Interaction between mineral composition, water content and mechanical properties of saturated cohesive soils

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Abstract: It is known that mechanical properties of cohesive soils, which are determined within the scope of geotechnical investigations, depend on the water content, while the latter mostly on the mineral composition of soils. These relationships are given as empirical expressions that are based on the results of laboratory tests performed on artificial monomineral clay mixtures. The aim of the investigations described in the article was to verify practical applications for the above-mentioned findings. Five samples of heterogeneous soils were tested for this purpose. The values of the geomechanical properties, which were calculated from selected mineralogical properties, are compared to experimentally tested values. The testing results show good correlation.

Key words: clays, mineral composition, undrained shear strength, compressibility

INTRODUCTION

It is known that mechanical properties of saturated cohesive soils, which are determined within the scope of geotechnical investigations, depend on the water content, while the latter mostly on the mineral composition of soils. Several researchers have tried to determine these relationships by comparing the quantity of water in soils to the sizes of grains, the specific surface, the quantity of clay fraction or the cationic exchange capacity. The results of their investigations, however, vary considerably, being valid for investigated soils only and not generally applicable. The reasons for the different conclusions reached by researchers in previous studies have been explained by Dolinar & Trauner (2003, 2004) as follows.

Clay and non-clay minerals are present in cohesive soils. To understand the influence of soils' composition on the water content it is necessary to know that clay minerals, as well as water, are not chemically inert; therefore they are subject to interaction. In non-expanding clay minerals, water is strongly adsorbed on the external surfaces of the grains (w_{ea}) whilst in expanding clays it is bonded to the external surfaces (w_{ea}) and internal surfaces (w_i) of the grains. In addition to strongly adsorbed water, the free pore water w_{ef}, occurs in saturated clays (Fig. 1). Using a standard method for measuring the water content w, the total quantity of intergrain water w_e ($w_e = w_{ef} + w_{ea}$) and interlayer water w_i can always be determined by drying at a temperature higher than 100 °C.

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The total quantity of water w is, therefore, equal to the intergrain water content in non-swelling clays (Eq. (1)), whilst it is higher for the portion of interlayer water in swelling clays (Eq. (2)).

$$w = w_e = w_{ef} + w_{ea} \quad (1)$$

$$w = w_e + w_i = w_{ef} + w_{ea} + w_i \quad (2)$$

Interaction forces between the external surfaces of the grains and the adsorbed water are the same for all clays due to equal surface structure. It leads to the conclusion that at an equal quantity of free pore water w_{ef}, the thickness of the adsorbed water film t_a is the same for different clay grains (MITCHELL, 1993). The quantity of water, adsorbed on the external surfaces of the clay grains w_{ea}, depends, in this case, on the surface of the clay grains expressed on the unit weight of clay grains A_{SC} (Eq. (3)).

$$w_{ea} = t_a \cdot A_{SC} \quad (3)$$

Whilst the quantity of intergrain water w_e depends on the sizes of the clay grains, the interlayer water quantity w_i mostly depends on the types and quantities of interlayer cations (GRIM, 1962). This means that it is impossible to express the total quantity of water w in expanding and non-expanding clays by considering the same mineralogical properties.

The interlayer water w_i is strongly adsorbed between the layers, therefore, it cannot influence the strength and compressibility of the clays. Any dependence between the water content and selected mechanical properties can be determined on the basis of the intergrain water content w_e only.

Taking into consideration the assumptions mentioned above the intergrain water content w_e depends on the free pore water quantity w_{ef}, and on the sizes of the clay grains. It can be expected that the quantity of free pore water

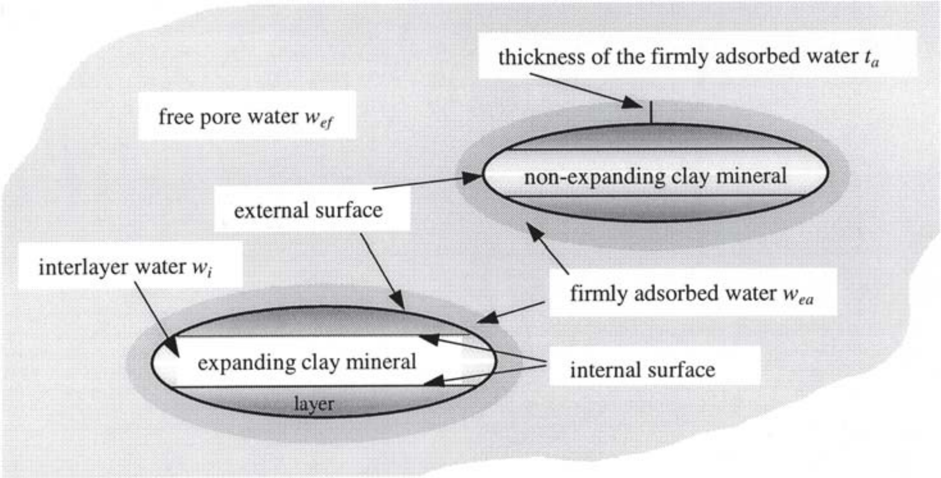

Figure 1. Graphical presentation of water distribution in fully saturated clays; the intergrain $w_e = w_{ef} + w_{ea}$.

Slika 1. Grafična predstavitev razporeditve vode v zasičenih glinah; medzrnska voda $w_e = w_{ef} + w_{ea}$.

only depends on stress state, which means that it must be the same for different clays at equal effective stress σ' and equal undrained shear strength c_u and also the thickness t_a of the adsorbed water film (Eq. (4)).

$$w_{e|\sigma',c_u} = w_{ef|\sigma',c_u} + t_{a|\sigma',c_u} \cdot A_{SC} \qquad (4)$$

Equation (4) is valid for soils that contain clay minerals only. In cases of clay- and non-clay minerals in soils, the findings of Mitchell (1993) and Seed et al. (1964) must be considered, i.e. water in soils is mainly bonded to clay grains. This means that the intergrain water quantity also depends on the weight portion p of the clay grains in the soil ($0 < p \leq 1$). In this case, the Eq. (4) is rewritten as follows:

$$w_{e|\sigma',c_u} = p \cdot (w_{ef|\sigma',c_u} + t_{a|\sigma',c_u} \cdot A_{SC}) \qquad (5)$$

where $p \cdot A_{SC} = A_S$; A_S (m^2/g) is the grain surface per unit weight of the dry soil.

In determining the relationship between the given mineralogical properties of soils, the intergrain water content w_e, and the undrained shear strength c_u, the researchers Dolinar et al. (2004) proceeded from the known experimentally defined relationship (Koumoto & Houlsby, 2001; Dolinar, 2004),

$$w_e = a_e \cdot c_u^{-b_e} \qquad (6)$$

where a_e and b_e are soil-dependent parameters. a_e (%) is the intergrain water content at the undrained shear strength c_u = 1 kPa, and b_e is the slope of the linear function which represents the ratio between the intergrain water content w_e (%) logarithm and the undrained shear strength c_u (kPa) logarithm.

It follows from equations (5) and (6) that the intergrain water content at the undrained shear strength c_u = 1 kPa can be expressed as

$$w_{e\,|\,c_u=1} = a_e = p \cdot w_{ef\,|\,c_u=1} + t_{a|c_u=1} \cdot A_S \qquad (7)$$

Equation (7) shows that the parameter a_e depends on the specific surface A_{SC} and the weight portion p of the clay grains in the soil, because $w_{ef\,|\,c_u=1}$ and $t_{a|\,c_u=1}$ are constants. It follows from equations (5), (6) and (7) that the parameter b_e depends on the specific surface A_{SC} only.

The relationships between the specific surfaces of grains and both parameters are given by Eqs. (8) and (9), where α = 33.70, β = 0.99, γ = 0.05 and λ = 0.27.

The coefficients in Eqs. (8), (9), (11) and (12) were determined by testing samples which had clearly defined chemical and mineral composition, a known structural formula and a specific surface.

$$a_e = p \cdot \alpha + \beta \cdot A_S \qquad (8)$$

$$b_e = \gamma \cdot A_{SC}^{\ \lambda} \qquad (9)$$

Equations (7) and (8) have the same structure, therefore, the first and the second terms in both of them have to be equal. Consequently, the quantity of free pore water $w_{ef\,|\,c_u=1} = \alpha$, the average thickness of the adsorbed water around the clay grains $t_{a|\,c_u=1kPa} = \beta$, and the adsorbed water quantity around the clay grains $w_{ea|\,c_u=1kPa} = t_{a|c_u=1kPa} \cdot A_{SC} = \beta \cdot A_{SC}$.

In determining the influence of mineralogical properties of soils on the intergrain water content w_e at the effective stresses σ' in the soils, DOLINAR ET AL. (2004) proceeded from the dependencies:

$$w_e = i_e \, \sigma'^{-j_e} \tag{10}$$

where i_e and j_e are soil dependent parameters.

In accordance with previous findings they can be expressed in dependence on the size and quantity of clay minerals in soil by Eqs. (11) and (12), where $\omega = 33.46$, $\theta = 1.39$, $\gamma = 0.05$, $\lambda = 0.27$ and p is the portion of the clay minerals in the sample.

$$i_e = p\omega + \theta \; A_{Se} \tag{11}$$

$$j_e = b_e = \gamma \cdot A_{SC}{}^{\lambda} \tag{12}$$

It should be noted, however, that in practical applications it is impossible to determine the structural formula of an individual clay mineral, which is needed for an accurate calculation of interlayer water. It is also difficult to determine an accurate quantitative mineral composition. The quantity of water in soils can also be influenced by organic substances, which are often found in soils, but their influence has not yet been investigated. So, the aim of the investigations described in this article, was to verify practical applications for the above-mentioned findings. Five samples of heterogeneous soils were tested for this purpose, in which the geomechanical properties were determined both experimentally and on the basis of mineralogical properties. This allowed a comparison between the measured and calculated values.

INVESTIGATION OF NATURAL SOILS

Experimental determination of parameters a_e, b_e, i_e, and j_e

We have determined the parameters a_e and b_e experimentally with five measurements of the undrained shear strength c_u (kPa) at different intergrain water content w_e (%). The relationship between c_u and w_e is linear in a double logarithmic scale, therefore a_e (%) is the water content at the undrained shear strength $c_u = 1$ kPa and b_e is the slope of the linear function, which enables easy determination of both parameters. The liquid limit *LL* and the plastic limit *PL*, which correspond to the undrained shear strength $c_u = 2.66$ kPa and $c_u = 266$ kPa, were also determined in the samples.

The quantity of intergrain water w_e in the saturated clays was measured under different effective stresses using an oedometer consolidation test. The initial moisture content in the samples was near the liquid limit. The corresponding water content after consolidation of the clays was determined at axial stresses σ' = 50 kPa, 100 kPa and 200 kPa. The relationship between the intergrain water quantity and the effective stress is completely linear when both variables are shown in a double logarithmic scale. In this case the parameter i_e represents the water content in the soil at σ' = 1 kPa and the parameter j_e the slope of the linear function.

In determining parameters a_e, b_e, i_e and j_e it was necessary to consider only the intergrain water content in the soils. The interlayer water in the tested samples, however, resulted from the presence of Ca-montmorillonite. The procedure of calculation is shown below.

The calculation of interlayer water in an expanding mineral

An example of the interlayer water content calculation is given for the sample of pure montmorillonite with the calcium exchangeable cation. The montmorillonite structure consists of an octahedral sheet sandwiched between two silica sheets. The layers formed in this way are continuous in the *a* and *b* directions and stacked one above the other in the *c* direction. Bonds between layers are weak, thus water or other polar molecules can enter between the unit layers causing the lattice to expand in the *c* direction. The basal spacing (spacing between the centre of two neighbouring layers) in the *c* direction which is d_1 = 0.96 nm for dry calcium montmorillonite (dried at 105 °C) rises to d_2 = 1.54 nm after the adsorbtion of water (Grim, 1962). In case of calcium exchangeable cation in montmorillonite the adsorption of water between layers is completed already at very low water content and the basal spacing is then practically constant. The volume of interlayer water V_{wi} can thus be calculated using the equation (Fink and Nakayama, 1972):

$$V_{wi} = \frac{A_{Si} \cdot (d_2 - d_1)}{2} \tag{13}$$

where A_{Si} (m²/g) is the internal specific surface of montmorillonite. With our samples we considered the internal specific surface of montmorillonite grains A_{Si} = 626.80 m²/g and adequate mass portions of this mineral in the individual soils.

Determination of mineralogical properties

The bulk mineral composition and the clay fraction of the samples were scanned by

Table 1. Mineral composition of the powder bulk samples.
Tabela 1. Mineralna sestava uprašenih vzorcev.

Sample	Mineral composition (%) and specific surface (m²/g)									
	I + Rx1	Ch + Rx2	Ka	Ca M	Q	Pl	Mic	Cal	G/H	A_S
1	25	8	5	14	34	9	5	0	0	30.1± 0.4
2	35	14	0	0	25	3	0	22	1	28.5± 0.4
3	28	16	0	0	42	9	4	0	0	16.7 ± 0.1
4	35	0	12	34	18	0	0	0	1	54.1 ± 0.3
5	34	18	0	0	43	3	3	0	0	32.6 ± 0.2

LEGEND:

I	illite	Q	quartz
Ch	chlorite	Pl	plagioclase
Rx1	mixed layer clays illite/montmorillonite	Mic	microcline
Rx2	mixed layer clays chlorite/montmorillonite	Cal	calcite
Ka	kaolinite	G/H	goethite/hematite
CaM	Ca - montmorillonite		

Table 2. Mineral composition of the fraction < 0.002 mm.
Tabela 2. Mineralna sestava frakcije < 0.002 mm.

Sample	Illite, MLC illite / Ca-montmorillonite, Ca-montmorillonite (%)									
	I	Il/ΣI1	Il/ΣI2	Il/ΣI3	Il/ΣI4	Il/ΣI5	Il/ΣI6	Il/ΣI7	Il/ΣI8	CaM
1	4	0	0	0	60/2	0	0	23/2	0	6
2	7	0	82/6	0	0	55/5	0	0	0	2
3	2	0	0	78/1	0	0	42/1	0	0	1
4	6	0	0	78/4	0	0	42/3	0	0	16
5	7	84/5	0	78/4	0	0	42/3	0	15/2	0

LEGEND:

I	illite
CaM	Ca-montmorillonite
Il	illite in mixed layer clays illite/montmorillonite
ΣI1 -I8	percentage of mixed layer clays illite/montmorillonite in soil

Table 2 (continued). Mineral composition of the fraction < 0.002 mm.
Tabela 2 (nadaljevanje). Mineralna sestava frakcije < 0.002 mm.

Sample	Kaolinite, Chlorite, MLC kaolinite/Ca-montmorillonite, MLC chlorite/Ca-montmorillonie [%]						
	Ka	Ch	K/ΣKao	C/ΣCh1	C/ΣCh2	C/ΣCh3	< 0.002 mm
1	1	0	0	0	0	0	14.54
2	3	2	0	0	68/1	0	26.21
3	0	3	0	76/2	0	63/2	12.84
4	2	0	90/2	0	0	0	33.02
5	4	0	0	0	0	0	24.96

LEGEND:

Ka	kaolinite
Ch	chlorite
K	kaolinite in mixed layer clays kaolinite/montmorillonite
C	chlorite in mixed layer clays chlorite/montmorillonite
ΣKao	percentage of kaolinite in mixed layer clays kaolinite/montmorillonite in fraction < 0.002 mm
ΣCh1 -Ch3	percentage of mixed layer clays chlorite/montmorillonite in soil

Table 3. Chemical composition.
Tabela 3. Kemična sestava.

Chemical composition (%)	Sample 1	2	3	4	5
SiO_2	64.15	46.83	72.10	51.71	67.48
Al_2O_3	15.83	13.45	12.09	20.38	12.77
TiO_2	0.89	0.61	0.78	0.80	0.88
Fe_2O_3	5.09	5.02	4.92	9.06	6.70
FeO	1.1	1.0	1.0	0.3	0.5
MnO	0.03	0.07	0.05	0.18	0.18
MgO	1.97	2.27	1.07	1.66	1.62
CaO	0.87	12.42	0.41	1.21	0.40
Na_2O	0.97	0.15	1.19	0.16	0.28
K_2O	2.20	2.75	1.95	2.52	2.64
P_2O_5	0.13	0.12	0.15	0.13	0.07
Cr_2O_3	0.017	0.015	0.012	0.024	0.027
TOT/C	0.27	3.03	0.40	0.41	0.30
TOT/S	0.01	0.06	0.03	0.06	< 0.01
Organic matter	2.37	2.02	1.26	2.70	1.82

Table 4. Portion *p* of the clay minerals in the soil, parameters a_e and b_e, interlayer water w_i, intergrain water at the liquid limit $w_{e|LL}$, liquid limit *LL*, intergrain water at the plastic limit $w_{e|PL}$, plastic limit *PL*, placticity index I_p.
Tabela 4. Delež glinenih mineralov v zemljini *p*, parametra a_e in b_e, medslojna voda w_i, medzrnska voda na meji židkosti $w_{e|LL}$, meja židkosti *LL*, medzrnska voda na meji plastičnosti $w_{e|PL}$, meja plastičnosti *PL*, indeks plastičnosti I_p.

	Sample									
	1		2		3		4		5	
	C	***E***	***C***	***E***	***C***	***E***	***C***	***E***	***C***	***E***
p	0.52		0.49		0.44		0.89		0.52	
a_e (%)	47.32	50.67	44.73	49.67	31.36	34.22	83.55	85.82	49.79	48.94
b_e	0.149	0.164	0.149	0.148	0.133	0.113	0.152	0.152	0.152	0.151
w_i (%)	4.05		1.02		0.55		8.14		0.93	
$w_{e\|LL}$ (%)	40.90	43.15	38.66	42.98	27.61	30.64	72.00	73.96	42.91	42.22
LL (%)	44.95	47.20	39.68	44.00	28.16	31.19	80.14	82.10	43.84	43.15
$w_{e\|PL}$ (%)	20.59	20.25	19.47	21.71	14.92	18.22	35.75	36.78	21.31	21.07
PL (%)	24.64	24.30	20.49	22.71	15.47	18.77	43.89	44.92	22.24	22.00
I_P (%)	20.31	22.90	19.19	21.29	12.69	12.42	36.25	37.18	21.60	21.15

Note: $a_e = 33.70 + 0.99 \cdot A_S$; $b_e = 0.05 \cdot A_{SC}^{0.27}$; $A_{SC} = p \cdot A_S$; $w_{e|LL} = a_e \cdot 2.66^{-b_e}$; $LL = w_{e|LL} + w_i$; $w_{e|PL} = a_e \cdot 266^{-b_e}$; $PL = w_{e|PL} + w_i$; $I_P = w_{e|LL} - w_{e|PL} = LL - PL$ because the interlayer water quantity is equal at the liquid and plastic limits.

Table 5. The specific surface A_S and the portion p of the clay minerals in the soil, the water content w before and after the test, the calculated interlayer water quantity w_i, the measured and calculated intergrain water quantity w_e after consolidation under different axial stresses s’, the calculated and experimentally determined soil dependent parameters i_e and j_e.

Tabela 5. Specifična površina A_S in delež glinenih mineralov p v zemljini, vsebnost vode w pred in po preiskavi, izračunana količina medslojne vode w_i, merjena in izračunana količina medzrnske vode w_e po konsolidaciji pri različnih efektivnih napetostih s’, izračunana in eksperimentalno določena parametra i_e in j_e.

	Sample				
	1	2	3	4	5
A_S (m^2/g)	30.1	28.5	16.7	54.1	32.6
p	0.52	0.49	0.44	0.89	0.52
w (%) before the test					
$\sigma' = 50$ kPa	49.36	45.92	31.20	70.13	37.77
$\sigma' = 100$ kPa	49.36	44.43	30.38	69.35	37.70
$\sigma' = 200$ kPa	44.99	44.05	27.95	66.22	35.25
w (%) after the test					
$\sigma' = 50$ kPa	37.47	36.76	25.82	66.04	34.72
$\sigma' = 100$ kPa	33.67	34.23	22.56	60.12	31.51
$\sigma' = 200$ kPa	30.85	32.68	23.42	55.16	27.87
w_i (%)	4.05	1.02	0.55	8.14	0.93
E - $w_e = w - w_i$ (%)					
$\sigma' = 50$ kPa	33.42	35.74	25.27	57.90	33.79
$\sigma' = 100$ kPa	29.62	33.21	22.01	51.98	30.58
$\sigma' = 200$ kPa	26.80	31.66	22.87	47.02	26.94
C - $w_e = i_e \sigma'^{-j_e}$ (%)					
$\sigma' = 50$ kPa	33.06	31.26	22.54	57.92	34.60
$\sigma' = 100$ kPa	29.82	28.20	20.50	52.13	31.14
$\sigma' = 200$ kPa	26.89	25.43	18.74	46.91	28.02
C - i_e (%)	59.23	56.01	37.93	104.98	62.71
E - i_e (%)	62.10	55.14	39.60	104.04	64.26
C - j_e	0.149	0.149	0.133	0.152	0.152
E - j_e	0.159	0.111	0.115	0.150	0.163

X-ray diffraction technique (XRD), using a Philips difractometer (PW 3710), a goniometer 1820, with an automatic divergence slit and a curved graphite monochromator, operating at 40 kV and 30 mA, with CuK$_\alpha$ radiation and an Ni filter. Powdered samples were used to determine the bulk sample composition (Tab. 1). Clay minerals were characterized with the help of oriented clay mineral aggregates on glass slides (Tabs. 2 and 3). The results of chemical analyses were used to determine the quantity of individual minerals in the soils (MIŠIČ, 1998), (Tab. 4). The external specific surface area was measured by the five-point BET method with N$_2$ (Tab. 1).

Comparison of laboratory determined and calculated values

The investigative results of parameters are given in Tables 4 and 5, where ***C*** indicates the values calculated from the mineralogical properties of the soils and ***E*** indicates the experimentally determined values. It is evident that all parameters, the quantity of water at the liquid and plastic limits, the plasticity index and the water quantity after consolidation under different vertical stresses calculated from the results of mineralogical analysis are almost equal to the experimentally obtained values. Minor differences probably appear due to the presence of organic substances in the soils.

Conclusion

This article describes the methods and results of investigations with which selected geomechanical properties of saturated cohesive soils were determined based on mineralogical properties of soils. The empirical expressions proposed by Dolinar & Trauner were used. Such a way of determining these properties has not been applied in engineering practice up to now.

The results of the calculated and measured geomechanical properties show very good correlation, which confirms the findings given in the first part of the paper. It should be noted that the values of these properties are influenced by temperature, organic additions, pore water chemical composition, and the fabric of soils, in addition to the compositions of the soils. These influences have not yet been investigated.

Povzetek

Interakcija med mineralno sestavo, vodo in mehanskimi lastnostmi zasičenih vezljivih zemljin

Znano je, da so mehanske lastnosti vezljivih zemljin, ki se določajo v okviru geomehanskih raziskav, odvisne od količine vsebujoče vode, slednja pa predvsem od mineralne sestave. Iz dostopne literature je razvidno, da je raziskav z opisanega področja zelo malo. Posamezni raziskovalci so geomehanske lastnosti zemljin poskušali povezati z nekaterimi mineraloškimi značilnostmi kot so vrsta mineralov v sestavi, velikost zrn in količina glinene frakcije, vendar so vse te odvisnosti zgolj približne in veljajo le za preiskovane zemljine. Povsem drugačen pristop k analiziranju obravnavane odvisnosti pa sta v novejšem času uporabila raziskovalca Dolinar in Trauner. Rezultat njunega dela je opredelitev splošno veljavnih razmerij med mineraloškimi in geomehanskimi lastnostmi zemljin, temelji pa na predhodni teoretični utemeljitvi mehanizmov, ki te odnose določajo.

V prispevku so prikazani rezultati preiskav, s katerimi smo ugotovitve študije omenjenih raziskovalcev, izvedene z vzorci monomineralnih glin, preizkusili na vzorcih heterogenih zemljin. Rezultati preiskav kažejo zelo dobro ujemanje med eksperimentalno določenimi vrednostmi geomehanskih lastnosti in izračunanimi na osnovi izbranih mineraloških lastnosti.

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