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A phenomenological lower bound for the Ξ_{cc}^{++} mass

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Abstract. We show that a simple interpolation between mesonic binding energies can give a good semiquantitative binding energy of the cc diquark and the Ξ_{cc}^{++} baryon. The mass of the Ξ_{cc}^{++} baryon is almost insensitive to widely different choices of the constituent quark masses.

1 Introduction

After the discovery of the Ξ_{cc}^{++} baryon at LHCb, there is a strong interest to verify whether the quark models which have been successful for light and single-heavy hadrons apply also to double-heavy hadrons; in particular, how rich spectrum we can expect. It is important to check whether we may use the same effective quark-quark interaction (apart from the colour factor and the mass-dependent spin-spin term): $V_{uu} = V_{cu} = V_{cc} = V_{c\bar{c}} = V_{bu} = V_{bb} = V_{b\bar{b}}$. For this purpose it is instructive to study some phenomenological models even if the results are only semiquantitative.

The present study is based on two assumptions:

(1) The quark-quark interaction in colour-triplet state is half the quark-antiquark interaction in colour-singlet state.

(2) The ccu baryon can be treated as a two-body system, the cc diquark plus the u quark, similar to the $\bar{c}$u or $\bar{b}$u meson.

These assumptions have been made already by several authors, for example [1,2]. The purpose of this presentation is to show a nice trick how to obtain easily the binding energies of meson-like systems by a simple interpolation between mesonic data [3].

2 The cc diquark interpolated between mesons

We compare the nonrelativistic Schrödinger equations for an (a$\bar{b}$) meson in the colour singlet state and for an (ab) diquark in a colour antitriplet state (with twice weaker interaction):

$$\left[\frac{p^2}{2m_{a\bar{b}}} + V_{a\bar{b}}\right]\psi = E_{a\bar{b}}\psi \equiv F(m_{a\bar{b}})\psi\,,$$

$$\left[\frac{p^2}{2m_{ab}} + V_{ab}\right]\psi = \left[\frac{p^2}{2m_{ab}} + \tfrac{1}{2}V_{a\bar{b}}\right]\psi = \tfrac{1}{2}\left[\frac{p^2}{2(m_{ab}/2)} + V_{a\bar{b}}\right]\psi$$
$$= E_{ab}\psi \equiv \tfrac{1}{2}F(\tfrac{1}{2}m_{ab})\psi\,.$$

Here the reduced masses are $m_{a\bar{b}} = m_a\, m_{\bar{b}}/(m_a + m_{\bar{b}})$ and $m_{ab} = m_a\, m_b/(m_a + m_b)$, respectively. The binding energy $F(m)$ is a smooth function of m as illustrated in Fig. 1.

Phenomenological binding energies of mesons are obtained from experimental meson masses M and model vales of constituent quark masses: $E_{a\bar{b}} = M_{a\bar{b}} - m_a - m_{\bar{b}}$. The diquark masses are then predicted (Table 1). The trick is to take for the diquark binding energy $\frac{1}{2}F(\frac{1}{2}m_{ab})$, according to the above Schrödinger equation.

The constituent quark masses in Fig. 1 and Table 1 are taken from Bhaduri [4]: $m_{u,d,s,c,b}$ = 337, 337, 600, 1870, 5259 MeV, and in Table 1 also from Karliner and Rosner [1]: $m_{u,d,s,c,b}$ = 310, 310, 483, 1663, 5004 MeV.

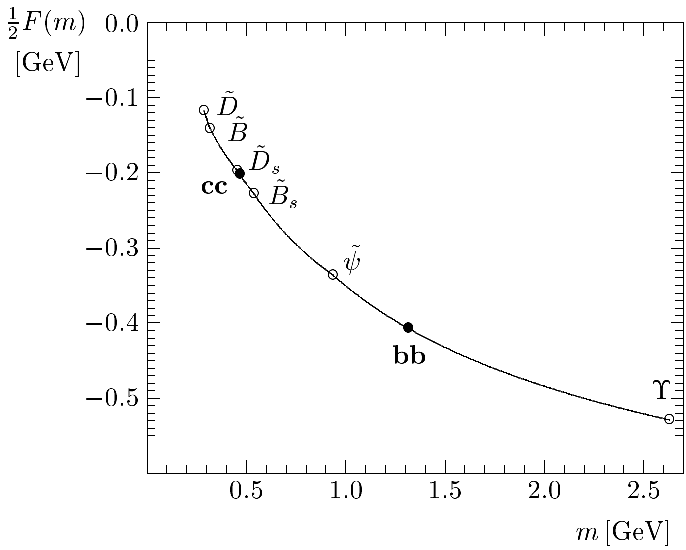

Fig. 1. The meson binding energy $F(m)$, multiplied by $\frac{1}{2}$, as a function of the reduced mass $m = m_{a\bar{b}}$. The diquark binding energies $\frac{1}{2}F(\frac{1}{2}m_{ab})$ are then predicted by interpolation. (From [3]).

3 The binding of the Ξ_{cc}^{++} baryon

The (cc)u baryon is treated as a two-body system. The reduced mass is $m = m_u\, M_{cc}/(m_u + M_{cc})$, where $M_{cc} = 2m_c + \frac{1}{2}F(\frac{1}{2}m_{cc})$ and the binding energy between the u quark and the (cc) diquark $E_{(cc)u} = F(m)$ is obtained by interpolation in Fig. 1 or Table 1. The mass of the Ξ_{cc}^{++} baryon is then $M_{(cc)u} = M_{cc} + m_u + E_{(cc)u} = 3605$ (3596) MeV for the choice of constituent masses of Badhuri (or Karliner-Rosner).

Table 1. The interpolation between mesons.
The tilde means spin average, Δ is the difference between the vector and scalar mesons, m is the reduced mass for mesons and half the reduced mass for diquarks; F is the meson or baryon binding energy and twice the diquark binding energy. Reduced masses refer to the constituent quark masses of Bhaduri [4] or Karliner-Rosner [1], respectively. Energies and masses are in MeV. In the 6th and 9th column are predictions for the diquark and double heavy baryons.

Meson	mass	Δ	m Bha	F	mass predict	m Kar-Ros	F	mass predict
$\widetilde{D}$	1973	141	286	-234		261	0	
$\widetilde{B}$	5314	46	317	-282		292	0	
$\widetilde{D_s}$	2076	144	454	-394		374	-70	
$\widetilde{B_s}$	5403	48	539	-456		440	-84	
$\widetilde{\psi}$	3069	113	935	-671		832	-257	
$\widetilde{\Upsilon}$	9445	61	2630	-1073		2502	-563	
$\widetilde{cc}$			467	-405	3538	416	-80	3286
$\widetilde{bb}$			1315	-819	10108	1251	-383	9817
$\widetilde{(cc)u}$			308	-268	3605	283	0	3596
$\widetilde{(bb)u}$			317	-282	10163	301	-4	10123

4 The hyperfine correction

So far, spin averages were taken for the diquark and baryon binding energies. The hyperfine splitting is obtained from the experimental differences between vector and scalar mesons. The cc diquark ($S = 1$) is therefore heavier by $(1/4)\Delta(\psi)/2$ = 113 MeV/8 = 14 MeV. (The extra (1/2) comes from the fact, that the potential in cc colour triplet state is twice weaker than in mesons.) On the other hand, the (cc)u ($S = \frac{1}{2}$) baryon is lighter by $\approx \Delta(D)\ (1870/3552)$ = -74 MeV. (The latter factor takes into account that the spin-spin interaction is inversely proportional to both masses, so instead of the u quark mass in the D meson one takes the (cc) mass. Also, it is convenient that the reduced mass of (cc)u is close to that of D and D_s mesons, so the interpolation is trivial.)

The result for the Ξ_{cc}^{++} mass is then 3545 MeV (Badhuri quark masses) or 3539 MeV (Karliner-Rosner quark masses).

5 A note on the binding energy of the DD* dimeson

We cannot estimate the binding energy of the DD* dimeson in the same way since the $(cc)\bar{u}\bar{b}$ (”tetraquark” or ”atomic” or ” He-like”) configuration is about 100 MeV above the D+D* threshold [3]. This is then only a minor configuration, the main configuration is a DD* ”molecule”, with a covalent bond like the H_2

molecule. In the restricted 4-body space with the two c quarks far apart and a general wavefunction of $\bar{u}$ and $\bar{d}$ the energy is also above the D+D* threshold, as presented by several authors.

Only combining both types of configurations brings the energy below the threshold, as shown by Janc and Rosina [5–7]. In the nonrelativistic calculation with the one-gluon exchange potential (including the chromomagnetic term) plus the linear confining potential they obtain the binding energy (DD*) - (D + D*) = - 2.7 MeV . The model parameters (Grenoble AL1) [8] fitted all relevant mesons and baryons and a rich 4-body space was used (Gaussian expansion at optimized distances, with 3 types of Jacobi coordinates).

We pose an important question ("to be discussed at the next Bled Workshop") whether the pion and sigma clouds between the u and d antiquarks can increase binding, in analogy with the deuteron. Is there a double counting? Would it be necessary to refit the model parameter so much that this extra binding would be compensated? If, however, the binding really becomes much stronger, at least below -6 MeV, the DDπ decay channel would be closed, the DD*system would live longer and would be easier to be recognized in experiment.

6 Conclusion

The phenomenological binding energies of the cc diquark and the Ξ_{cc}^{++} baryon can be obtained by interpolation between the mesonic data. The mass of the Ξ_{cc}^{++} baryon is a lower bound, further corrections (eg. the Coulomb energy and the finite size of the cc diquark) would raise it, possibly close to the experimental value.

It is instructive to see that the final result depends only very weakly on the choice of quark constituent masses. In the binding energy, larger constituent masses (larger by as much as 200 MeV) are compensated by a stronger attractive potential.

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