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3 Revisiting Trace Anomalies in Chiral Theories

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Abstract. This is a report on work in progress about gravitational trace anomalies. We review the problem of trace anomalies in chiral theories in view of the possibility that such anomalies may contain not yet considered CP violating terms. The research consists of various stages. In the first stage we examine chiral theories at one-loop with external gravity and show that a (CP violating) Pontryagin term appears in the trace anomaly in the presence of an unbalance of left and right chirality. However the imaginary coupling of such term implies a breakdown of unitarity, putting a severe constraint on such type of models. In a second stage we consider the compatibility of the presence of the Pontryagin density in the trace anomaly with (local) supersymmetry, coming to an essentially negative conclusion.

Povzetek. To je poročilo o raziskavah gravitacijskih slednih anomalij. Pri tem nas posebej zanima, kaj lahko sledne anomalije prispevajo h kršitvi simetrije CP. Najprej obravnavamo kiralne teorije v prisotnosti (zunanjega) gravitacijskega polja v enozančnem približku. Pokažemo, da se v sledni anomaliji pojavi Pontrjaginov člen, ta krši CP simetrijo, kadar število levoročnih in desnoročnih brezmasnih delcev ni v ravnovesju. Vendar modeli z imaginarno sklopitvijo takega člena s poljem niso unitarni. V drugem koraku obravnavamo skladnost Pontrjaginove gostote v sledni anomaliji v modelih z lokalno supersimetrijo in to možnost v bistvu zavrnemo.

3.1 Introduction

We revisit trace anomalies in theories coupled to gravity, an old subject brought back to people's attention thanks to the importance acquired recently by conformal field theories both in themselves and in relation to the AdS/CFT correspondence. What has stimulated specifically this research is the suggestion by [1] that trace anomalies may contain a CP violating term (the Pontryagin density). It is well known that a basic condition for baryogenesis is the existence of CP nonconserving reactions in an early stage of the universe. Many possible mechanisms for this have been put forward, but to date none is completely satisfactory. The appearance of a CP violating term in the trace anomaly of a theory weakly coupled to gravity may provide a so far unexplored new mechanism for baryogenesis.

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Let us recall that the energy-momentum tensor in field theory is defined by $T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}}$. Under an infinitesimal local rescaling of the matrix: $\delta g_{\mu\nu} = 2\sigma g_{\mu\nu}$ we have

$$\delta S = \frac{1}{2} \int d^4x \sqrt{-g} T_{\mu\nu} \delta g^{\mu\nu} = - \int d^4x \sqrt{-g} \sigma T_\mu{}^\mu.$$

If the action is invariant, classically $T_\mu{}^\mu = 0$, but at one loop (in which case S is replaced by the one-loop effective action W) the trace of the e.m. tensor is generically nonvanishing. In D=4 it may contain, in principle, beside the Weyl density (square of the Weyl tensor)

$$\mathcal{W}^2 = \mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2 \tag{3.1}$$

and the Gauss-Bonnet (or Euler) one,

$$E = \mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 4\mathcal{R}_{nm}\mathcal{R}^{nm} + \mathcal{R}^2, \tag{3.2}$$

another nontrivial piece, the Pontryagin density,

$$P = \frac{1}{2} \left(\epsilon^{nmlk} \mathcal{R}_{nmpq} \mathcal{R}_{lk}{}^{pq} \right) \tag{3.3}$$

Each of these terms appears in the trace with its own coefficient:

$$T_\mu{}^\mu = aE + c\mathcal{W}^2 + eP \tag{3.4}$$

The coefficient a and c are known at one-loop for any type of matter. The coefficient of (3.3) has not been sufficiently studied yet. The purpose of this paper is to fill up this gap. The plan of our research consists of three stages. To start with we analyse the one loop calculation of the trace anomaly in chiral models. Both the problem and the relevant results are not new: the trace anomaly contains beside the square Weyl density and the Euler density also the Pontryagin density. What is important is that the e coefficient is purely imaginary. This entails a violation of unitarity at one-loop and, consequently, introduces an additional criterion for a theory to be acceptable. The latter is similar to the analogous criterion for chiral gauge and gravitational anomalies, which is since long a selection criterion for acceptable theories. A second stage of our research concerns the compatibility between the appearance of the Pontryagin term in the trace anomaly and supersymmetry. Since it is hard to supersymmetrize the above three terms and relate them to one another in a supersymmetric context, the best course is to consider a conformal theory in 4D coupled to (external) $N = 1$ supergravity formulated in terms of superfields and find all the potential superconformal anomalies. This will allow us to see whether (3.3) can be accommodated in an anomaly supermultiplet as a trace anomaly member. The result of our analysis seems to exclude this possibility. Finally, a third stage of our research is to analyse the possibility that the Pontryagin density appears in the trace anomaly in a nonperturbative way, for instance via an AdS/CFT correspondence as suggested in [1].

In this contribution we will consider the first two issues above. In the next section we will examine the problem of the one-loop trace anomaly in a prototype chiral theory. Section 3.3 is devoted to the compatibility of the Pontryagin term in the trace anomaly with supersymmetry.

3.2 One-loop trace anomaly in chiral theories

The model we will consider is the simplest possible one: a left-handed spinor coupled to external gravity in 4D. The action is

$$S = \int d^4x\, \sqrt{|g|}\, i\bar{\psi}_L \gamma^m (\nabla_m + \frac{1}{2}\omega_m)\psi_L \tag{3.5}$$

where $\gamma^m = e^m_a \gamma^a$, ∇ ($m, n, \ldots$ are world indices, $a, b, \ldots$ are flat indices) is the covariant derivative with respect to the world indices and ω_m is the spin connection:

$$\omega_m = \omega_m^{ab}\Sigma_{ab}$$

where $\Sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$ are the Lorentz generators. Finally $\psi_L = \frac{1+\gamma_5}{2}\psi$. Classically the energy-momentum tensor

$$T_{\mu\nu} = \frac{i}{2}\bar{\psi}_L \gamma_\mu \overset{\leftrightarrow}{\nabla}_\nu \psi_L \tag{3.6}$$

is both conserved on shell and traceless. At one loop to make sense of the calculations one must introduce regulators. The latter generally break both diffeomorphism and conformal invariance. A careful choice of the regularization procedure may preserve diff invariance, but anyhow breaks conformal invariance, so that the trace of the e.m. tensor takes the form (3.4), with specific nonvanishing coefficients a, c, e. There are various techniques to calculate the latter: cutoff, point splitting, Pauli-Villars, dimensional regularizations. Here we would like to briefly recall the heat kernel method utilized in [2] and in references cited therein (a more complete account will appear elsewhere). Denoting by D the relevant Dirac operator in (3.5) one can prove that

$$\delta W = -\int d^4x \sqrt{-g}\sigma T_\mu{}^\mu = -\frac{1}{16\pi^2}\int d^4x \sqrt{-g}\sigma b_4\left(x, x; D^\dagger D\right).$$

Thus

$$T_\mu{}^\mu = b_4\left(x, x; D^\dagger D\right) \tag{3.7}$$

The coefficient $b_4\left(x, x; D^\dagger D\right)$ appear in the heat kernel. The latter has the general form

$$K(t, x, y; \mathcal{D}) \sim \frac{1}{(4\pi t)^2} e^{-\frac{\sigma(x,y)}{2t}} \left(1 + t b_2(x, y; \mathcal{D}) + t^2 b_4(x, y; \mathcal{D}) + \cdots\right),$$

where $\mathcal{D} = D^\dagger D$ and $\sigma(x, y)$ is the half square length of the geodesic connecting x and y, so that $\sigma(x, x) = 0$. For coincident points we therefore have

$$K(t, x, x; \mathcal{D}) \sim \frac{1}{16\pi^2}\left(\frac{1}{t^2} + \frac{1}{t} b_2(x, x; \mathcal{D}) + b_4(x, x; \mathcal{D}) + \cdots\right). \tag{3.8}$$

This expression is divergent for $t \to 0$ and needs to be regularized. This can be done in various ways. The finite part, which we are interested in, has been calculated first by DeWitt, [3], and then by others with different methods. The results are reported in [2]. For a spin $\frac{1}{2}$ left-handed spinor as in our example one gets

$$b_4\left(x,x;D^{\dagger}D\right)=\frac{1}{180\times16\pi^2}\int d^4x\sqrt{-g}\left(a\,E_4+c\,W^2+e\;P\right)\tag{3.9}$$

with

$$a=\frac{11}{4},\qquad c=-\frac{9}{2},\qquad e=\frac{15}{4}\tag{3.10}$$

This result was obtained with an entirely Euclidean calculation. Turning to the Minkowski the actual e.m trace at one loop is

$$T_{\mu}{}^{\mu}=\frac{1}{180\times16\pi^2}\left(\frac{11}{4}\,E+c\,\mathcal{W}^2+i\frac{15}{4}\,P\right)\tag{3.11}$$

As pointed out above the important aspect of (3.11) is the i appearing in front of the Pontryagin density. The origin of this imaginary coupling is easy to trace. It comes from the trace of gamma matrices including a γ_5 factor. In 4d, while the trace of an even number of gamma matrices, which give rise to first two terms in the RHS of (3.11), is a real number, the trace of an even number of gamma's multiplied by γ_5 is always imaginary. The Pontryagin term comes precisely from the latter type of traces. It follows that, as a one loop effect, the energy momentum tensor becomes complex, and, in particular, since T_0^0 is the Hamiltonian density, we must conclude that unitarity is not preserved in this type of theories. Exactly as chiral gauge theories with nonvanishing chiral gauge anomalies are rejected as sick theories, also chiral models with complex trace anomalies are not acceptable theories. For instance the old-fashioned standard model with massless left-handed neutrinos is in this situation. This model, provided it has an UV fixed point, has a complex trace anomaly and breaks unitarity. This is avoided in the modern formulation of the electroweak interactions by the addition of a right-handed neutrino (for each flavor), or, alternatively, by using Majorana neutrinos. So, in hindsight, one could have predicted massive neutrinos.

In general we can say that in models with a chirality unbalance a problem with unitarity may arise due to the trace anomaly and has to be carefully taken into account.

3.3 Pontryagin density and supersymmetry

In this section we discuss the problem posed by the possible appearance of the Pontryagin term in the trace anomaly: is it compatible with supersymmetry? It is a well known fact that trace anomalies in supersymmetric theories are members of supermultiplets, to which also the Abelian chiral anomaly belongs. Thus one way to analyse this issue would be to try and supersymmetrize the three terms (3.1,3.2) and (3.3) and see whether they can be accommodated in supermultiplets. This direct approach, however, is far from practical. What we will do, instead, is to consider a conformal theory in 4D coupled to (external) supergravity formulated in terms of superfields, and find all the potential superconformal anomalies. This will allow us to see whether (3.3) can be accommodated in an anomaly supermultiplet as a trace anomaly member.

3.3.1 Minimal supergravity

The most well known model of $N = 1$ supergravity in $D = 4$ is the so-called *minimal supergravity*, see for instance [4]. The superspace of $N = 1$ supergravity is spanned by the supercoordinates $Z^M = (x^m, \theta^\mu, \bar{\theta}_{\dot{\mu}})$. In this superspace one introduces a superconnection, a supertorsion and the relevant supercurvature. To determine the dynamics one imposes constraints on the supertorsion. Such constraints are not unique. A particular choice of the latter, the *minimal* constraints, define the minimal supergravity model, which can be formulated in terms of the superfields $R(z)$, $G_a(z)$ and $W_{\alpha\beta\gamma}(z)$. R and $W_{\alpha\beta\gamma}$ are chiral while G_a is real. One also needs the antichiral superfields $R^+(z)$ and $\bar{W}_{\dot{\alpha}\dot{\beta}\dot{\gamma}}(z)$. $W_{\alpha\beta\gamma}$ is completely symmetric in the spinor indices $\alpha, \beta, \ldots$. These superfields satisfy themselves certain constraints. Altogether the independent degrees of freedom are 12 bosons + 12 fermions. One can define superconformal transformations in terms of a parameter superfield σ. For instance

$$\begin{aligned}
\delta R &= (2\bar{\sigma} - 4\sigma)R - \frac{1}{4}\nabla_{\dot{\alpha}}\nabla^{\dot{\alpha}}\bar{\sigma} \\
\delta G_a &= -(\sigma + \bar{\sigma})G_a + i\nabla_a(\bar{\sigma} - \sigma) \\
\delta W_{\alpha\beta\gamma} &= -3\sigma W_{\alpha\beta\gamma}
\end{aligned}$$

To find the possible superconformal anomalies we use a cohomological approach. Having in mind a superconformal matter theory coupled to a $N = 1$ supergravity, we define the functional operator that implements these transformations, i.e.

$$\Sigma = \int_{x\theta} \delta\chi_i \frac{\delta}{\delta\chi_i}$$

where χ_i represent the various superfields in the game and $_{x\theta}$ denotes integration $d^4x d^4\theta$. This operator is nilpotent: $\Sigma^2 = 0$. As a consequence it defines a cohomology problem. The cochains are integrated local expressions of the superfields and their superderivatives, invariant under superdiffeomorphism and local super-Lorentz transformations. Candidates for superconformal anomalies are nontrivial cocycles of Σ which are not coboundaries, i.e. integrated local functionals Δ_σ, linear in σ, such that

$$\Sigma\,\Delta_\sigma = 0, \qquad \text{and} \qquad \Delta_\sigma \neq \Sigma\,\mathcal{C}$$

for any integrated local functional $\mathcal{C}$ (not containing σ).

The complete analysis of all the possible nontrivial cocycles of the operator Σ was carried out in [5]. It was shown there that the latter can be cast into the form

$$\Delta_\sigma = \int_{x\theta} \left[\frac{E(z)}{-8R(z)}\,\sigma(z)\,\mathcal{S}(z) + \text{h.c.}\right] \tag{3.12}$$

where $\mathcal{S}(z)$ is a suitable chiral superfield, and all the possibilities for $\mathcal{S}$ were classified. For supergravity alone (without matter) the only nontrivial possibilities turn out to be:

$$\mathcal{S}_1(z) = W^{\alpha\beta\gamma}W_{\alpha\beta\gamma} \qquad \text{and} \qquad \mathcal{S}_2(z) = (\bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}} - 8R)(G_a G^a + 2RR^+) \tag{3.13}$$

(the operator $(\bar{\nabla}_{\dot{\alpha}}\bar{\nabla}^{\dot{\alpha}} - 8R)$ maps a real superfield into a chiral one).

It is well-known that the (3.12) cocycles contain not only the trace anomaly, but a full supermultiplet of anomalies. The local expressions of the latter are obtained by stripping off the corresponding parameters from the integrals in (3.12).

In order to recognize the ordinary field content of the cocycles (3.13) one has to pass to the component form. This is done by choosing the lowest components of the supervielbein as follows:

$$E_M{}^A(z)\big|_{\theta=\bar{\theta}=0} = \begin{pmatrix} e_m{}^a(x) & \frac{1}{2}\psi_m{}^\alpha(x) & \frac{1}{2}\bar{\psi}_{m\dot{\alpha}}(x) \\ 0 & \delta_\mu{}^\alpha & 0 \\ 0 & 0 & \delta^{\dot{\mu}}{}_{\dot{\alpha}} \end{pmatrix}$$

where $e_m{}^a$ are the usual 4D vierbein and $\psi_m{}^\alpha(x), \bar{\psi}_{m\dot{\alpha}}(x)$ the gravitino field components. Similarly one identifies the independent components of the other superfields (the lowest component of R and G_a). For σ we have

$$\sigma(z) = \omega(x) + i\alpha(x) + \sqrt{2}\Theta^\alpha\chi_\alpha(x) + \Theta^\alpha\Theta_\alpha(F(x) + iG(x)) \tag{3.14}$$

where Θ^α are Lorentz covariant anticommuting coordinates, [4]. The component fields of (3.14) identify the various anomalies in the cocycles (3.13). In particular ω is the parameter of the ordinary conformal transformations and α the parameter of the chiral transformations. They single out the corresponding anomalies. At this point it is a matter of algebra to write down the anomalies in component. Retaining for simplicity only the metric we obtain the *ordinary* form of the cocycles. This is

$$\Delta_\sigma^{(1)} \approx \int_x e\Big\{\omega\Big(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2\Big) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\} \tag{3.15}$$

for the first cocycle ($\approx$ denotes precisely the ordinary form), and

$$\Delta_\sigma^{(2)} = 4\int_x e\,\omega\big(\frac{2}{3}\mathcal{R}^2 - 2\mathcal{R}_{nm}\mathcal{R}^{nm}\big) \tag{3.16}$$

for the second. Taking a suitable linear combination of the two we get

$$\Delta_\sigma^{(1)} + \frac{1}{2}\Delta_\sigma^{(2)} \approx \int_x e\Big\{\omega\big(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 4\mathcal{R}_{nm}\mathcal{R}^{nm} + \mathcal{R}^2\big) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\} \tag{3.17}$$

We see that (3.15) contain $\mathcal{W}^2$ while (3.17) contains the Euler density in the terms proportional to ω (trace anomaly). They both contain the Pontryagin density in the term proportional to α (chiral anomaly).

In conclusion $\Delta_\sigma^{(1)}$ corresponds to a multiplet of anomalies, whose first component is the Weyl density multiplied by ω, accompanied by the Pontryagin density (the Delbourgo-Salam anomaly) multiplied by α. On the other hand $\Delta_\sigma^{(2)}$ does not contain the Pontryagin density and the part linear in ω is a combination of the Weyl and Gauss-Bonnet density. None of them contains the Pontryagin density in the trace anomaly part. Therefore we must conclude that, as far as $N = 1$ minimal supergravity is concerned, our conclusion about the compatibility between the Pontryagin density as a trace anomaly terms and local supersymmetry, is negative.

3.3.2 Other nonminimal supergravities

As previously mentioned the minimal model of supergravity is far from unique. There are many other choices of the supertorsion constraints, beside the minimal one. Most of them are connected by field redefinitions and represent the same theory. But there are choices that give rise to different dynamics. This is the case for the nonminimal 20+20 and 16+16 models. In the former case one introduces two new spinor superfields T_α and $\bar{T}_{\dot\alpha}$, while setting $R = R^+ = 0$. This model has 20+20 degrees of freedom. The bosonic dofs are those of the minimal model, excluding R and R^+, plus 10 additional ones which can be identified with the lowest components of the superfields $S = \mathcal{D}^\alpha T_\alpha - (n+1)T^\alpha T_\alpha$ and $\bar{S}$, $\bar{\mathcal{D}}_{\dot\alpha} T_\alpha$ and $\mathcal{D}_\alpha T_{\dot\alpha}$. The superconformal parameter is a generic complex superfield Σ constrained by the condition

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha + (n+1)T^\alpha \mathcal{D}_\alpha)\left[3n(\bar\Sigma - \Sigma) - (\bar\Sigma + \Sigma)\right] = 0$$

where n is a numerical parameter. It is easy to find a nontrivial cocycle of this symmetry

$$\Delta^{(1)}_{n.m.} = \int_{x,\theta} E\,\Sigma\, W^{\alpha\beta\gamma} W_{\alpha\beta\gamma} \frac{\bar{T}_{\dot\alpha}\bar{T}^{\dot\alpha}}{\bar{S}^2} + \text{h.c.}$$

and to prove that its ordinary component form is, up to a multiplicative factor,

$$\Delta_\Sigma^{(1)} \approx \frac{1}{4}\int_x e\Big\{\omega\Big(\mathcal{R}_{nmkl}\mathcal{R}^{nmkl} - 2\mathcal{R}_{nm}\mathcal{R}^{nm} + \frac{1}{3}\mathcal{R}^2\Big) - \frac{1}{2}\,\alpha\,\epsilon^{nmlk}\mathcal{R}_{nmpq}\mathcal{R}_{lk}{}^{pq}\Big\}$$

where $\omega + i\alpha$ is the lowest component of the superfield Σ. That is, the same ordinary form as $\Delta_\sigma^{(1)}$. As for other possible cocycles they can be obtained from the minimal supergravity ones by way of superfield redefinitions. To understand this point one should remember what was said above: different models of supergravity are defined by making a definite choice of the torsion constraints and, after such a choice, by identifying the dynamical degrees of freedom. This is the way minimal and nonminimal models are introduced. However it is possible to transform the choices of constraints into one another by means of linear transformations of the supervierbein and the superconnection, [7,8]:

$$E'_M{}^A = E_M{}^B X_B{}^A, \qquad E'_A{}^M = X^{-1}{}_A{}^B E_B{}^M, \qquad \Phi'_{MA}{}^B = \Phi_{MA}{}^B + \chi_{MA}{}^B$$

for suitable $X_A{}^B$ and $\chi_{MA}{}^B$. This was done in [6] and will not be repeated here. The result is a very complicated form for the cocycle $\Delta^{(2)}_{n.m.}$, derived from $\Delta^{(2)}_{\sigma}$. However the ordinary component form is the same for both.

As for the 16+16 nonminimal supergravity, it is obtained from the 20+20 model by imposing

$$T_\alpha = \mathcal{D}_\alpha \psi, \qquad T_{\dot\alpha} = \mathcal{D}_{\dot\alpha} \psi$$

where ψ is a (dimensionless) real superfield. The independent bosonic dofs are the lowest component of $S, \bar{S}$, $c_{\alpha\dot\alpha}$ and $G_{\alpha\dot\alpha}$, beside the metric. The superconformal transformation are expressed in terms of a real vector superfield L and an arbitrary chiral superfield Λ satisfying the constraint

$$\left(\mathcal{D}^\alpha \mathcal{D}_\alpha + (n+1) T^\alpha \mathcal{D}_\alpha\right)(2L + (3n+1)\Lambda) = 0.$$

The derivation of the nontrivial superconformal cocycles is much the same as for the previous model. The end result is two cocycles whose form, in terms of superfields, is considerably complicated, but whose ordinary form is the same as $\Delta^{(1)}_{\sigma}$ and $\Delta^{(2)}_{\sigma}$.

At this point we must clarify whether the cocycles we have found in 20+20 and 16+16 nonminimal supergravities are the only ones. In [6] a systematic cohomological search of such nontrivial cocycles has not been done, the reason being that when dimensionless fields, like ψ and $\bar\psi$, are present in a theory a polynomial analysis is not sufficient (and a non-polynomial one is of course very complicated). But we can argue as follows: consider a nontrivial cocycle in nonminimal or 16+16 nonminimal supergravity; it can be mapped to a minimal cocycle which either vanishes or coincides with the ones classified in [5]. There is no other possibility because in minimal supergravity there are no dimensionless superfields (apart from the vielbein) and the polynomial analysis carried out in [5] is sufficient to identify all cocycles. We conclude that the 20+20 and 16+16 nonminimal nontrivial cocycles, which reduce in the ordinary form to a nonvanishing expression, correspond to $\Delta^{(1)}_{\sigma}$ and $\Delta^{(2)}_{\sigma}$ in minimal supergravity and only to them.

None of these cocycles contains the Pontryagin density in the trace anomaly part. Therefore we must conclude that, as far as $N = 1$ minimal and nonminimal supergravity is concerned, our conclusion about the compatibility between the Pontryagin density as a trace anomaly terms and local supersymmetry, is negative.

3.4 Conclusion

A component of the trace anomaly which appear in chiral theories (the Pontryagin density) may have interesting implications. It is a CP violating term and, as such, it could be an interesting mechanism for baryogenesis. At one loop, as we have seen, this term violates unitarity and the only use we can make of it is as a selection criterion for phenomenological models with an UV fixed point. If, on the other hand, by some other kind of mechanism still to be discovered, this term appears in the trace of the em tensor with a real coefficient, it may become very interesting as a CP violating term. In the last section we have seen that, however, this is incompatible with supersymmetry. In other words, if such mechanism exists, it can become effective only after supersymmetry breaking. The search for the P term continues.

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