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CALCULATION OF THE LUBRICANT LAYER FOR A COARSE SURFACE OF A BAND AND ROLLS

IZRAČUN SLOJA MAZIVA NA GROBI POVRŠINI TRAKU IN VALJEV

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The effect of the average roughness of a lubricated band caused by dressing processes is analysed by applying the Reynolds differential equation for lubrication with the incorporated average roughness and evolution in the Fourier series to the third member. The analysis has shown that the average roughness has two effects on the lubricant-layer thickness in the entering section of the deformation zone. For a small surface roughness, the nominal lubricant-layer thickness decreases slowly (if the process is treated as occurring on a smooth surface) and the thickness grows again with an increase in the roughness. The basis for the analysis was the numerical Monte-Carlo method and the developed approximate analytical solution was in acceptable agreement with the numerical method.

Keywords: surface roughness, lubricant-layer thickness, Reynolds equation, Monte-Carlo method, Fourier series

Analiziran je vpliv povprečne hrapavosti mazanega traku pri procesih dresiranja. Podlaga analize je Reynoldsova diferencialna enačba za mazanje z vključeno povprečno hrapavostjo in obravnavo s Fourierovo vrsto do tretjega člena. Analiza je pokazala, da ima povprečna hrapavost dva učinka na debelino plasti maziva v vhodnem preseku področja deformacije. Pri majhni začetni hrapavosti se nominalna debelina plasti maziva počasi zmanjšuje (če se proces obravnava, kot da poteka na gladki površini) in znova raste, če se povečuje hrapavost. Podlaga za analizo je bila numerična metoda Monte Carlo, razvita pa je bila tudi približna analitična rešitev, ki se zadovoljivo ujema z numerično.

Ključne besede: hrapavost površine, debelina plasti maziva, Reynoldsova enačba, metoda Monte Carlo, Fourierova vrsta

1 INTRODUCTION

This technology is strongly associated with the quality of technological lubricants as it:

- diminishes the contact friction,
- removes the heat, cools the tool and diminishes the wear,
- diminishes the deformation resistance and the deformation work,
- diminishes the sticking to the tool and keeps the surface of the product clean.

The basic groups examined in this work[1–3] are:

- liquid emulsions,
- fats and compounds,
- consistent lubricants,
- transparent/glass lubricants,
- powder lubricants and
- metallic lubricants.

Technological lubricants must meet a series of requirements, beginning with a high lubricity – the ability to form a flat, firm layer separating the contact surfaces – then there are thermal consistency and stability that prevent the damaging effect of the product corrosion, the properties not posing any health and environmental risks, etc.

The liquid emulsions, whose compounds are mixtures of vegetable and mineral oils, are especially used in the cold rolling of 0.3–0.4 mm thick sheets and strips.

In the cold rolling of sheets and strips, the dressing process is also used with an application of liquid lubricants to reduce undulation.

2 MATHEMATICAL MODELLING

Mathematical modelling is a requirement of today's metallurgy[4,5] and it is also used in the field of plastic deformation of metals. For an analysis of smooth surfaces[6,7] the following equation is used:

$$\frac{\mathrm{d}p}{\mathrm{d}x} = \frac{6\mu(v_0 + v_\mathrm{R})}{\varepsilon^2(x)} - \frac{12\mu Q}{\varepsilon^2(x)} \tag{1}$$

$$Q(x) = \int_0^{\varepsilon(x)} u\,dy = \frac{1}{12\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\varepsilon^3(x) + \left(\frac{v_0 + v_\mathrm{R}}{2}\right)\varepsilon(x) \tag{2}$$

The geometry of the lubricant contact[8] and the length of the lubricant wedge are described with the relations (3), (4) and (5):

$$\varepsilon(x) = \varepsilon_0 + R\left[\cos\alpha - \sqrt{1 - \left(\sin\alpha - \frac{x}{R}\right)^2}\,\right] \tag{3}$$

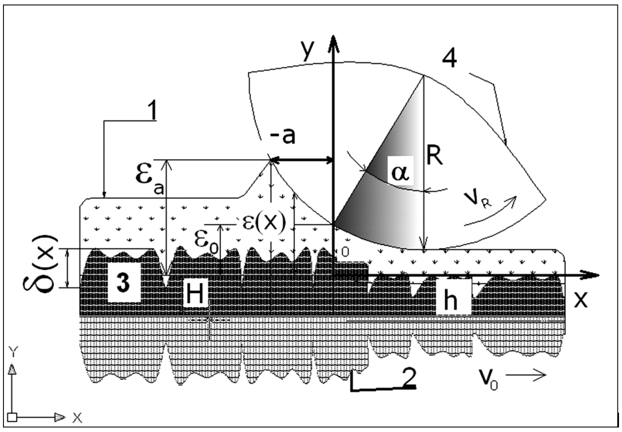

Figure 1: Model of the tribomechanical system; 1- lubricant layer $\varepsilon(x)$ – nominal thickness for smooth surfaces, 2- band – in dressing processes the adhering angle α is low, 3- average band roughness $\delta(x)$ – casual sheet roughness, 4- roll defined by surface smoothness. In **Table 1** the roughness is R_z = 8 µm.

Slika 1: Model tribomehanskega sistema; 1- plast maziva $\varepsilon(x)$ – nominalna debelina za gladke površine, 2- trak – pri procesu dresiranja pri majhnem kotu stika α, 3- povprečma hrapavost traku $\delta(x)$ – slučajna hrapavost traku, 4- valj z definirano gladkostjo površine. V **Tabeli 1** je njegova hrapavost R_z = 8 µm.

$$a = R\left[\sqrt{1-\left(\cos\alpha-\frac{\varepsilon_a}{R}+\frac{\varepsilon_0}{R}\right)^2}-\sin\alpha\right] \qquad (4)$$

$$\varepsilon(x)=\varepsilon_0-\alpha x+\frac{x^2}{2R}-\frac{\alpha x^3}{2R^2}+\frac{x^4}{8R^3} \qquad (5)$$

For the average sheet roughness[9], the mathematical relation in accordance with **Figure 1** is:

$$\left\langle\frac{\mathrm{d}p}{\mathrm{d}x_0}\right\rangle=6\mu(v_0+v_R)\left[\left\langle\frac{1}{\varepsilon^2(x_0)}\right\rangle-\frac{\left\langle\frac{1}{\varepsilon_0^2}\right\rangle}{\left\langle\frac{1}{\varepsilon_0^3}\right\rangle}\left\langle\frac{1}{\varepsilon^3(x_0)}\right\rangle\right] \qquad (6)$$

$$\left\langle\varepsilon(x_0)\right\rangle=\varepsilon(x)+\delta(x) \qquad (7)$$

Reflexion of sheet roughness is added, as ε_0, to the lubricant wedge (4). The calculation is possible only with numerical mathematical methods and, in the program MATHEMATICA, the numerical method Monte Carlo was used. In the theoretical calculations regarding the model of the average roughness, the following function developed to the third term of Fourier series was applied:

$$\delta(x)=\frac{4}{\pi}\left(\sin x+\frac{1}{3}\sin 3x+\frac{1}{5}\sin 5x\right)R_z \qquad (8)$$

3 RESULTS AND DISCUSSION

In **Table 1** the standard values of geometrical, rheological and kinematic characteristics of the processes of theoretical investigations are given according to the Russian-Ukrainian[10,11] authors.

Table 1: Standard lubricant characteristics for theoretical calculations
Tabela 1: Standardne značilnosti maziva za teoretične izračune

Parameter	Value	Unit
γ- piezo coefficient of viscosity	2.18E-7	Pa^{-1}
p_0- rolling pressure	20E6	Pa
v_R- circumferential roll speed	10	m/s
v_0- sheet speed	6	m/s
R- roll radius	0.35 (0.25)	m
μ_0- lubricant dynamic viscosity $\mu = \mu_0 \exp(\gamma * p_0)$ Barussa equation	0.024–0.048	Pa s
α- gripping angle	0–0.02	rad
ε_a- lubricant thickness on sheet	0.001–0.00001	m
A- technological parameter	1965512 (3934525)	m^{-1}
$A = (1-\exp(-\gamma * p_0)/6\mu_0\gamma(v_0+v_R))$		
$R_z \approx 6\delta$	R_z = 1–10	µm

The parameters in **Table 1** are of two groups:

1- lubricant rheological characteristics (μ_0, γ)

2- geometrical characteristics of the technological process (R, α, R_z)

3- kinematics (v_0, v_R)

Table 2: Lubricant-layer exit results (µm)
Tabela 2: Izhodni rezultati za plast maziva (µm)

	0	1	2	3	4	5	6	7
0	10.102	10.095	10.087	10.075	10.059	10.039	9.92	9.207
1	10.003	9.996	9.988	9.976	9.959	9.934	9.819	9.099
2	9.941	9.935	9.926	9.914	9.898	9.872	9.757	9.031
3	9.92	9.914	9.905	9.893	9.876	9.851	9.735	9.008
4	9.94	9.933	9.925	9.913	9.896	9.87	9.755	9.03
5	9.998	9.991	9.983	9.971	9.954	9.929	9.814	9.093
6	10.09	10.083	10.075	10.063	10.046	10.021	9.907	9.193
7	10.21	10.204	10.195	10.184	10.167	10.143	10.03	9.322
8	10.354	10.348	10.339	10.328	10.312	10.287	10.175	9.475
9	10.517	10.51	10.502	10.491	10.474	10.45	10.339	9.645
10	10.693	10.686	10.678	10.667	10.651	10.627	10.517	9.828
11								
12								

4- compounds (A, roughness space angle)

The solutions of differential equation (6) are partially given in **Table 2**.

The examined roughness is classified[12,13] in 10 vertical classes and the band profile roughness in 32 horizontal classes.

In principle, with a decreasing band-lubricant thickness (ε_a in **Figure 1**) the lubricant thickness in the entering section of the metal deformation zone is also decreased (ε_0). As shown in[14], the lubricant wedge has the ideal geometry and can give economic savings of the lubricant in the metalworking technology.

The numerical integration of equation (6) was checked with the approximate[15–17] analytical solutions possible in the case of practical interest, which is found in equations (9), (10a)–(10e) and (11). Equation (9) is the simplest analytical solution that does not consider the thickness of the band lubricant layer, $\varepsilon_a >> \varepsilon_0$. With a clear complexity, equation (11) corrects this deficiency:

$$315AR^3\alpha^7 - 168R^2\alpha^4 - 1824\delta^2 = 0$$
$$\varepsilon_0 = 0.5R\alpha^2 \ldots A = \frac{1 - exp(-\gamma p_0)}{6\mu\gamma(v_0 + v_R)} \qquad (9)$$

$$W_1 = A - \left[\frac{4}{3\alpha^3 R} - 4\frac{R^2}{3\left[R\sqrt{1-\left(\cos\alpha - \frac{\varepsilon_a}{R} + \frac{\varepsilon_0}{R}\right)^2} - R(\sin\alpha - \alpha)\right]^3} \right] \qquad (10a)$$

$$-W_2 = 3\delta^2 \left[\frac{16}{7\alpha^7 R^3} - 16\frac{R^4}{3\left[R\sqrt{1-\left(\cos\alpha - \frac{\varepsilon_a}{R} + \frac{\varepsilon_0}{R}\right)^2} - R(\sin\alpha - \alpha)\right]^7} \right] \qquad (10b)$$

$$W_3 = \frac{\varepsilon_0^3 + 3\varepsilon_0\delta^2}{\varepsilon_0^2 + 6\delta^2} \qquad (10c)$$

$$W_4 = \frac{8}{5\alpha^5 R^2} - 8\frac{R^3}{5\left[R\sqrt{1-\left(\cos\alpha - \frac{\varepsilon_a}{R} + \frac{\varepsilon_0}{R}\right)^2} - R(\sin\alpha - \alpha)\right]^5} \qquad (10d)$$

$$-W_5 = 32\frac{R^5}{5\left[R\sqrt{1-\left(\cos\alpha - \frac{\varepsilon_a}{R} + \frac{\varepsilon_0}{R}\right)^2} - R(\sin\alpha - \alpha)\right]^9} \qquad (10e)$$

$$W_1 + W_2 + W_3 * (W_4 + W_5) = 0 \qquad (11)$$

In **Table 3** approximate numerical and analytical solutions are compared. The approximate numerical solutions can be compared with numerical integration only for the entering roughness profile, thus, at the entering section of the deformation zone with $x = 0$.

It is clear from **Table 3** that the simple analytical form of equation (9) with numerous approximations describes well the lubricant layer for the case of a lubricant excess on the sheet and the rolls.

Table 3: Comparison of approximate analytical and numerical Monte-Carlo solutions for one point of the graph crossing from **Figure 2**

Tabela 3: Primerjava približnih analitičnih in numeričnih rešitev Monte Carlo za eno točko prereza grafa na **sliki 2**

Case conditions	Approximate analytical solutions, eq. (11) and (9)	Monte-Carlo method, eq. (6)
$x = 0$ (initial roughness profile) $R_z = 1$ μm $R_z \approx 6\ \delta$ $\alpha = 0.00918759$ rad $A = 1965512$ m^{-1} $R = 0.35$ m	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 14.721$ μm (11) $\varepsilon_0 = 14.771$ μm (9) $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 13.834$ μm (11)	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 14.772$ μm $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 13.761$ μm
$x = 0$ (initial roughness profile) $R_z = 10$ μm $R_z \approx 6\ \delta$ $\alpha = 0.0092867$ rad $A = 1965512$ m^{-1} $R = 0.35$ m	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 15.024$ μm (11) $\varepsilon_0 = 15.092$ μm (9) $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 13.511$ μm (11)	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 15.077$ μm $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 13.429$ μm
$x = 0$ (initial roughness profile) $R_z = 10$ μm $R_z \approx 6\ \delta$ $\alpha = 0.00840867$ rad $A = 3934525$ m^{-1} $R = 0.25$ m	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 8.776$ μm (11) $\varepsilon_0 = 8.838$ μm (9) $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 8.464$ μm (11)	$\varepsilon_a = 0.001$ m $\varepsilon_0 = 8.755$ μm $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 8.429$ μm

Table 4: Effect of the two-sided roughness of the sheet and rolls, congruous for ε_0

Tabela 4: Vpliv dvostranske hrapavosti traku in valjev, kongruenten za ε_0

$x = 0$ (initial roughness profile) $R_z = 10$ μm, average roughness, horizontal (transversal) $R_z = 8$ μm, longitudinal roll roughness $R_z \approx 6\ \delta$ (GOST 2789-73) $\alpha = 0.00840867$ rad $A = 3934525$ m^{-1} $R = 0.25$ m	Monte-Carlo method $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 9.299$ μm $\varepsilon_0 = 8.429$ μm	
	One-sided roughness of the sheet $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 8.429$ μm	Two-sided roughness of the sheet and roll $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 8.919$ μm
	$R_z \rightarrow 0$ $\varepsilon_a = 0.0001$ m $\varepsilon_0 = 7.877$ μm	

The longitudinal band profile on abscissa in shown in 66 classes and on ordinate in 11 classes for roughness (0–10 μm). It is useful to calculate the lubricant thickness ε_0 in the range of 8.5–12.5 μm in the area of I-I. Q, K and W designations connect the specific areas of the network diagram with the contour plot (an aircraft picture of the network diagram).

B and C are the left and right sides of the band roughness defined as a sine evolution function in the Fourier series: B in the range of (π–2π) rad and C in (0–π) rad.

Line P in **Figure 2** represents the nominal lubricant-layer thickness on side C, thus, by having the thickness for $R_z \approx 6$ μm, an equivalent to the lubricant-layer thickness on a smooth surface is obtained. Side B does not have this property.

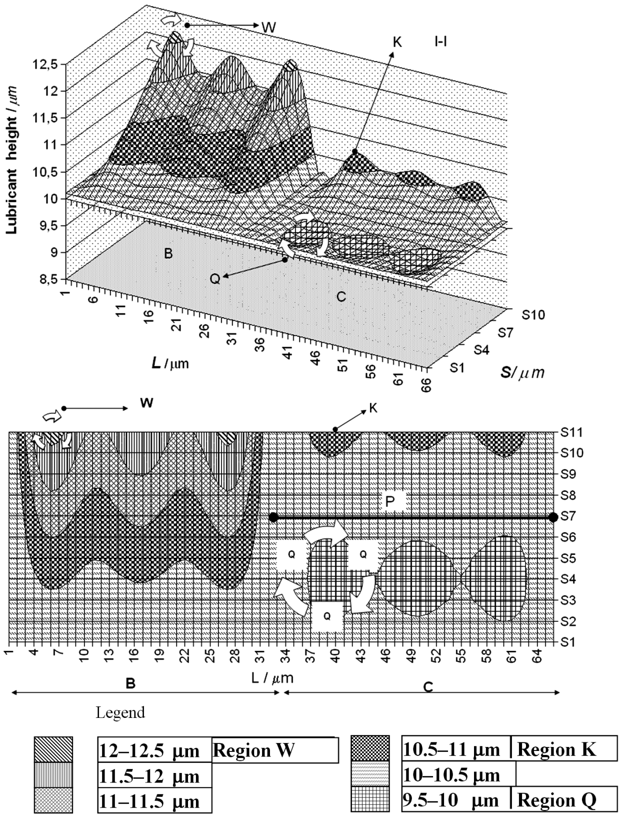

Figure 2: Effect of the average sheet roughness and smooth rolls on ε_0
Slika 2: Vpliv povprečne hrapavosti traku in gladkih valjev na ε_0

In **Figure 3** both sides of the roll longitudinal roughness C from **Figure 2** are shown. The average roughness conserves the same properties as in **Figure 2**. The longitudinal roughness profile in the range of classes 33 to 66 gives a more stable hydrodynamic lubrication, while for classes 1 to 33 the hydrodynamic lubrication is already seriously impaired by the low roughness of the band and rolls. The lubricant layer decreases rapidly and spreads to fractal areas. A stable lubrication can be achieved on small band segments and around class 4 of the longitudinal sheet profile and around classes 10 and 30. The complex shapes of the lubrication space are probably determined by the band and roll roughness in the entering section of the deformation zone that determines a different lubrication layer than in the case of smooth-sheet and roll surfaces.

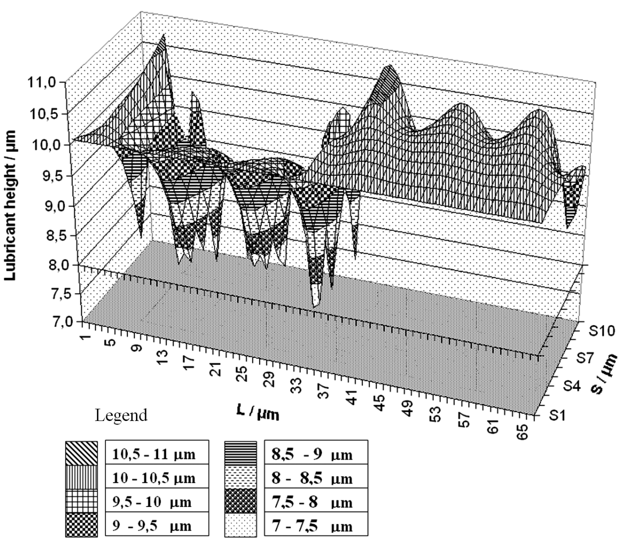

Figure 3: Effect of the average sheet roughness and longitudinal roll roughness on ε_0 (**Table 4**)
Slika 3: Vpliv povprečne hrapavosti traku in longitudinalne hrapavosti valjev na ε_0 (**tabela 4**)

4 CONCLUSIONS

Based on the results of theoretical analyses of the effect of the band roughness on the lubrication dressing processes, the following conclusions are proposed:

- The average band roughness has a critical value when it starts to affect positively the lubricant layer with its increase in comparison with a smooth surface. Up to line P in **Figure 2**, the lubricant layer has a tendency to increase and to decrease the formation of sunk baskets in area *Q*. The theoretical explanation for this is that the surface roughness determines the shape of the lubricant layer for every value of R_z. This is the range of a stable lubrication.
- If congruous roll roughness is added to the average band roughness, forming a longitudinal roll roughness with the positive side in the range of (0–π), the thickness of the lubricant layer in the entering section of the band deformation zone will increase its longitudinal profile from class 33 to 66 (**Figure 3** and **Table 4**) and will approach the boundary lubrication.
- The developed approximate analytical solutions agree with the numerical integration of equation (6) and ensure a reliable approach to the analysis.
- If the technological process was performed with a nominal lubricant-layer thickness marked with line P in **Figure 2** the best roll rhythm would be obtained without significant fluctuations of the lubricant thickness, especially in the case of the boundary-lubrication proximity. This includes the control of the roll roughness.

5 SYMBOLS AND FIGURES

Symbol	Unit	Comment
ε_0	m, (μm)	Lubricant thickness in the entering section of the deformation zone (**Figure 1**)
$\varepsilon(x)$	m	Lubricant thickness in the range of [–a : 0], **Figure 1**, equations (3) and (5)
ε_a	m	Lubricant thickness ahead of the entering section of the deformation zone
a	m	Length of the lubricant wedge (**Figure 1**), equation (4)
α	rad	Band dressing angle
v_R	m/s	Circumferential roll speed

v_T	m/s	Mandrel speed
R	m	Roll radius
R_z	m	Roughness of the band surface, equation (8)
δ^2		Dispersion roughness of the sheet and rolls according to equation (9)
δ_x		Casual lubricant thickness depending on the band roughness (and rolls)
$< >$		Operative mathematical expectation
x, y		Descartes coordinates
$Q(x)$	–	Volume use of lubricant (on the band perimeter)
μ_0	Pa s	Lubricant dynamic viscosity by the rolling pressure
μ	Pa s	Lubricant dynamic viscosity by the air pressure
u	m/s	Lubricant rate on the abscissa
γ	m^2/N	Piezo coefficient of lubricant viscosity
p	Pa	Rolling pressure
Q	m^2/s	Use of lubricant on the mandrel perimeter – a one-dimensional model
dp/dx	Pa/m	Pressure gradient in the lubricant layer, equation (1)
sin α	rad	Marking the trigonometric function for the griping alpha angle
H	m	Enter band thickness
h	m	Exit band thickness
A	m^{-1}	Technological parameter: $A = [1-\exp(-\gamma p)] / [6\mu_0\gamma(v_R + v_0)]$
exp, π [14] 1 μm	2.718 1–1 10^{-6} m	Base of natural logarithm (3.141) Reference Micrometre
S	μm	Band- and roll-roughness classes
L	μm	Longitudinal holding-band profile
Q, K, W		Markers for **Figure 2**

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