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6 Phenomenological Mass Matrices With a Democratic Texture

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Abstract. Taking into account all available data on the mass sector, we obtain unitary rotation matrices that diagonalize the quark matrices by using a specific parametrization of the Cabibbo-Kobayashi-Maskawa mixing matrix. The form of the resulting mass matrices is consistent with a democratic scheme with a well-defined, stepwise breaking of the initial flavour symmetry.

Povzetek. Avtorica izbere parametrizacijo mešalne matrike Cabibba, Kobayashija in Maskawe, poišče zanjo unitarne rotacijske matrike, ki pri tej parametrizaciji diagonalizirajo masne matrike kvarkov. Izmerjene mase kvarkov zavrti v startni masni matriki, ki sta skladni z demokratično shemo matrik z dobro definirano in postopno zlomljeno začetno simetrije.

Keywords: Mass matrices, CKM matrix, Democratic texture

6.1 Mass states and flavour states

In this work, we take a very phenomenological approach on the fermion mass matrices, by assuming that the quark mass matrices can be derived from a (naive) factorization of the Cabbibo-Kobayashi-Maskawa (CKM) mixing matrix V [1], which appears in the charged current Lagrangian

$$\mathcal{L}_{cc} = -\frac{g}{2\sqrt{2}}\bar{\varphi}_L\gamma^\mu V\varphi'_L W_\mu + \text{h.c.} \tag{6.1}$$

where φ and φ' are quark fields with charges Q and $Q-1$, correspondingly.

From the perspective of weak interactions, $\mathcal{L}_{cc}$ describes an interaction between left-handed flavour states. From the point of view of all other interactions, the interaction takes place between mixed physical particle states - where "physical particles" refer to mass eigenstates of the mass matrices M and M' appearing in the mass Lagrangian

$$\mathcal{L}_{mass} = \bar{f}Mf + \bar{f}'M'f'$$

where f, f' are fermion flavour states of charge 2/3 and -1/3, respectively, with the corresponing mass matrices denoted as $M = M(2/3)$ and $M' = M'(-1/3)$. Our

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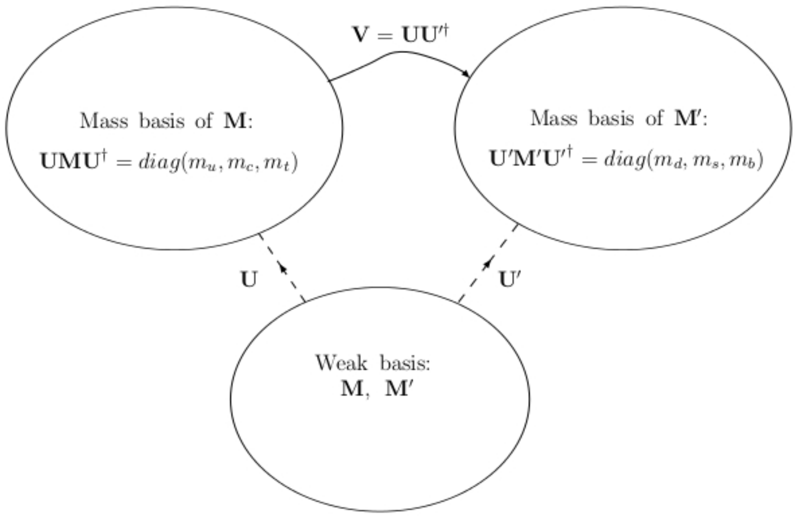
dilemma is in a way how to understand the relation between physical particles and flavour states.

We imagine that all the flavour states live in the same "weak basis" in flavour space, while the mass states of the 2/3-sector and the -1/3-sector live in their separate "mass bases". We go between the weak basis and the mass bases of the two charge sectors by rotating with the unitary matrices U and U', which are factors of the CKM-matrix, $V = UU'^{\dagger}$.

$$M \to UMU^{\dagger} = D = diag(m_u, m_c, m_t) \tag{6.2}$$

$$M' \to U'M'U'^{\dagger} = D' = diag(m_d, m_s, m_b)$$

Since $V \neq \mathbf{1}$, the up-sector mass basis is different from the down-sector mass basis, the CKM matrix thus bridges the two mass bases.



The mass Lagagrangian reads

$$\mathcal{L}_{mass} = \bar{f}Mf + \bar{f}'M'f' = \bar{\psi}D\psi + \bar{\psi}'D'\psi' \tag{6.3}$$

where f, f' are the flavour states and ψ, ψ' are the mass states. We of course know the diagonal mass matrices $D(2/3)$ and $D'(-1/3)$, it is $M(2/3)$ and $M'(-1/3)$ that we are looking for, in the hope that their form can shed light on (the mechanism behind) the mysterious, hierarchical fermion mass spectra.

Whereas the quark mass eigenstates are perceived as "physical", and the weakly interacting flavour states are percieved as mixings of physical particles, in the lepton sector the situation is somewhat different, due to the fact that neutrino mass eigenstates don't ever appear in interactions - they merely propagate in free space. In the realm of neutral leptons it is actually the flavour states ν_e, ν_μ, ν_τ

that we perceive as "physical", since they are the only neutrinos that we "see", as they appear together with the charged leptons. As the charged leptons e, μ, τ are assumed to be both weak eigenstates and mass eigenstates, the only mixing matrix that appears in the lepton sector is the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U which only operates on neutrino states,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{(PMNS)} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

where (ν_1, ν_2, ν_3) are mass eigenstates, and $(\nu_e, \nu_\mu, \nu_\tau)$ are the weakly interacting "flavour states". In the lepton sector, the charged currents are thus interpreted as charged lepton flavours (e, μ, τ) interacting with the neutrino flavour states $(\nu_e, \nu_\mu, \nu_\tau)$.

6.2 Factorizing the weak mixing matrix

The usual procedure in establishing an ansatz for the quark mass matrices is based on some argument or model. Here we follow a rather phenomenaological approach, looking for a factorization of the Cabbibo-Kobayashi-Maskawa mixing matrix, which would give the 'right' mass matrices. The CKM matrix can of course be parametrized and factorized in many different ways, and different factorizations correspond to different rotation matrices U and U', and correspondingly to different mass matrices M and M'.

We choose what we perceive as the most obvious and "symmetric" factorization of the CKM mixing matrix is, following the standard parametrization [2] with three Euler angles α, β, 2θ,

$$V = \begin{pmatrix} c_\beta c_{2\theta} & s_\beta c_{2\theta} & s_{2\theta} e^{-i\delta} \\ -c_\beta s_\alpha s_{2\theta} e^{i\delta} - s_\beta c_\alpha & -s_\beta s_\alpha s_{2\theta} e^{i\delta} + c_\beta c_\alpha & s_\alpha c_{2\theta} \\ -c_\beta c_\alpha s_{2\theta} e^{i\delta} + s_\beta s_\alpha & -s_\beta c_\alpha s_{2\theta} e^{i\delta} - c_\beta s_\alpha & c_\alpha c_{2\theta} \end{pmatrix} = UU'^{\dagger} \tag{6.4}$$

with the diagonalizing rotation matrices for the up- and down-sectors

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} W(\rho) =$$

$$= \begin{pmatrix} c_\theta e^{-i\gamma} & 0 & s_\theta e^{-i\gamma} \\ -s_\alpha s_\theta e^{i\gamma} & c_\alpha & s_\alpha c_\theta e^{i\gamma} \\ -c_\alpha s_\theta e^{i\gamma} & -s_\alpha & c_\alpha c_\theta e^{i\gamma} \end{pmatrix} W(\rho) \tag{6.5}$$

and

$$U' = \begin{pmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{-i\gamma} & & \\ & 1 & \\ & & e^{i\gamma} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix} W(\rho) =$$

$$= \begin{pmatrix} c_\beta c_\theta e^{-i\gamma} & -s_\beta & -c_\beta s_\theta e^{-i\gamma} \\ s_\beta c_\theta e^{-i\gamma} & c_\beta & -s_\beta s_\theta e^{-i\gamma} \\ s_\theta e^{i\gamma} & 0 & c_\theta e^{i\gamma} \end{pmatrix} W(\rho) \tag{6.6}$$

respectively, where $W(\rho)$ is a unitary matrix which is chosen is such a way that γ is the only phase in either of the mass matrices,

$$\begin{pmatrix} 0 & \cos\rho & \pm\sin\rho \\ 1 & 0 & 0 \\ 0 & \mp\sin\rho & \cos\rho \end{pmatrix}, \quad \begin{pmatrix} \cos\rho & 0 & \pm\sin\rho \\ 0 & 1 & 0 \\ \mp\sin\rho & 0 & \cos\rho \end{pmatrix}, \quad \begin{pmatrix} \cos\rho & \pm\sin\rho & 0 \\ 0 & 0 & 1 \\ \mp\sin\rho & \cos\rho & 0 \end{pmatrix}$$

Here ρ is unknown, whereas α, β, θ and γ correspond to the parameters in the standard parametrization, with $\gamma = \delta/2$, $\delta = 1.2 \pm 0.08$ rad, and $2\theta = 0.201 \pm 0.011°$, while $\alpha = 2.38 \pm 0.06°$ and $\beta = 13.04 \pm 0.05°$. In this factorization scheme, α and β are rotation angles operating in the up-sector and the down-sector, respectively.

With the rotation matrices $U(\alpha, \theta, \gamma, \rho)$ and $U'(\beta, \theta, \gamma, \rho)$, we obtain the the up- and down-sector mass matrices

$$M = U^\dagger \mathrm{diag}(m_u, m_c, m_t) U \quad \text{and} \quad M' = U'^\dagger \mathrm{diag}(m_d, m_s, m_b) U',$$

such that

$$M = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{pmatrix} =$$
$$= W^\dagger(\rho) \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ Zs_\theta\, e^{i\gamma} & Y - 2Z\cot 2\alpha & -Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & -Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} W(\rho) \tag{6.7}$$

where $X = m_u$, $Z = (m_t - m_c)\sin\alpha\cos\alpha$ and $Y = m_c \sin^2\alpha + m_t \cos^2\alpha$; and

$$M' = \begin{pmatrix} M'_{11} & M'_{12} & M'_{13} \\ M'_{21} & M'_{22} & M'_{23} \\ M'_{31} & M'_{32} & M'_{33} \end{pmatrix} =$$
$$= W^\dagger(\rho) \begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & Y' + 2Z'\cot 2\beta & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix} W(\rho) \tag{6.8}$$

where $X' = m_b$, $Z' = (m_s - m_d)\sin\beta\cos\beta$ and $Y' = m_d\cos^2\beta + m_s\sin^2\beta$. The two mass matrices thus have similar textures.

From $Y = m_c\sin^2\alpha + m_t\cos^2\alpha$, $Z = (m_t - m_c)\sin\alpha\cos\alpha$, $Y' = m_d\cos^2\beta + m_s\sin^2\beta$ and $Z' = (m_s - m_d)\sin\beta\cos\beta$, we moreover have

$$\begin{aligned} m_u &= X, \quad m_c = Y - Z\cot\alpha, \quad m_t = Y + Z\tan\alpha \\ m_d &= Y' - Z'\tan\beta, \quad m_s = Y' + Z'\cot\beta, \quad m_b = X' \end{aligned} \tag{6.9}$$

6.3 The matrix W

We choose the matrix $W(\rho)$ as

$$W(\rho) = \begin{pmatrix} \cos\rho & -\sin\rho & 0 \\ 0 & 0 & 1 \\ \sin\rho & \cos\rho & 0 \end{pmatrix}, \tag{6.10}$$

which gives the up-sector mass matrix

$$M = W^\dagger \begin{pmatrix} Xc_\theta^2 + Ys_\theta^2 & Zs_\theta\, e^{-i\gamma} & (X-Y)c_\theta s_\theta \\ Zs_\theta\, e^{i\gamma} & Y - 2Z\cot 2\alpha & -Zc_\theta\, e^{i\gamma} \\ (X-Y)c_\theta s_\theta & -Zc_\theta\, e^{-i\gamma} & Xs_\theta^2 + Yc_\theta^2 \end{pmatrix} W =$$

$$= W^\dagger \begin{pmatrix} A & Zs_\theta\, e^{-i\gamma} & H \\ Zs_\theta\, e^{i\gamma} & F & -Zc_\theta\, e^{i\gamma} \\ H & -Zc_\theta\, e^{-i\gamma} & K \end{pmatrix} W =$$

$$= \begin{pmatrix} Ac_\rho^2 + Ks_\rho^2 + H\sin 2\rho & \frac{1}{2}(K-A)\sin 2\rho + H\cos 2\rho & -Ze^{-i\gamma}\sin(\rho-\theta) \\ \frac{1}{2}(K-A)\sin 2\rho + H\cos 2\rho & As_\rho^2 + Kc_\rho^2 - H\sin 2\rho & -Ze^{-i\gamma}\cos(\rho-\theta) \\ -Ze^{i\gamma}\sin(\rho-\theta) & -Ze^{i\gamma}\cos(\rho-\theta) & F \end{pmatrix}, \tag{6.11}$$

With

$A = Xc_\theta^2 + Ys_\theta^2$, $H = (X-Y)c_\theta s_\theta$ and $K = Xs_\theta^2 + Yc_\theta^2$,

we get

$$M = \begin{pmatrix} X\cos^2\mu + Y\sin^2\mu & (Y-X)\sin\mu\cos\mu & -Z\sin\mu\, e^{-i\gamma} \\ (Y-X)\sin\mu\cos\mu & X\sin^2\mu + Y\cos^2\mu & -Z\cos\mu\, e^{-i\gamma} \\ -Z\sin\mu\, e^{i\gamma} & -Z\cos\mu\, e^{i\gamma} & F \end{pmatrix} \tag{6.12}$$

where $\mu = \rho - \theta$, and as before, $X = m_u$, $Z = (m_t - m_c)\sin\alpha\cos\alpha$, $Y = m_c\sin^2\alpha + m_t\cos^2\alpha$, and $F = Xs_\theta^2 + Yc_\theta^2 = Y - 2Z\cot 2\alpha = trace(M) - X - Y$.

Now, depending on the value of $\mu = \rho - \theta$, we get different matrix textures, e.g.

$\mu = \rho - \theta$	0 or π	$\pi/4$	$\pi/2$
$M_{11} = Xc_\mu^2 + Ys_\mu^2$	X	$(X+Y)/2$	Y
$M_{12} = \frac{1}{2}(Y-X)s_{2\mu}$	0	$(Y-X)/2$	0
$M_{13} = -Zs_\mu\, e^{-i\gamma}$	0	$-Ze^{-i\gamma}/\sqrt{2}$	$-Ze^{-i\gamma}$
$M_{22} = Xs_\mu^2 + Yc_\mu^2$	Y	$(X+Y)/2$	X
$M_{23} = -Zc_\mu\, e^{-i\gamma}$	$-Ze^{-i\gamma}$	$-Ze^{-i\gamma}/\sqrt{2}$	0
$M_{33} = F$	$Y - 2Z\cot 2\alpha$	$Y - 2Z\cot 2\alpha$	$Y - 2Z\cot 2\alpha$

So for $\rho - \theta = 0$ or π, we get the simple form

$$M(0,\pi) = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & -Ze^{-i\gamma} \\ 0 & -Ze^{i\gamma} & F \end{pmatrix}, \tag{6.13}$$

and for $\rho - \theta = \pi/2$, equally simple

$$M(\pi/2) = \begin{pmatrix} Y & 0 & -Ze^{-i\gamma} \\ 0 & X & 0 \\ -Ze^{i\gamma} & 0 & F \end{pmatrix} \tag{6.14}$$

Applying the same procedure on the down-sector, we get the down-sector mass matrix

$$M' = W(\rho)^\dagger \begin{pmatrix} X's_\theta^2 + Y'c_\theta^2 & Z'c_\theta\, e^{i\gamma} & (X'-Y')c_\theta s_\theta \\ Z'c_\theta\, e^{-i\gamma} & Y' + 2Z'\cot 2\beta & -Z's_\theta\, e^{-i\gamma} \\ (X'-Y')c_\theta s_\theta & -Z's_\theta\, e^{i\gamma} & X'c_\theta^2 + Y's_\theta^2 \end{pmatrix} W(\rho) =$$

$$= \begin{pmatrix} X'\sin^2\mu' + Y'\cos^2\mu' & (X'-Y')\sin\mu'\cos\mu' & Z'\cos\mu'\, e^{i\gamma} \\ (X'-Y')\sin\mu'\cos\mu' & X'\cos^2\mu' + Y'\sin^2\mu' & -Z'\sin\mu'\, e^{i\gamma} \\ Z'\cos\mu'\, e^{-i\gamma} & -Z'\sin\mu'\, e^{-i\gamma} & F' \end{pmatrix} \tag{6.15}$$

where $\mu' = \rho + \theta$, and as before, $X' = m_b$, $Z' = (m_s - m_d)\sin\beta\cos\beta$, $Y' = m_d\cos^2\beta + m_s\sin^2\beta$, and $F' = Y' + 2Z'\cot 2\beta = trace(M') - X' - Y'$.

Depending on the value of $\mu' = \rho + \theta$, we get different matrix textures.

$\mu' = \rho + \theta$	0 or π	$\pi/4$	$\pi/2$
$M'_{11} = X's^2_{\mu'} + Y'c^2_{\mu'}$	Y'	$(X'+Y')/2$	X'
$M'_{12} = \frac{1}{2}(X'-Y')s_{2\mu'}$	0	$(X'-Y')/2$	0
$M'_{13} = Z'c_{\mu'}\, e^{i\gamma}$	$Z'e^{i\gamma}$	$Z'e^{i\gamma}/\sqrt{2}$	0
$M'_{22} = X'c^2_{\mu'} + Y's^2_{\mu'}$	X'	$(X'+Y')/2$	Y'
$M'_{23} = -Z's_{\mu'}\, e^{i\gamma}$	0	$-Z'e^{i\gamma}/\sqrt{2}$	$-Z'e^{i\gamma}$
$M'_{33} = F'$	$Y' + 2Z'\cot 2\beta$	$Y' + 2Z'\cot 2\beta$	$Y' + 2Z'\cot 2\beta$

So for $\mu' = \rho + \theta = 0$ or π, we get

$$M'(0,\pi) = \begin{pmatrix} Y' & 0 & Z'e^{i\gamma} \\ 0 & X' & 0 \\ Z'e^{-i\gamma} & 0 & F' \end{pmatrix} \tag{6.16}$$

and for $\mu' = \rho + \theta = \pi/2$, we get

$$M'(\pi/2) = \begin{pmatrix} X' & 0 & 0 \\ 0 & Y' & -Z'e^{i\gamma} \\ 0 & -Z'e^{-i\gamma} & F' \end{pmatrix} \tag{6.17}$$

6.4 Texture Zero Mass Matrices

The textures (6.13) and (6.14), as well as (6.16) and (6.17), make us wonder if our scheme implies quark mass matrices of texture zero.

Texture zero matrices can be said to have come about because of the need to reduce the number of free parameters, since the fermion mass matrices are 3x3 complex matrices, which without any constraints contain 36 real free parameters. It is however always possible to perform a unitary transformation that renders an arbitrary mass matrix Hermitian [5], so there is no loss of generality to assume that the mass matrices be Hermitian, reducing the number of free parameters to 18. This is still a very big number, which in the end of the 1970-ies prompted Fritzsch

[4], [6] to introduce "texture zero matrices", mass matrices where a certain number of the entries are zero.

Since then, a huge amount of articles have appeared, with analyses of the very large number of (different types of) texture zero matrices and their phenomenology. In the course of this work, a number of of texture zero matrices has been ruled out, singling out a smaller subset of matrices as viable [7]. Among the texture 4 zero matrices the only matrices that are found to be viable are:

$$\begin{pmatrix} A & B & 0 \\ B^* & D & C \\ 0 & C^* & 0 \end{pmatrix}, \begin{pmatrix} A & B & C \\ B^* & D & 0 \\ C^* & 0 & 0 \end{pmatrix}, \begin{pmatrix} A & 0 & B \\ 0 & 0 & C \\ B^* & C^* & D \end{pmatrix}, \begin{pmatrix} 0 & C & 0 \\ C^* & A & B \\ 0 & B^* & D \end{pmatrix}, \begin{pmatrix} 0 & 0 & C \\ 0 & A & B \\ C^* & B^* & D \end{pmatrix}, \begin{pmatrix} D & C & B \\ C^* & 0 & 0 \\ B^* & 0 & A \end{pmatrix}$$

while

$$\begin{pmatrix} A & 0 & 0 \\ 0 & C & B \\ 0 & B^* & D \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A & 0 & B \\ 0 & C & 0 \\ B^* & 0 & D \end{pmatrix}$$

are among the matrices that are ruled out. In our scheme this precisely corresponds to the matrices (6.13), (6.14), (6.16) and (6.17), which means that our mass matrices M and M' are not of texture zero. This can be expressed as a constraint on the values of the angle ρ,

$$\rho \neq \frac{1}{2}N\pi \pm \theta \tag{6.18}$$

where $N \in \mathcal{Z}$, ruling out the matrices $M(\frac{1}{2}N\pi - \theta)$ and $M'(\frac{1}{2}N\pi + \theta)$, so our mass matrices M and M' are not of texture zero. Instead, they display a democratic texture.

6.5 Democratic mass matrices

Initially, we were looking for mass matrices with a democratic structure [3], where the assumption is that both the up- and down-sector mass matrices start out from a form of the type $M_0 = k\mathbf{N}$ and $M'_0 = k'\mathbf{N}$ where

$$\mathbf{N} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The underlying philosophy is that in the Standard Model, where the fermions get their masses from the Yukawa couplings by the Higgs mechanism, there is no reason why there should be a different Yukawa coupling for each fermion. The couplings to the gauge bosons of the strong, weak and electromagnetic interactions are identical for all the fermions in a given charge sector, it thus seems like a natural assumption that they should also have identical Yukawa couplings. The difference is that the weak interactions take place in a specific flavour space basis, while the other interactions are flavour independent.

A matrix of the form $M = k\mathbf{N}$ moreover has the mass spectrum $(0, 0, 3k)$, reflecting the phenomenology of the fermion mass spectra with one very big, and two much smaller mass values. In the weak basis $M = k\mathbf{N}$ is however totally flavour symmetric, which means that the (weak) flavours f_i are indistinguishible ("absolute democracy").

In the assumed initial stage, since the up-sector mass matrix and the down sector mass matrix are identical except for the dimensional coefficients k and k', the mixing matrix is equal to unity, so there is no CP-violation. In order to obtain the final mass spectra with the three hierarchical non-zero values, the initial flavour symmetry displayed by the matrices M_0 and M_0' must be broken, in such a way that the mixing matrix becomes the observed CKM matrix (with a CP-violating phase).

An "ansatz" within the democratic scenario then consists of a specific choice of a flavour symmetry breaking scheme. And it is precisely what we are looking for: a credible flavour symmetry breaking scheme that gives the observed mass spectra.

Our initial assumption is that the rotation matrices (6.5), (6.6) which diagonalize the up-sector and down-sector mass matrices, are given by the factorization of the Cabibbi-Koabayashi-Maskawa matrix (6.4), with well-known angles. The only "steering-parameter parameter" is then ρ, in the sense that different values of ρ correspond to mass matrices of different form.

6.5.1 A democratic substructure

We now reparametrize the mass matrices (6.12) and (6.15),

$$M = \begin{pmatrix} Xc_\mu^2 + Ys_\mu^2 & (Y-X)s_\mu c_\mu & -Zs_\mu\, e^{-i\gamma} \\ (Y-X)s_\mu c_\mu & Xs_\mu^2 + Yc_\mu^2 & -Zc_\mu\, e^{-i\gamma} \\ -Zs_\mu\, e^{i\gamma} & -Zc_\mu\, e^{i\gamma} & F \end{pmatrix}$$

and

$$M' = \begin{pmatrix} X's_{\mu'}^2 + Y'c_{\mu'}^2 & (X'-Y')s_{\mu'}c_{\mu'} & Z'c_{\mu'}\, e^{i\gamma} \\ (X'-Y')s_{\mu'}c_{\mu'} & X'c_{\mu'}^2 + Y's_{\mu'}^2 & -Z's_{\mu'}\, e^{i\gamma} \\ Z'c_{\mu'}\, e^{-i\gamma} & -Z's_{\mu'}\, e^{-i\gamma} & F' \end{pmatrix},$$

in a way that reveals their "democratic substructure":

$$M = \begin{pmatrix} P & & \\ & R & \\ & & Se^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P & & \\ & R & \\ & & Se^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & Q \end{pmatrix} \tag{6.19}$$

and

$$M' = \begin{pmatrix} P' & & \\ & R' & \\ & & S'e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P' & & \\ & R' & \\ & & S'e^{i\gamma} \end{pmatrix} + \begin{pmatrix} X' & & \\ & X' & \\ & & Q' \end{pmatrix} \tag{6.20}$$

where

$$P = \sqrt{|Y-X|}\sin(\rho-\theta), \quad R = \sqrt{|Y-X|}\cos(\rho-\theta), \quad S = \frac{-Z}{\sqrt{|Y-X|}}, \quad Q = F - S^2,$$

and

$P' = \sqrt{|Y' - X'|}\cos(\rho + \theta)$, $R' = -\sqrt{|Y' - X'|}\sin(\rho + \theta)$, $S' = \frac{Z'}{\sqrt{|Y'-X'|}}$, $Q' = F' - S'^2$.

These matrices can in their turn be rewritten as

$$M = B \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\mu & & \\ & \cos\mu & \\ & & Ge^{-i\gamma} \end{pmatrix} + \begin{pmatrix} X & & \\ & X & \\ & & Q \end{pmatrix} \quad (6.21)$$

where

$$\mu = \rho - \theta, \quad B = Y - X, \quad G = -Z/(Y - X), \quad Q = F - BG^2.$$

Likewise,

$$M' = B' \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G'e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \cos\mu' & & \\ & -\sin\mu' & \\ & & G'e^{i\gamma} \end{pmatrix} + \begin{pmatrix} X' & & \\ & X' & \\ & & Q' \end{pmatrix} \quad (6.22)$$

where

$$\mu' = \rho + \theta, \quad B' = Y' - X', \quad G' = Z'/(Y' - X'), \quad Q' = F' - B'G'^2.$$

So without any assumptions about an initial democratic texture, we get a mass matrix structure that can be interpreted as originating from a democratic mass matrix, where the flavour symmetry has subsequently been broken in a very specific manner.

6.6 Flavour symmetry breaking mechanisms

The goal of our investigation is to get some hint about the form that the quark mass matrices take in the weak basis - and the hint we get from the matrices (6.21) and (6.22) is that the mass matrices come about from a kind of democratic scenario where the initial flavour symmetry is broken in a stepwise fashion.

Flavour symmetries relate the different flavours f_j, and in the democratic scenario, where the initial form of the mass matrices is taken to be

$$M_0 = k\mathbf{N} = k \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (6.23)$$

the mass Lagrangian reads

$$\mathcal{L}_{mass} = k\bar{f}\mathbf{N}f = \sum_{i=1,j=1}^{3} k\, \bar{f}_i f_j$$

This means that in the democratic scheme, all the flavours f_j are initially indistinguishible, with the same Yukawa coupling for all the flavours: a totally flavour symmetric situation.

Following the hint given by our approach, we now postulate that the mass matrices originate from a democratic form (6.23), and that the initial overall flavour symmetries have subsequently undergone a stepwise breaking. To show how this works, we start with a generic matrix M_0, and take the first symmetry breaking step to be

$$M_0 = k\mathbf{N} \to M_1 = \begin{pmatrix} E & & \\ & E & \\ & & J \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} E & & \\ & E & \\ & & J \end{pmatrix} \tag{6.24}$$

Here the mass spectrum is basically unchanged even though the flavour symmetry is partially broken, with the mass Lagrangian

$$\mathcal{L}_{mass} = k\bar{f}M_1 f = E^2\bar{\chi}\chi + EJ(\bar{\chi}f_3 + \bar{f}_3\chi) + J^2\bar{f}_3 f_3$$

where $\chi = f_1 + f_2$; thus the flavour symmetry $f_1 \Leftrightarrow f_2$ is still unbroken. In the next step, we lift the remaining flavour symmetry by rotating the two equal terms,

$$(E, E) \to (L\sin\eta, L\cos\eta),$$

which gives

$$M_1 = k\mathbf{N} \to M_2 = L^2 \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & T \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & T \end{pmatrix}, \tag{6.25}$$

where L^2 is the only dimensional parameter, and $T = J/L$. In order to account for CP-violation, we moreover introduce a phase γ, in a way that reflects that CP-violation is connected to the presence of three families (with only two families there is no CP-violation):

$$M_2 \to M_3 = L^2 \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & Te^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & Te^{-i\gamma} \end{pmatrix}, \tag{6.26}$$

where the CP-breaking phase is connected to the third family, as it should. We know nothing about the values of L, η, T, but by the assumption that the trace of the mass matrix is constant through all the flavour symmetry breaking steps, we get

$$L^2 + T^2 = 3k$$

But the matrix M_3 still has determinant zero, and a mass spectrum with two vanishing and one non-zero mass value. We therefore add an extra term to M_3, which like in (6.21) and (6.22), is of diagonal form. This gives us the final mass matrix

$$M_3 \to M_4 = L^2 \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & Te^{i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \sin\eta & & \\ & \cos\eta & \\ & & Te^{-i\gamma} \end{pmatrix} + \Lambda \tag{6.27}$$

where Λ is a diagonal matrix.

Our scheme thus reads:

- We start with the democratic matrix $M_0 = k\mathbf{N}$, $k = Trace(M)/3$, with total flavour symmetry $f_1 \Leftrightarrow f_2 \Leftrightarrow f_3$ in the weak basis.
- Assumption: the trace of the matrix M is constant throughout every flavour symmetry breaking step.
- First flavour breaking step (6.24): $M_0 \rightarrow M_1$. The flavour symmetry $f_1 \Leftrightarrow f_2$ still remains, and there is still only one non-zero mass value, but f_3 is singled out.
- Next flavour breaking step (6.25): $M_1 \rightarrow M_2$, lifting the flavour symmetry $f_1 \Leftrightarrow f_2$.
- Introducing a CP-violating phase (6.26): $M_2 \rightarrow M_3$.
- Last step (6.27): adding a diagonal matrix to M_3, $M_3 \rightarrow M_4 = M_3 + \Lambda$, whereby we get the three observed non-zero mass values.

6.7 Conclusion

Without introducing any new assumptions, by just factorizing the "standard parametrization" of the CKM weak mixing matrix in a specific way, we obtain mass matrices with a specific type of democratic texture and a well-defined scheme for breaking the initial flavour symmetry. Our approach thus hints at a democratic scenario, which comes from the formalism without any other assumptions than a very natural and straightforward way of factorizing the weak mixing matrix.

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