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Distance formula for direct-co-direct product in the case of disconnected factors*

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Abstract

Direct-co-direct product $G \circledast H$ of graphs G and H is a graph on the vertex set $V(G) \times V(H)$. Two vertices (g, h) and (g', h') are adjacent if $gg' \in E(G)$ and $hh' \in E(H)$ or $gg' \notin E(G)$ and $hh' \notin E(H)$. We show that if at most one factor of $G \circledast H$ is connected, then the distance between two vertices of $G \circledast H$ is bounded by three unless some small number of exceptions. All the exceptions are completely described which yields the distance formula.

Keywords: Direct-co-direct product, distance, eccentricity, disconnected graphs.

Math. Subj. Class.: 05C12, 05C76

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Formula za razdaljo med dvema vozliščema v direktnem-ko-direktnem produktu, kadar so faktorji nepovezani*

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Povzetek

Direktni-ko-direktni produkt $G \circledast H$ grafov G in H je graf na množici vozlišč $V(G) \times V(H)$. Dve vozlišči (g, h) in (g', h') tega produkta sta sosedni, če je $gg' \in E(G)$ in $hh' \in E(H)$ ali $gg' \notin E(G)$ in $hh' \notin E(H)$. Pokažemo, da velja: če je največ en faktor produkta $G \circledast H$ povezan, potem je razdalja med dvema vozliščema v njem omejena s tri, razen za nekaj malega izjem. Vse izjeme natančno opišemo, na podlagi česar lahko izpeljemo formulo za razdaljo.

Ključne besede: Direktni-ko-direktni produkt, razdalja, ekscentričnost, nepovezani grafi.

Math. Subj. Class.: 05C12, 05C76

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