta}(k)}{\omega_\alpha(k) + \varepsilon^\alpha_{i\beta}}\; f^i_{\alpha\beta}\,,$$

$$\xi^\beta_{\alpha i}(k') = \frac{V^\beta_{i\alpha}(k')}{\omega_\beta(k') + \varepsilon^\beta_{i\alpha}}\,, \qquad \varepsilon^\beta_{i\alpha} = \frac{m_i^2 - m_\alpha^2 - \mu_\beta^2}{2E_\alpha}\,,$$

where i runs over intermediate N and Δ, f are the corresponding spin-isospin factors, $V^\alpha_{i\beta}$ corresponds to the decay of the baryon in channel β into the intermediate baryon and the meson in channel α, and m (E) and μ (ω) stand for the baryon and the meson mass (energy), respectively. $\mathcal{K}_{\alpha\beta}(k, k')$ reduces to the u-channel exchange potential when either k or k' takes its on-shell value. This type of approximation has been used in our previous calculations and has lead to consistent results. Let us mention that neglecting the integral terms in (2) and (3) corresponds to the so called K-matrix approximation.

[1] χ is proportional to the K matrix (satisfying $S = (1 + iK)/(1 - iK)$) by a kinematical factor which is not relevant for the present discussion.

Equation (2) and (3) can be solved exactly by the ansatz:

$$\mathcal{V}_{\alpha N}(k) = V^{(0)}_{\alpha N}(k) + \sum_{\beta i} x^{\alpha}_{\beta i}\, \varphi^{\alpha}_{\beta i}(k)\,, \tag{5}$$

$$\mathcal{D}_{\alpha\delta}(k) = \mathcal{K}_{\alpha\delta}(k, k_\delta) + \sum_{\beta i} z^{\alpha\delta}_{\beta i}\, \varphi^{\alpha}_{\beta i}(k)\,, \tag{6}$$

with coefficients x and z satisfying sets of algebraic equations of the form

$$\sum_{\gamma j} A^{\beta}_{\alpha i,\gamma j}\, x^{\beta}_{\gamma j} = b^{\beta}_{\alpha i}\,, \qquad \sum_{\gamma j} A^{\beta}_{\alpha i,\gamma j}\, z^{\beta\delta}_{\gamma j} = c^{\beta\delta}_{\alpha i}\,.$$

Note that both equations involve the same matrix $\mathsf{A} = \mathsf{I} + \mathsf{M}$, $\mathsf{M} = [M]^{\beta}_{\alpha i,\gamma j}$ where

$$M^{\beta}_{\alpha i,\gamma j} = -\int dk\, \frac{\xi^{\beta}_{\alpha i}(k)\varphi^{\beta}_{\gamma j}(k)}{\omega_\beta(k) + E_\beta(k) - W}\,. \tag{7}$$

For sufficiently strong interaction, the matrix A becomes singular and one or more poles appear in the background part of the K matrix which signals the emergence of a dynamically generated state. In fact, poles at the *same* energies appear also in the corresponding resonant terms of the K matrix, in addition to the nucleon pole and the (optional) pole at m_R. The mechanism of this process can be studied by performing the *singular value decomposition* $\mathsf{A} = \mathsf{U}\mathsf{W}\mathsf{V}^{\mathsf{T}}$ where W is a diagonal matrix containing the singular values w_i. The singular values remain close to unity with exception of one which approaches zero as the interaction increases (Fig. 1 a) and eventually becomes negative for sufficiently strong $g_{\sigma NN}$ (Fig. 2 a). We claim that it is this value, $w_{\min}$, and the corresponding singular vector $U_{\min}$, that determine the properties of the quasi-bound molecular state. This state is dominated by the σN component. For the invariant energies W for which $w_{\min}$ is close to zero, the solutions (5) and (6), in the absence of the resonant state R, can be cast in the form

$$\mathcal{V}_{\alpha N}(k_\alpha) \approx V^{(0)}_{\alpha N}(k_\alpha) + \frac{a_\alpha}{w_{\min}}\,, \qquad \mathcal{D}_{\alpha\delta}(k_\alpha, k_\delta) \approx \mathcal{K}_{\alpha\delta}(k_\alpha, k_\delta) + \frac{d_{\alpha\delta}}{w_{\min}}\,. \tag{8}$$

Similarly, the nucleon self energy acquires the form

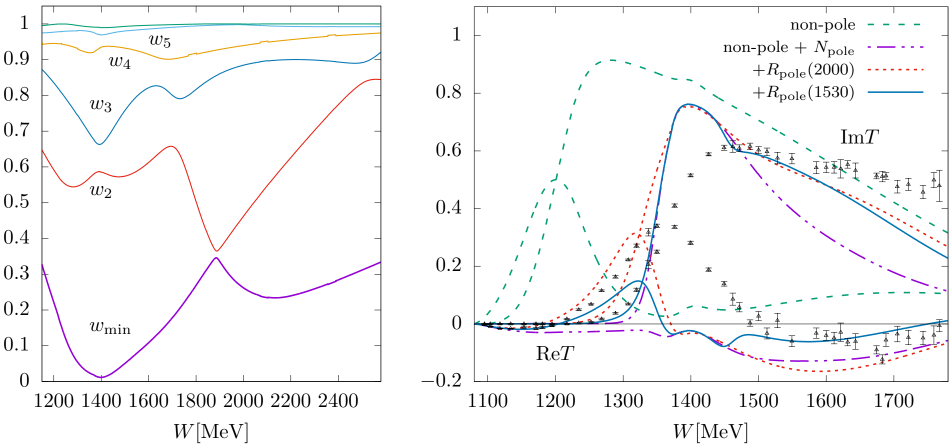
$$\Sigma_N(W) = \sum_\beta \int dk\, \frac{\mathcal{V}_{\beta N}(k) V^{(0)}_{\beta N}(k)}{\omega_\beta(k) + E_\beta(k) - W} \approx (m_N - W)\left(\Sigma'_N(W) + \frac{b}{w_{\min}}\right)\,. \tag{9}$$

Just above the πN threshold, the $\mathcal{D}$ term is dominated by the u-channel N exchange processes which is reflected in a large peak in ImT (the non-pole term in Fig. 1 b). This term has the opposite sign with respect to the nucleon-pole term; these two terms almost cancel each other. In the energy region where $w_{\min}$ reaches its minimum the second terms in (8) and (9) dominate and the leading contribution to the K matrix reads

$$K_{\alpha\delta} \approx \frac{a_\alpha a_\delta}{b} \frac{1}{(m_N - W)\, w_{\min}} + \frac{d_{\alpha\delta}}{w_{\min}}\,.$$

The two terms generate a resonance peak at the minimum of w_{min} (dashed-dotted line in Fig. 1 b); the real part, ReW_p, of the corresponding S-matrix pole in Table 1 appears slightly below W of the minimum of w_{min}. Increasing $g_{\sigma NN}$, w_{min} crosses zero twice and two poles of the S-matrix appear with ReW_p close to the intersections (see Fig. 1 a and Table 1 for $g_{\sigma NN} = 2.05$).

If we include the resonant state by imposing a fixed value for m_R in the second term of (1), the position of the peak almost does not change for a value of m_R as low as 1530 MeV (solid line in Fig. 1 b). The effect of the resonant state is reflected in the increased width of the resonance rather than in the change of its position. This general scenario does not change if we decrease $g_{\pi NN}$ in order to reproduce the experimental values of ReT and ImT (Fig. 2 b). While the peak in ImT moves to somewhat higher W, the position of the minimum of w_{min} as well as of the real part of the S-matrix pole stay almost at the same value (see Table 1). Also, varying the value of m_R between 1520 MeV and 2000 MeV has almost no influence on the behaviour of the amplitudes and the position of the S-matrix pole.



a) b)

Fig. 1. (Color online) a) The six lowest singular eigenvalues of the A matrix for $g_{\sigma NN} = 2.0$. b) The real and imaginary parts of the T matrix calculated from the background (non-pole) term alone (dashed lines), from the background plus the nucleon pole term (dash-dotted lines), and from including the resonant state either at $m_R = 1530$ MeV (solid lines), or at $m_R = 2000$ MeV (short-dashed lines) for $g_{\sigma NN} = 2.0$.

We can summarize the results obtained in our simplified model as follows:

- The main mechanism for the Roper resonance formation is the dynamical generation through a quasi-bound meson-baryon state around $W \approx 1400$ MeV dominated by the σN component. Its mass is rather insensitive to variations of the $g_{\pi NN}$ coupling.

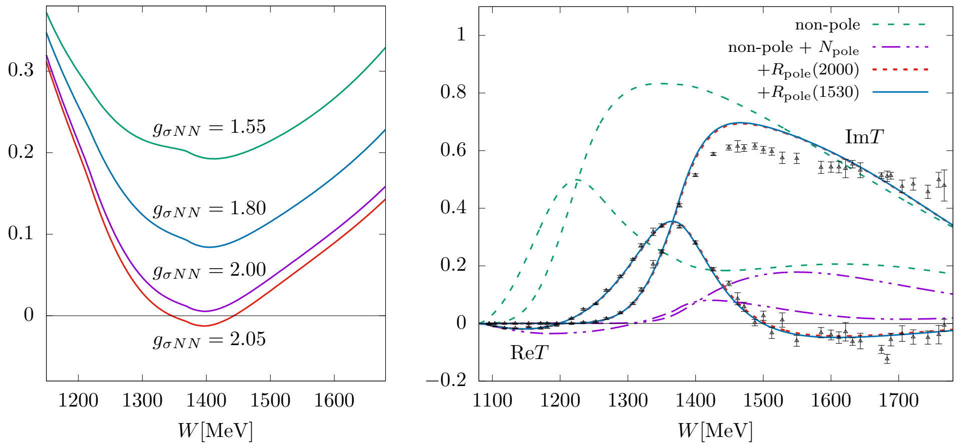


Fig. 2. (Color online) a) The lowest singular value of the W matrix, w_{min}, for four values of $g_{\sigma NN}$. b) Same as Fig. 1 b, except for $g_{\sigma NN} = 1.55$.

Table 1. S-matrix pole position and modulus for the model without the resonant state ($m_R = \infty$), and the model with the resonant state for two values of the K-matrix pole mass. The PDG values are taken from [10].

$g_{\sigma NN}$	m_R [MeV]	$\mathrm{Re}W_p$ [MeV]	$-2\mathrm{Im}W_p$ [MeV]	$\|r\|$	ϑ
PDG		1370	180	46	$-90°$
1.80	∞	1397	157	11.2	$-78°$
2.00	∞	1358	111	20.6	$-81°$
2.05	∞	1331	44	7.3	$-62°$
		1438	147	18.6	$-17°$
2.00 $g_{\pi N\Delta} = 0$	∞	1342	285	18.8	$-11°$
1.55	2000	1368	180	48.0	$-87°$
1.55	1530	1367	180	47.5	$-86°$

- The real part of the S-matrix pole, $\mathrm{Re}W_p$, remains close to or slightly below the mass of the quasi-bound state and is almost insensitive to the presence of a three-quark resonant state, while the PDG value of the imaginary part, $\mathrm{Im}W_p$, is reproduced only if the three-quark resonant state is included.
- The S-matrix pole emerges with $\mathrm{Re}W_p$ close to the minimum of w_{min} even if (positive) w_{min} stays relatively far from zero; in this case the corresponding pole is not present in the K matrix.

- The mass of the quasi-bound molecular state is most strongly influenced by the σN component and lies ~ 100 MeV below the nominal σN threshold; removing the $\pi\Delta$ component has little influence on the mass (see $g_{\pi N\Delta} = 0$ entry in Table 1).

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