



4 Can We Predict the Fourth Family Masses for Quarks and Leptons?

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Abstract. In the ref. [1–4] four massless families of quarks and leptons before the electroweak break are predicted. Mass matrices of all the family members demonstrate in this proposal the same symmetry, determined by the family groups. There are scalar fields - two $SU(2)$ triplets, the gauge fields of the family quantum numbers, and three singlets, the gauge fields of the three charges (Q , Q' and Y')- all doublets with respect to the weak charge, which determine mass matrices on the tree level and, together with other contributions, also beyond the tree level. The symmetry of mass matrices remains unchanged for all loop corrections. The three singlets are, in loop corrections also together with other contributors, responsible for the differences in properties of the family members. Taking into account by the *spin-charge-family* theory proposed symmetry of mass matrices for all the family members and simplifying study by assuming that mass matrices are Hermitian and real and mixing matrices real, we fit free parameters of mass matrices to experimental data within the experimental accuracy. Calculations are in progress.

Povzetek. Teorija spina-nabojev-družin napoveduje [1–4], preden se zlomi elektrošibka simetrija, štiri brezmasne družine kvarkov in leptonov. Masne matrike vseh članov družin imajo po zlomitvi enako simetrijo, ki jo določajo družinska kvantna števila: Vsak spinor nosi družinski kvantni števili dveh grup $SU(2)$, nosi pa tudi kvantna števila člana družine. Pri zlomitvi simetrije sodelujejo skalarna polja, ki so tripletna umeritvena polja bodisi ene od dveh grup $SU(2)$ (družinska simetrija), ali pa singletna umeritvena polja treh nabojev (Q , Q' in Y'), ki razlikujejo med člani posamezne družine. Vsa skalarna polja so dubleti glede na šibki naboj, nosijo pa tudi hiper naboj. Simetrije masnih matrik se ohranjajo v vseh zančnih popravkih. Trije singleti določajo, v zančnih popravkih skupaj z ostalimi prispevki, razlike v lastnostih članov družin. Problem poenostavimo s predpostavko, da so masne matrike hermitske in realne in mešalne matrike realne. Zahtevana simetrija masnih matrik ima enako število prostih parametrov kot je doslej izmerjenih podatkov (dvakrat po tri mase in mešalna matrika - za kvarke in leptone). Napake podatkov omogočijo določitev le intervalov za vrednosti parametrov. Iz masnih matrik določimo lastnosti članov četrte družine. Računi so v teku.

4.1 Introduction

There are several attempts in the literature to reconstruct mass matrices of quarks and leptons out of the observed masses and mixing matrices and correspondingly to learn more about properties of fermion families [8]. The most popular is the

$n \times n$ matrix close to the democratic one, predicting that $(n - 1)$ families must be very light in comparison with the n^{th} one. Most of attempts treat neutrinos differently than the other family members, relying on the Majorana part, the Dirac part and the "sea-saw" mechanism. Most often are the number of families taken to be equal to the number of the so far observed families, while symmetries of mass matrices are chosen in several different ways [9]. Also possibilities with four families are discussed [12].

In this paper we follow the prediction of the *spin-charge-family* theory [1–4,7] that there are four massless families above the electroweak break and that the scalar fields - the two triplets carrying the family charges in the adjoint representations and the three singlets carrying the charges of the family members (Q , Q' and Y') - all doublets with respect to the weak charge, cause (after getting nonzero vacuum expectation values) the electroweak break. Assuming that the contributions of all the scalar (and in loop corrections also of other) fields to mass matrices of fermions are real and symmetric, we are left with the following symmetry of mass matrices

$$\mathcal{M}^\alpha = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^\alpha, \quad (4.1)$$

the same for all the family members $\alpha \in \{u, d, \nu, e\}$. In appendix 4.5.1 the evaluation of this mass matrix is presented and the symmetry commented. A change of phases of the left handed and the right handed basis - there are $(2n - 1)$ free choices - manifests in a change of phases of mass matrices.

The differences in the properties of the family members originate in the different charges of the family members and correspondingly in the different couplings to the corresponding scalar and gauge fields.

We fit (sect. 4.3.2) the mass matrix Eq. (4.1) with 6 free parameters of any family member 6 to the so far observed properties of quarks and leptons within the experimental accuracy. That is: *For a pair of either quarks or leptons, we fit twice 6 free parameters of the two mass matrices to twice three so far measured masses and to the corresponding mixing matrix.* Since we have the same number of free parameters (two times 6 for each pair, since the mass matrices are assumed to be real) as there are measured quantities (two times 3 masses and 6 angles of the orthogonal mixing matrix under a simplification that the mixing matrix is real and Hermitian), we would predict the fourth family masses uniquely, provided that the measured quantities are accurate. The $n - 1$ submatrix of any unitary matrix determine the unitary matrix uniquely for $n \geq 4$. The experimental inaccuracy enable to determine only the interval for the fourth family masses.

If the prediction of the *spin-charge-family* theory, that there are four families which manifest in the massless basis the symmetry of Eq. (4.1), is correct, we expect that enough accurate experimental data for the properties of the so far observed three families will offer narrow enough intervals for the fourth family masses.

We treat all the family members, the quarks and the leptons, equivalently. We also estimate the contributions of the fourth family members to the mesons decays in dependence of the fourth family masses, taking into account also the

estimations of the refs. [15]¹. However, we must admit that our estimations are so far pretty rough.

In sect. 4.3.1 we check on a toy model how accurate must be the experimental data that enable the prediction of the fourth family masses: For two “known” mass matrices, obeying the symmetry of Eq. (4.1), which lead approximately to the experimental data, we calculate masses and the mixing matrix. Then, taking the mixing matrix and twice three lower masses as an input, we look back for the starting two mass matrices with the required symmetry, allowing for the three lower families “experimental” inaccuracy. In the same section we then estimate the fourth family masses. So far the results are preliminary. Although we spent quite a lot of efforts to make the results transparent and trustable, the numerical procedure to take into account the experimental inaccuracy of data is not yet good enough to allow us to determine the interval of the fourth family masses, even not for quarks, so that all the results are very preliminary.

Still we can say that the so far obtained support the prediction of the *spin-charge-family* theory that there are four families of quarks and leptons, the mass matrices of which manifest the symmetry determined by the family groups – the same for all the family members, quarks and leptons. The mass matrices are quite close to the “democratic” ones, in particular for leptons.

Since the mass matrices offer an insight into the properties of the scalar fields, which determine mass matrices (together with other fields), manifesting effectively as the observed Higgs and the Yukawa couplings, we hope to learn about the properties of these scalar fields also from the mass matrices of quarks and leptons.

In appendix 4.5 we offer a very brief introduction into the *spin-charge-family* theory, which the reader, accepting the proposed symmetry of mass matrices without knowing the origin of this symmetry, can skip.

In sect. 4.2 the procedure to fit free parameters of mass matrices (Eq. (4.1) to the experimental data is discussed. We comment our studies in sect. 4.4.

4.2 Procedure used to fit free parameters of mass matrices to experimental data

Matrices, following from the *spin-charge-family* theory might not be Hermitian (appendix 4.6). We, however, simplify our study, presented in this paper, by assuming that the mass matrix for any family member, that is for the quarks and the leptons, is real and symmetric. We take the simplest phases up to signs, which depend on the choice of phases of the basic states, as discussed in appendices 4.5.1².

¹ M.I.Vysotsky and A.Lenz comment in their very recent papers that the fourth family is excluded provided that one assumes the *standard model* with one scalar field (the Higgs) and extends the number of families from three to four while using loop corrections when evaluating the decay properties of the Higgs. We have, however, several scalar fields and first estimates show that the fourth family quarks might have masses close to 1 TeV.

² In the ref. [17] we made a similar assumption, except that we allow that the symmetry on the tree level of mass matrices might be changed in loop corrections. We got in that study dependence of mass matrices and correspondingly mixing matrices for quarks on masses of the fourth family.

The matrix elements of mass matrices, with the loop corrections in all orders taken into account, manifesting the symmetry of Eq. (4.1), are in this paper taken as free parameters.

Let us first briefly overview properties of mixing matrices, a more detailed explanation of which can be found in subsection 4.2.1 of this section.

Let M^α , α denotes the family member ($\alpha = u, d, \nu, e$), be the mass matrix in the massless basis (with all loop corrections taken into account). Let $V_{\alpha\beta} = S^\alpha S^{\beta\dagger}$, where α represents either the u-quark and β the d-quark, or α represents the ν -lepton and β the e-lepton, denotes a (in general unitary) mixing matrix of a particular pair.

For $n \times n$ matrix ($n = 4$ in our case) it follows:

- i. If a known submatrix $(n-1) \times (n-1)$ of an unitary matrix $n \times n$ with $n \geq 4$ is extended to the whole unitary matrix $n \times n$, the n^2 unitarity conditions determine $(2(2(n-1)+1))$ real unknowns completely. If the submatrix $(n-1) \times (n-1)$ of an unitary matrix is made unitary by itself, then we loose the information.
- ii. If the mixing matrix is assumed to be orthogonal, then the $(n-1) \times (n-1)$ submatrix contains all the information about the $n \times n$ orthogonal matrix to which it belongs and the $n(n+1)/2$ conditions determine the $2(n-1)+1$ real unknowns completely for any n .

If the submatrix of the orthogonal matrix is made orthogonal by itself, then we loose the information.

We make in this paper, to simplify the present study, several assumptions [7], presented already in the introduction. In what follows we present the procedure used in our study and repeat the assumptions.

1. If the mass matrix M^α is Hermitian, then the unitary matrices S^α and T^α , introduced in appendix 4.6 to diagonalize a non Hermitian mass matrix, differ only in phase factors depending on phases of basic vectors and manifesting in two diagonal matrices, $F^{\alpha S}$ and $F^{\alpha T}$, corresponding to the left handed and the right handed basis, respectively. For Hermitian mass matrices we therefore have: $T^\alpha = S^\alpha F^{\alpha S} F^{\alpha T \dagger}$. By changing phases of basic vectors we can change phases of $(2n-1)$ matrix elements.
2. We take the diagonal matrices \mathcal{M}_d^α and the mixing matrices $V_{\alpha\beta}$ from the available experimental data. The mass matrices M^α in Eq. (4.1) have, if they are Hermitian and real, 6 free real parameters ($a^\alpha, a_1^\alpha, a_2^\alpha, b^\alpha, e^\alpha, d^\alpha$).
3. We limit the number of free parameters of the mass matrix of each family member α by taking into account n relations among free parameters, in our case $n = 4$, determined by the invariants

$$\begin{aligned} I_1^\alpha &= - \sum_{i=1,4} m_i^\alpha, & I_2^\alpha &= \sum_{i>j=1,4} m_i^\alpha m_j^\alpha, \\ I_3^\alpha &= - \sum_{i>j>k=1,4} m_i^\alpha m_j^\alpha m_k^\alpha, & I_4^\alpha &= m_1^\alpha m_2^\alpha m_3^\alpha m_4^\alpha, \end{aligned} \quad (4.2)$$

which are expressions appearing at powers of $\lambda_\alpha, \lambda_\alpha^4 + \lambda_\alpha^3 I_1 + \lambda_\alpha^2 I_2 + \lambda_\alpha I_3 + \lambda_\alpha^0 I_4 = 0$, in the eigenvalue equation. The invariants are fixed, within the experimental accuracy of the data, by the observed masses of quarks and leptons

and by the fourth family mass, if we make a choice of it. In appendix 4.2.2 we present the relations among the reduced number of free parameters for a chosen m_4^α . There are $(6 - 4)$ free parameters left for each mass matrix.

4. The diagonalizing matrices S^α and S^β , each depending on the reduced number of free parameters, are for real and symmetric mass matrices orthogonal. They follow from the procedure

$$\begin{aligned} M^\alpha &= S^\alpha \mathbf{M}_d^\alpha T^{\alpha\dagger}, \quad T^\alpha = S^\alpha F^\alpha S F^{\alpha T\dagger}, \\ \mathbf{M}_d^\alpha &= (m_1^\alpha, m_2^\alpha, m_3^\alpha, m_4^\alpha), \end{aligned} \quad (4.3)$$

provided that S^α and S^β fit the experimentally observed mixing matrices $V_{\alpha\beta}^\dagger$ within the experimental accuracy and that M^α and M^β manifest the symmetry presented in Eq. (4.1). We keep the symmetry of the mass matrices accurate. One can proceed in two ways.

$$\begin{aligned} \text{A. : } S^\beta &= V_{\alpha\beta}^\dagger S^\alpha, & \text{B. : } S^\alpha &= V_{\alpha\beta} S^\beta, \\ \text{A. : } V_{\alpha\beta}^\dagger S^\alpha \mathbf{M}_d^\beta S^{\alpha\dagger} V_{\alpha\beta} &= M^\beta, & \text{B. : } V_{\alpha\beta} S^\beta \mathbf{M}_d^\alpha S^{\beta\dagger} V_{\alpha\beta}^\dagger &= M^\alpha. \end{aligned} \quad (4.4)$$

In the case A. one obtains from Eq. (4.3), after requiring that the mass matrix M^α has the desired symmetry, the matrix S^α and the mass matrix $M^\alpha (= S^\alpha \mathbf{M}_d^\alpha S^{\alpha\dagger})$, from where we get the mass matrix $M^\beta = V_{\alpha\beta}^\dagger S^\alpha \mathbf{M}_d^\beta S^{\alpha\dagger} V_{\alpha\beta}$. In case B. one obtains equivalently the matrix S^β , from where we get $M^\alpha (= V_{\alpha\beta} S^\beta \mathbf{M}_d^\alpha S^{\beta\dagger} V_{\alpha\beta}^\dagger)$. We use both ways iteratively taking into account the experimental accuracy of masses and mixing matrices.

5. Under the assumption of the present study that the mass matrices are real and symmetric, the orthogonal diagonalizing matrices S^α and S^β form the orthogonal mixing matrix $V_{\alpha\beta}$, which depends on at most $6 (= \frac{n(n-1)}{2})$ free real parameters (appendix 4.6). Since, due to what we have explained at the beginning of this section, the experimentally measured matrix elements of the 3×3 submatrix of the 4×4 mixing matrix (if not made orthogonal by itself) determine the 4×4 mixing matrix - within the experimental accuracy - completely, also the fourth family masses are determined, again within the experimental accuracy. We must not forget, however, that the assumption of the real and symmetric mass matrices, leading to orthogonal mixing matrices, might not be an acceptable simplification, since we do know that the 3×3 submatrix of the mixing matrix has one complex phase, while the unitary 4×4 has three complex phases. (In the next step of study, with hopefully more accurate experimental data, we shall relax conditions on hermiticity of mass matrices and correspondingly on orthogonality of mixing matrices). We expect that too large experimental inaccuracy leave the fourth family masses in the present study quite undetermined, in particular for leptons.
6. We study quarks and leptons equivalently. The difference among family members originate on the tree level in the eigenvalues of the operators $(Q^\alpha, Q'^\alpha, Y'^\alpha)$, which in loop corrections together with other contributors in all orders con-

tribute to all mass matrix elements and cause the difference among family members³.

Let us conclude. If the mass matrix of a family member obeys the symmetry required by the *spin-charge-family* theory, which in a simplified version (as it is taken in this study) is real and symmetric, the matrix elements of the mixing matrices of quarks and leptons are correspondingly real, each of them with $\frac{n(n-1)}{2}$ free parameters. These six parameters of each mixing matrix are, within the experimental inaccuracy, determined by the three times three experimentally determined submatrix. After taking into account three so far measured masses of each family member, the six parameters of each mass matrix reduce to three. Twice three free parameters are within the experimental accuracy correspondingly determined by the 3×3 submatrix of the mixing matrix. The fourth family masses are correspondingly determined - within the experimental accuracy.

The assumption that the two 3×3 mixing matrices are unitary would lead to the loss of the information about the 4×4 mixing matrix. This is the case also if we take the orthogonalized version of the 3×3 mixing matrices.

Since neither the measured masses nor the measured mixing matrices are determined accurately enough to reproduce the 4×4 mixing matrices, we can expect that the masses and mixing matrix elements of the fourth family will be determined only within some quite large intervals.

4.2.1 Submatrices and their extensions to unitary and orthogonal matrices

In this appendix well known properties of $n \times n$ matrices, extended from $(n-1) \times (n-1)$ submatrices are discussed. We make a short overview of the properties, needed in this paper, although all which will be presented here, is the knowledge on the level of text books.

Any $n \times n$ complex matrix has $2n^2$ free parameters. The $n + 2n(n-1)/2$ unitarity requirements reduce the number of free parameters to n^2 ($= 2n^2 - (n + 2n(n-1)/2)$).

Let us assume a $(n-1) \times (n-1)$ known submatrix of the unitary matrix. The submatrix can be extended to the unitary matrix by $(2 \times [2(n-1) + 1])$ real parameters of the last column and last line. The n^2 unitarity conditions on the whole matrix reduce the number of unknowns to $(2(2n-1) - n^2)$. For $n = 4$ and higher the $(n-1) \times (n-1)$ submatrix contains all the information about the unitary $n \times n$ matrix.

The ref. [6] proposes a possible extension of an $(n-1) \times (n-1)$ unitary matrix $V_{(n-1)(n-1)}$ into $n \times n$ unitary matrices V_{nn} .

The choice of phases of the left and the right basic states which determine the unitary matrix (like this is the case with the mixing matrices of quarks and leptons) reduces the number of free parameters for $(2n-1)$. Correspondingly is the number of free parameters of such an unitary matrix equal to $n^2 - (2n-1)$, which manifests

³ There are also Majorana like terms contributing in higher order loop corrections [3] which might strongly influence in particular the neutrino mass matrix.

in $\frac{1}{2}n(n-1)$ real parameters and $\frac{1}{2}(n-1)(n-2)$ ($= n^2 - \frac{1}{2}n(n-1) - (2n-1)$) phases (which determine the number of complex parameters).

Any real $n \times n$ matrix has n^2 free parameters which the $\frac{1}{2}n(n+1)$ orthogonality conditions reduce to $\frac{1}{2}n(n-1)$. The $(n-1) \times (n-1)$ submatrix of this orthogonal matrix can be extended to this $n \times n$ orthogonal matrix with $[2(n-1) + 1]$ real parameters. The $\frac{1}{2}n(n+1)$ orthogonality conditions reduce these $[2(n-1) + 1]$ free parameters to $(2n-1 - \frac{1}{2}n(n+1))$, which means that the $(n-1) \times (n-1)$ submatrix of an $n \times n$ orthogonal matrix determine properties of its $n \times n$ orthogonal matrix completely. Any $(n-1) \times (n-1)$ submatrix of an orthogonal matrix contains all the information about the whole matrix for any n . Making the submatrix of the orthogonal matrix orthogonal by itself one loses the information about the $n \times n$ orthogonal matrix.

4.2.2 Free parameters of mass matrices after taken into account invariants

It is useful for numerical evaluation purposes to take into account for each family member its mass matrix invariants (sect. 4.2), expressible with three within the experimental accuracy known masses, while we keep the fourth one as a free parameter. We shall make a choice of a^α instead of the fourth family mass.

We shall skip in this section the family member index α and introduce new parameters as follows

$$a, b, \quad f = d + e, \quad g = d - e, \quad q = \frac{a_1 + a_2}{\sqrt{2}}, \quad r = \frac{a_1 - a_2}{\sqrt{2}}. \quad (4.5)$$

After making a choice of $a \frac{I_1}{4}$, that is of the fourth family mass, four invariants of Eq. (4.2) reduce the number of free parameters to 2. The four invariants therefore relate six parameters leaving three of them, the a included as a free parameter, undetermined. There are for each pair of family members the measured mixing matrix elements, assumed in this paper to be orthogonal and correspondingly determined by six parameters, which then fixes these two times 3 parameters. The (accurately enough) measured 3×3 submatrix of the (assumed to be orthogonal) 4×4 mixing matrix namely determines these 6 parameters within the experimental accuracy.

Using the starting relation among the invariants and introducing into them new parameters (a, b, f, g, q, r) from Eq. (4.5) we obtain

$$\begin{aligned} a &= \frac{I_1}{4}, \\ I'_2 &= -I_2 + 6a^2 - q^2 - r^2 - 2b^2 = f^2 + g^2, \\ I'_3 &= -\frac{1}{2b}(I_3 - 2aI_2 + 4a^2) = f^2 - g^2, \\ I'_4 &= I_4 - aI_3 + a^2I_2 - 3a^4 \\ &= \frac{1}{4}(q^2 - r^2)^2 + (q^2 + r^2)b^2 + \frac{1}{2}(q^2 - r^2) \cdot (\pm) \cdot [\pm] 2gf \\ &\quad + b^2(f^2 + g^2) + \frac{1}{4}(2gf)^2. \end{aligned} \quad (4.6)$$

We eliminate, using the first two equations, the parameters f and g , expressing them as functions of I'_2 and I'_3 , which depend, for a particular family member, on the three known masses, the parameter a and the three parameters r , q and b . We are left with the four free parameters (a , b , q , r) and the below relation among these parameters

$$\begin{aligned} & \left\{ -\frac{1}{2}(q^4 + r^4) + (-2b^2 + \frac{1}{2}(-I_2 + 6a^2 - 2b^2))(q^2 + r^2) \right. \\ & \left. + (I'_4 - \frac{1}{4}((-I_2 + 6a^2 - 2b^2)^2 + I_3'^2) + b^2(-I_2 + 6a^2 - 2b^2)) \right\}^2 \\ & = -\frac{1}{4}(q^2 - r^2)^2((-I_2 + 6a^2 - 2b^2 - (q^2 + r^2))^2 - I_3'^2), \end{aligned} \quad (4.7)$$

which reduces the number of free parameters to 3. These 3 free parameters must be determined, together with the corresponding three parameters of the partner, from the measured mixing matrix.

We eliminate one of the 4 free parameters in Eq. (4.7) by solving the cubic equation for, let us make a choice, q^2

$$\alpha q^6 + \beta q^4 + \gamma q^2 + \delta = 0. \quad (4.8)$$

Parameter $(\alpha, \beta, \gamma, \delta)$ depend on the 3 remaining free parameters (a, b, r) and the three, within experimental accuracy, known masses.

To reduce the number of free parameters from the starting 6 in Eq. (4.1) to the 3 left after taking into account invariants of each mass matrix, we look for the solution of Eq (4.8) for all allowed values for (a, b, r) . We make a choice for a in the interval of (a_{\min}, a_{\max}) , determined by the requirement that a , which solves the equations, is a real number. Allowing only real values for parameters f and g we end up with the equation

$$-I_2 + 6a^2 - 2b^2 - (q^2 + r^2) > \left| \frac{I_3 + 8a^3 - 2aI_2}{2b} \right|, \quad (4.9)$$

which determines the maximal positive b for $q = 0 = r$ and also the minimal positive value for b . For each value of the parameter a the interval (b_{\min}, b_{\max}) , as well as the interval $(r_{\min} = 0, r_{\max})$, follow when taking into account experimental values for the three lower masses.

4.3 Numerical results

Taking into account the assumptions and the procedure explained in sect. 4.2 and in the ref. [7] we are looking for the 4×4 in this paper taken to be real and symmetric mass matrices for quarks and leptons, obeying the symmetry of Eq. (4.1) and manifesting properties – masses and mixing matrices – of the so far observed three families of quarks and leptons in agreement with the experimental limits for the appearance of the fourth family masses and mixing matrix elements to the lower three families, as presented in the refs. [16,15]. We also take into account our so far made rough estimations of possible contributions of the fourth family members to the decay of mesons. More detailed estimations are in progress.

We hope that we shall be able to learn from the mass matrices of quarks and leptons also about the properties of the scalar fields, which cause masses of quarks and leptons, manifesting effectively so far as the measured Higgs and Yukawa couplings.

First we test the predicting power of our model in dependence of the experimental inaccuracy of masses and mixing matrices on a toy model: Starting with two known mass matrices with the symmetry of Eq. (4.1) we calculate masses and from the two diagonalizing matrices also the mixing matrix. From the known masses and mixing matrix, for which we allow "experimental inaccuracy", we check how does the reproducibility of the two starting mass matrices depend on the "experimental inaccuracy" and how does the "experimental inaccuracy" influence the fourth family masses.

Then we take the 3×3 measured mixing matrices for quarks and leptons and the measured masses, all with the experimental inaccuracy. Taking into account that the 3×3 submatrix of the unitary 4×4 matrix determines, if measured accurately enough, the 4×4 matrix, we look for the twice 4×4 mass matrices with the symmetry of Eq. (4.1), and correspondingly for the fourth family masses, for quarks and leptons.

When extending the two so far measured 3×3 submatrices of the 4×4 mixing matrices we try to take into account as many experimental data as possible.

4.3.1 Checking on a toy model how much does the symmetry of mass matrices (Eq. (4.1)) limit the fourth family properties

We check in this subsection on a toy model the reproducibility of the starting two mass matrices from the known two times three lower masses (say $m_{u_i}, m_{d_i}, i = (1, 2, 3)$) and the 3×3 submatrix (say $(V_{ud})_{i,j}, i, j = (1, 2, 3)$) of the 4×4 unitary mixing matrix in dependence of the inaccuracy allowed for $m_{u_i}, m_{d_i}, i = (1, 2, 3)$ and $(V_{ud})_{i,j}, i, j = (1, 2, 3)$.

We take the following two mass matrices, chosen so that they reproduce to high extent the measured properties of quarks (masses and mixing matrix) for some experimentally acceptable values for the fourth family masses and also the corresponding mixing matrix elements.

$$\begin{aligned} \mathcal{M}^{\text{toy}_u} &= \begin{pmatrix} 220985. & 119365. & 120065. & 204610. \\ 119365. & 218355. & 204610. & 120065. \\ 120065. & 204610. & 192956. & 119365. \\ 204610. & 120065. & 119365. & 190325. \end{pmatrix}, \\ \mathcal{M}^{\text{toy}_d} &= \begin{pmatrix} 175825. & 174262. & 174290. & 175709. \\ 174262. & 175839. & 175709. & 174290. \\ 174290. & 175709. & 175640. & 174262. \\ 175709. & 174290. & 174262. & 175654. \end{pmatrix}. \end{aligned} \quad (4.10)$$

Diagonalizing these two mass matrices we find the following twice four masses

$$\begin{aligned} \mathbf{M}_d^{\text{toy}_u} / \text{MeV}/c^2 &= (1.3, 620., 172000., 650000.), \\ \mathbf{M}_d^{\text{toy}_d} / \text{MeV}/c^2 &= (2.9, 55., 2900., 700000.). \end{aligned} \quad (4.11)$$

and the mixing matrix

$$V_{\text{toy}_{u,d}} = \begin{pmatrix} -0.97286 & -0.22946 & -0.02092 & 0.02134 \\ 0.23019 & -0.97205 & -0.04607 & -0.00287 \\ 0.00976 & 0.04965 & -0.99872 & -0.00045 \\ 0.02143 & 0.00213 & -0.00013 & 0.99977 \end{pmatrix}. \quad (4.12)$$

In order to simulate experimental inaccuracies (intervals of values for twice three lower masses and for the matrix elements of the 3×3 submatrix of the above unitary 4×4 matrix) and test the influence of these inaccuracies on the fourth family masses, we change the fourth family mass m_{u_4} in the interval $((300 - 1200))$ GeV and check the accuracy with which the matrix elements of the 3×3 submatrix of the 4×4 unitary matrix are reproduced. We measure the averaged inaccuracy in σ 's⁴. We keep in Table 4.1 the d_4 mass equal to 700 GeV.

m_{u_4}/GeV	300	500	600	650	700	800	1200
"exp. inacc"/ σ	4.0	1.0	0.29	0.0	0.25	0.66	1.6

Table 4.1. The average inaccuracy in σ of the mixing matrix elements of the 3×3 submatrix of the unitary quark mixing matrix (Eq.(4.12)) in dependence of the fourth family mass of the $m_{\text{toy}_{u_4}}$ -quark. $m_{\text{toy}_{d_4}}$ mass is kept equal to 700 GeV.

Let us add that the accuracy, with which the 3×3 submatrix of the 4×4 mixing matrix is reproduced, depends much less on $m_{\text{toy}_{d_4}}$ than it does on $m_{\text{toy}_{u_4}}$ in this toy model case.

We use this experience when evaluating intervals, within which the fourth family masses appear when taking into account the inaccuracies of the experimental data.

4.3.2 Numerical results for the observed quarks and leptons with mass matrices obeying Eq. (4.1)

We take for the quark and lepton masses the experimental values [16], recalculated to the Z boson mass scale. We take from [16] also the experimentally declared inaccuracies for the so far measured 3×3 mixing matrices, taken in our calculations as submatrices of the 4×4 unitary mixing matrices and pay attention on the experimentally allowed values for the fourth family masses and other limitations presented in refs. [15]. We also have started to make our own rough estimations for limitations which follow from the meson decays to which the fourth family members participate. Our estimations are in progress.

The numerical procedure, tested in the toy model and working well in this case, must still be adapted to take experimental inaccuracies into account in a way to be able to see which values within the experimentally allowed ones are the

⁴ We define σ as the difference of the reproduced mixing matrix elements and the exact matrix elements, following from the starting two mass matrices.

most trustable from the point of view of the symmetries of the 4×4 mass matrices predicted by the *spin-charge-family* theory.

Although the accurate enough mixing matrices and masses of quarks and leptons are essential for the prediction of the fourth family members masses, we still hope that even with the present accuracy of the experimental data the intervals for the fourth family masses shall not be too large, in particular not for quarks, for which the data are much more accurate than for leptons. Let us point out that from so far obtained results we are not yet able to predict the fourth family mass intervals, which would be reliable enough.

We therefore present some preliminary results. Let us point out that all the mass matrices manifest within a factor less than 2 the "democratic" view. This is, as expected, more and more the case, the higher might be the fourth family masses, and in particular is true for the leptons.

- For quarks we take [16]:

1. The quark mixing matrix [16] $V_{ud} = S^u S^{d\dagger}$

$$|V_{ud}| = \begin{pmatrix} 0.97425 \pm 0.00022 & 0.2252 \pm 0.0009 & 0.00415 \pm 0.00049 & |V_{u_1 d_4}| \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 & |V_{u_2 d_4}| \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 & |V_{u_3 d_4}| \\ |V_{u_4 d_1}| & |V_{u_4 d_2}| & |V_{u_4 d_3}| & |V_{u_4 d_4}| \end{pmatrix}, \quad (4.13)$$

determining for each assumed and experimentally allowed set of values for the mixing matrix elements of the 3×3 submatrix the corresponding fourth family mixing matrix elements ($|V_{u_i d_4}|$ and $|V_{u_4 d_j}|$) from the unitarity condition for the 4×4 mixing matrix.

2. The masses of quarks are taken at the energy scale of M_Z , while we take the fourth family masses as free parameters. We allow the values from 300 GeV up to more than TeV to see the influence of the experimental inaccuracy on the fourth family masses.

$$\begin{aligned} \mathbf{M}_d^u / \text{MeV}/c^2 &= (1.27 + 0.50 - 0.42, 619 \pm 84, 171\,700. \pm 3\,000., \\ &\quad m^{u_4} > 335\,000.), \\ \mathbf{M}_d^d / \text{MeV}/c^2 &= (2.90 + 1.24 - 1.19, 55 + 16 - 15, 2\,890. \pm 90., \\ &\quad m^{d_4} > 300\,000.). \end{aligned} \quad (4.14)$$

- For leptons we take [16]:

1. We evaluate 3×3 matrix elements from the data [16]

$$\begin{aligned} 7.05 \cdot 10^{-17} &\leq \Delta(\mathbf{m}_{21}/\text{MeV}/c^2)^2 \leq 8.34 \cdot 10^{-17}, \\ 2.07 \cdot 10^{-15} &\leq \Delta(\mathbf{m}_{(31),(32)}/\text{MeV}/c^2)^2 \leq 2.75 \cdot 10^{-15}, \\ 0.25 &\leq \sin^2 \theta_{12} \leq 0.37, \quad 0.36 \leq \sin^2 \theta_{23} \leq 0.67, \\ \sin^2 \theta_{13} &< 0.035(0.056), \quad \sin^2 2\theta_{13} = 0.098 \pm 0.013, \end{aligned} \quad (4.15)$$

which means that $\frac{\pi}{4} - \frac{\pi}{10} \leq \theta_{23} \leq \frac{\pi}{4} + \frac{\pi}{10}$, $\frac{\pi}{5.4} - \frac{\pi}{10} \leq \theta_{12} \leq \frac{\pi}{4} + \frac{\pi}{10}$, $\theta_{13} < \frac{\pi}{13}$.

This reflects in the lepton mixing matrix $V_{\nu e} = S^\nu S^{e\dagger}$

$$|V_{\nu e}| = \begin{pmatrix} 0.8224 & 0.5200 & 0.1552 & |V_{\nu_1 e_4}| \\ 0.3249 & 0.7239 & 0.6014 & |V_{\nu_2 e_4}| \\ 0.4455 & 0.4498 & 0.7704 & |V_{\nu_3 e_4}| \\ |V_{\nu_4 e_1}| & |V_{\nu_4 e_2}| & |V_{\nu_4 e_3}| & |V_{\nu_4 e_4}| \end{pmatrix}, \quad (4.16)$$

determining for each assumed value for any mixing matrix element within the experimentally allowed inaccuracy the corresponding fourth family mixing matrix elements ($|V_{\nu_i e_4}|$ and $|V_{\nu_4 e_j}|$) from the unitarity condition for the 4×4 mixing matrix.

2. The masses of leptons are taken from [16] while we take the fourth family masses as free parameters, checking how much does the experimental inaccuracy influence a possible prediction for the fourth family leptons masses and how does this prediction agree with experimentally allowed values [16,15] for the fourth family lepton masses.

$$\begin{aligned} \mathbf{M}_d^\nu/\text{MeV}/c^2 &= (1 \cdot 10^{-9}, 9 \cdot 10^{-9}, 5 \cdot 10^{-8}, m^{\nu_4} > 90\,000.), \\ \mathbf{M}_d^e/\text{MeV}/c^2 &= (0.486\,570\,161 \pm 0.000\,000\,042, \\ &102.718\,135\,9 \pm 0.000\,009\,2, 1746.24 \pm 0.20, m^{e_4} > 102\,000). \end{aligned} \quad (4.17)$$

Following the procedure explained in sect. 4.2 we look for the mass matrices for the u-quarks and the d-quarks and the ν -leptons and the e-leptons by requiring that the mass matrices reproduce experimental data while manifesting symmetry of Eq. (4.1), predicted by the *spin-charge-family* theory.

We look for several properties of the obtained mass matrices: **i.** We test the influence of the experimentally declared inaccuracy of the 3×3 submatrices of the 4×4 mixing matrices and of the twice 3 measured masses on the prediction of the fourth family masses. **ii.** We look for how could different choices for the masses of the fourth family members limit the inaccuracy of particular matrix elements of the mixing matrices or the inaccuracy of the three lower masses of family members. **iii.** We test how close to a democratic mass matrix are the obtained mass matrices in dependence of the fourth family masses.

The numerical procedure, used in this contribution, is designed for quarks and leptons.

In the two next subsections 4.3.2, 4.3.2 we present some preliminary results for 4×4 mass matrices as they follow from the *spin-charge-family* theory for quarks and leptons, respectively.

Mass matrices for quarks Searching for mass matrices with the symmetries of Eq. (4.1) to determine the interval for the fourth family quark masses in dependence of the values of the mixing matrix elements within the experimental inaccuracy, we have not yet found a trustable way to extract which experimental inaccuracies of the mixing matrix elements should be taken more and which less "seriously". We also need to evaluate more accurately the experimental limitations for the fourth family masses, originating in decay properties of mesons and other

experiments. Although in the toy model case the "inaccuracy" of the matrix elements leads very clearly to the right fourth family masses, this is not the case when the experimental data for the 3×3 mixing matrix elements are known within the accuracy from 0.02% to 12%. The so far obtained results can not yet make the choice among less or more trustable experimental values: We can not yet make more accurate choice for those data which have large experimental inaccuracies.

We are still trying to improve our the procedure of searching for the masses of the fourth family quarks.

Let us still present two cases to demonstrate how do quark mass matrices change with respect to the fourth family masses: The first two mass matrices lead to the fourth family masses $m_{u_4} = 300$ GeV and $m_{d_4} = 700$ GeV, while the second two lead to the fourth family masses $m_{u_4} = 1\,200$ GeV and $m_{d_4} = 700$ GeV.

•

$$M^u = \begin{pmatrix} 402673. & 256848. & 267632. & 329419. \\ 256848. & 402393. & 329419. & 267632. \\ 267632. & 329419. & 283918. & 256848. \\ 329419. & 267632. & 256848. & 283638. \end{pmatrix},$$

$$M^d = \begin{pmatrix} 176784. & 174262. & 174524. & 175473. \\ 174262. & 176816. & 175473. & 174524. \\ 174524. & 175473. & 174663. & 174262. \\ 175473. & 174524. & 174262. & 174695. \end{pmatrix}, \quad (4.18)$$

$$V_{ud} = \begin{pmatrix} 0.97365 & 0.22296 & 0.00225 & -0.04782 \\ 0.22276 & -0.97412 & 0.03818 & -0.00444 \\ 0.01071 & -0.03671 & -0.99927 & -0.0001 \\ 0.04761 & 0.00634 & 0.00018 & 0.99885 \end{pmatrix}. \quad (4.19)$$

The corresponding masses are

$$\mathbf{M}_d^u/\text{MeV}/c^2 = (1.29957, 620.002, 172\,000., 300\,000.),$$

$$\mathbf{M}_d^d/\text{MeV}/c^2 = (2.88508, 55.024, 2\,899.99, 700\,000.). \quad (4.20)$$

•

$$M^u = \begin{pmatrix} 351427. & 256907. & 257179. & 342730. \\ 256907. & 342353. & 342730. & 257179. \\ 257179. & 342730. & 343958. & 256907. \\ 342730. & 257179. & 256907. & 334884. \end{pmatrix},$$

$$M^d = \begin{pmatrix} 175762. & 174263. & 174289. & 175708. \\ 174263. & 175581. & 175708. & 174289. \\ 174289. & 175708. & 175898. & 174263. \\ 175708. & 174289. & 174263. & 175717. \end{pmatrix}, \quad (4.21)$$

$$V_{ud} = \begin{pmatrix} -0.9743 & 0.22521 & -0.00366 & 0.00383 \\ 0.22515 & 0.97325 & -0.04567 & 0.00299 \\ -0.00672 & -0.04532 & -0.99895 & -0.00019 \\ 0.00305 & -0.00378 & -0.00004 & 0.99999 \end{pmatrix}. \quad (4.22)$$

The corresponding masses are

$$\begin{aligned} \mathbf{M}_d^u/\text{MeV}/c^2 &= (1.29957, 620.002, 172\,000., 1\,200\,000.), \\ \mathbf{M}_d^d/\text{MeV}/c^2 &= (2.88508, 55.024, 2\,899.99, 700\,000.). \end{aligned} \quad (4.23)$$

We notice:

- i. In both cases the required symmetry, Eq. (4.1), is (on purpose) kept very accurate.
- ii. In both cases the mass matrices of quarks look quite close to the "democratic" matrix, in the second case slightly more than in the first case.
- iii. The mixing matrix elements are in the second case much closer (within the experimental values are V_{11} , V_{12} , V_{13} and V_{32} , almost within the experimental values are V_{21} , V_{22} and V_{33}) to the experimentally allowed values, than in the first case (almost within the experimentally allowed values are only V_{21} , V_{22} and V_{23}).

These results suggest that the fourth family masses $m_{u_4} = 1\,200$ GeV and $m_{d_4} = 700$ GeV are much more trustable than $m_{u_4} = 300$ GeV and $m_{d_4} = 700$ GeV.

Mass matrices for leptons We present here results for leptons, manifesting properties of the lepton mass matrices. These results are less informative than those for quarks, since the experimental results are for leptons mixing matrix much less accurate than in the case of quarks and also masses are known less accurately.

We have

$$\begin{aligned} M^\nu &= \begin{pmatrix} 14\,021. & 14\,968. & 14\,968. & -14\,021. \\ 14\,968. & 15\,979. & 15\,979. & -14\,968. \\ 14\,968. & 15\,979. & 15\,979. & -14\,968. \\ -14\,021. & -14\,968. & -14\,968. & 14\,021. \end{pmatrix}, \\ M^e &= \begin{pmatrix} 28\,933. & 30\,057. & 29\,762. & -27\,207. \\ 30\,057. & 32\,009. & 31\,958. & -29\,762. \\ 29\,762. & 31\,958. & 32\,009. & -30\,057. \\ -27\,207. & -29\,762. & -30\,057. & 28\,933. \end{pmatrix}, \end{aligned} \quad (4.24)$$

which leads to the mixing matrix $V_{\nu e}$

$$V_{\nu e 1} = \begin{pmatrix} 0.82363 & 0.54671 & -0.15082 & 0. \\ -0.50263 & 0.58049 & -0.64062 & 0. \\ -0.26268 & 0.60344 & 0.75290 & 0. \\ 0. & 0. & 0. & 0. \end{pmatrix}, \quad (4.25)$$

and the masses

$$\begin{aligned} \mathbf{M}_d^\nu/\text{MeV}/c^2 &= (5 \cdot 10^{-9}, 1 \cdot 10^{-8}, 4.9 \cdot 10^{-8}, 60\,000.), \\ \mathbf{M}_d^e/\text{MeV}/c^2 &= (0.510999, 105.658, 1\,776.82\,120\,000). \end{aligned} \quad (4.26)$$

We did not adapt lepton masses to Z_m mass scale. Zeros (0.) for the matrix elements concerning the fourth family members means that the values are less than 10^{-5} .

We notice:

- i. The required symmetry, Eq. (4.1), is kept very accurate.

- ii. The mass matrices of leptons are very close to the "democratic" matrix.
- iii. The mixing matrix elements among the first three and the fourth family members are very small, what is due to our choice, since the matrix elements of the 3×3 submatrix of the 4×4 unitary matrix, predicted by the *spin-charge-family* theory are very inaccurately known.

4.4 Discussions and conclusions

One of the most interesting open questions in the elementary particle physics is: Where do the family originate? Explaining the origin of families would answer the question about the number of families possibly observable at the low energy regime, about the origin of the scalar field(s) and Yukawa couplings and would also explain differences in the fermions properties - the differences in masses and mixing matrices among family members – quarks and leptons.

Assuming that the prediction of the *spin-charge-family* theory that there are four rather than so far observed three coupled families, the mass matrices of which demonstrate in the massless basis the $SU(2) \times SU(2)$ symmetry of Eq. (4.1), the same for all the family members - the quarks and the leptons - we look in this paper for:

- i. The origin of differences in the properties of the family members - quarks and leptons.
- ii. The allowed intervals for the fourth family masses.
- iii. The matrix elements in the mixing matrices among the fourth family members and the three already measured ones.

Our calculations presented here are preliminary and in progress.

Let us tell that there are two kinds of the scalar fields in the *spin-charge-family* theory, responsible for the masses and mixing matrices of quarks and leptons (and consequently also for the masses of the weak gauge fields): The ones which distinguish among the family members and the other ones which distinguish among the families. The differences between quarks and leptons and between u and d quarks and between ν and e leptons originate in the first kind of the scalar fields, which carry Q, Q' (the two charges which, like in the *standard model*, originate in the weak and hyper charge) and Y' (which originates in the hypercharge and in the fermion quantum number, similarly as in the SO(10) models).

The existence of four coupled families seems almost unavoidable for the explanation of the properties of the neutrino families if all the family members should start from the massless basis in an equivalent way: The 4×4 mass matrix, very close to a democratic one, offers three almost massless (in comparison with the observed quarks and charged leptons masses) families and a very massive one.

Taking the symmetry of, to simplify the calculations assumed to be real and symmetric, 4×4 mass matrices, we determine 6 free parameters of any of the mass matrices by requiring that the mass matrices lead to the observed properties of quarks and leptons. In both cases the 2 times three masses and the (in this

simplified study) orthogonal mixing matrix with 6 parameters, determine the 2×6 parameters (as required by the *spin-charge-family* theory) of the two mass matrices within the experimental accuracy.

The same procedure is used to study either quarks or leptons. Expected results are not only the mass matrices, but also the intervals within which masses of the fourth families should be observed and the corresponding mixing matrices.

We developed a special procedure to extract the dependence of the fourth family masses on the experimental inaccuracy of masses and mixing matrices. Our test of this procedure on a toy model, in which we first postulate two mass matrices (leading to masses and mixing matrices very close to those of quarks), calculate the masses and the mixing matrix, and then from three lowest masses and the 3×3 sub matrix of the unitary 4×4 mixing matrix calculate back the starting mass matrices and the fourth family masses, showed that the procedure leads very accurately to the starting mass matrices.

When we use the same procedure to extract the properties of the fourth family members from the experimental data within the experimental inaccuracies, the procedure was not selective enough to make useful predictions. We are improving the procedure to be able to extract the intervals of the fourth family masses in dependence of the accuracy of particular data. *Yet the here presented preliminary results show, that the masses of the fourth families quarks with $m_{u_4} > 1\text{TeV}$ lead to the mixing matrix much closer to the experimental data than does $m_{u_4} \approx 300\text{GeV}$.*

Let us conclude this report by pointing out that even if we shall not be able to limit the mass intervals for the fourth family members strongly enough to be predictive, yet the accurate enough data for the 3×3 submatrix of the unitary mass matrix will sooner or later determine the 4×4 unitary matrix so that the predictions will be accurate enough.

4.5 APPENDIX: A brief presentation of the *spin-charge-family* theory

We present in this section a very brief introduction into the *spin-charge family* theory [1–4]. The reader can skip this appendix taking by the *spin-charge family* theory required symmetry of mass matrices of Eq. (4.1) as an input to the study of properties of the 4×4 mass matrices – with the parameters which depend on charges of the family members – and can come to this part of the paper, if and when would like to learn where do families and scalar fields possibly originate from.

Let us start by directing attention of the reader to one of the most open questions in the elementary particle physics and cosmology: Why do we have families, where do they originate and correspondingly where do scalar fields, manifesting as Higgs and Yukawa couplings, originate? The *spin-charge-family* theory is offering a possible explanation for the origin of families and scalar fields, and in addition for the so far observed charges and the corresponding gauge fields.

There are, namely, two (only two) kinds of the Clifford algebra objects: One kind, the Dirac γ^a , takes care of the spin in $d = (3 + 1)$, while the spin in $d \geq 4$ (rather than the total angular momentum) manifests in $d = (3 + 1)$ in the low

energy regime as the charges. In this part the *spin-charge family* theory is like the Kaluza-Klein theory, unifying spin (in the low energy regime, otherwise the total angular momentum) and charges, and offering a possible answer to the question about the origin of the so far observed charges and correspondingly also about the so far observed gauge fields. The second kind of the Clifford algebra objects, forming the equivalent representations with respect to the Dirac kind, recognized by one of the authors (SNMB), is responsible for the appearance of families of fermions.

There are correspondingly also two kinds of gauge fields, which appear to manifest in $d = (3 + 1)$ as the so far observed vector gauge fields (the number of - obviously non yet observed - gauge fields grows with the dimension) and as the scalar gauge fields. The scalar fields are responsible, after gaining nonzero vacuum expectation values, for the appearance of masses of fermions and gauge bosons. They manifest as the so far observed Higgs [5] and the Yukawa couplings.

All the properties of fermions and bosons in the low energy regime originate in the *spin-charge-family* theory in a simple starting action for massless fields in $d = [1 + (d - 1)]$. Fermions interact with the vielbeins f^α_a and correspondingly with the two kinds of the spin connection fields: with $\omega_{abc} = f^\alpha_c \omega_{ab\alpha}$ which are the gauge fields of $S^{ab} = \frac{i}{4} (\gamma^a \gamma^b - \gamma^b \gamma^a)$ and with $\tilde{\omega}_{abc} = f^\alpha_c \tilde{\omega}_{ab\alpha}$ which are the gauge fields of $\tilde{S}^{ab} = \frac{i}{4} (\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a)$. α, β, \dots is the Einstein index and a, b, \dots is the flat index. The starting action is the simplest one

$$\begin{aligned}
 S &= \int d^d x \, E \, \mathcal{L}_f + \int d^d x \, E \, (\alpha R + \tilde{\alpha} \tilde{R}), \\
 \mathcal{L}_f &= \frac{1}{2} (\bar{\Psi} \gamma^\alpha p_{0\alpha} \Psi) + \text{h.c.} \\
 p_{0\alpha} &= f^\alpha_a p_{0\alpha} + \frac{1}{2E} \{p_\alpha, E f^\alpha_a\}_-, \\
 p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab} \omega_{ab\alpha} - \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{ab\alpha}, \\
 R &= \frac{1}{2} \{f^{\alpha[a} f^{\beta b]} (\omega_{ab\alpha, \beta} - \omega_{c\alpha\alpha} \omega^c_{b\beta})\} + \text{h.c.}, \\
 \tilde{R} &= \frac{1}{2} f^{\alpha[a} f^{\beta b]} (\tilde{\omega}_{ab\alpha, \beta} - \tilde{\omega}_{c\alpha\alpha} \tilde{\omega}^c_{b\beta}) + \text{h.c.} \quad (4.27)
 \end{aligned}$$

Fermions, coupled to the vielbeins and the two kinds of the spin connection fields, *manifest* (after several breaks of the starting symmetries) *before the electroweak break four massless families of quarks and leptons*, the left handed fermions are weak charged and the right handed ones are weak chargeless. The vielbeins and the two kinds of the spin connection fields manifest effectively as the observed gauge fields and (those with the scalar indices in $d = (1 + 3)$) as several scalar fields. The mass matrices of the four family members (quarks and leptons) are after the electroweak break expressible on a tree level by the vacuum expectation values of the two kinds of the spin connection fields and the corresponding vielbeins with the scalar indices ([4,13]):

i. One kind originates in the scalar fields $\tilde{\omega}_{abc}$, manifesting as the two $SU(2)$ triplets - $\tilde{A}_s^{\tilde{N} i}$, $i = (1, 2, 3)$, $s = (7, 8)$; $\tilde{A}_s^{\tilde{I} i}$, $i = (1, 2, 3)$, $s = (7, 8)$; - and one singlet - $\tilde{A}_s^{\tilde{4}}$, $s = (7, 8)$ - contributing equally to all the family members.

ii. The second kind originates in the scalar fields ω_{abc} , manifesting as three singlets $-A_s^Q, A_s^{Q'}, A^{Y'}$, $s = (7, 8)$ – contributing the same values to all the families and distinguishing among family members. Q and Q' are the quantum numbers from the *standard model*, Y' originates in the second SU(2) (a kind of a right handed "weak") charge.

All the scalar fields manifest, transforming the right handed quarks and leptons into the corresponding left handed ones⁵ and contributing also to the masses of the weak bosons, as doublets with respect to the weak charge. Loop corrections, to which all the scalar and also gauge vector fields contribute coherently, change contributions of the off-diagonal and diagonal elements on the tree level, keeping the tree level symmetry of mass matrices unchanged⁶.

4.5.1 Mass matrices on the tree level and beyond which manifest SU(2) × SU(2) symmetry

Let us make a choice of a massless basis ψ_i , $i = (1, 2, 3, 4)$, for a particular family member α . And let us take into account the two kinds of the operators, which transform the basis vectors into one another

$$\tilde{N}_L^i, i = (1, 2, 3), \quad \tilde{\tau}_L^i, i = (1, 2, 3), \quad (4.28)$$

with the properties

$$\begin{aligned} \tilde{N}_L^3(\psi_1, \psi_2, \psi_3, \psi_4) &= \frac{1}{2}(-\psi_1, \psi_2, -\psi_3, \psi_4), \\ \tilde{N}_L^+(\psi_1, \psi_2, \psi_3, \psi_4) &= (\psi_2, 0, \psi_4, 0), \\ \tilde{N}_L^-(\psi_1, \psi_2, \psi_3, \psi_4) &= (0, \psi_1, 0, \psi_3), \\ \tilde{\tau}^3(\psi_1, \psi_2, \psi_3, \psi_4) &= \frac{1}{2}(-\psi_1, -\psi_2, \psi_3, \psi_4), \\ \tilde{\tau}^+(\psi_1, \psi_2, \psi_3, \psi_4) &= (\psi_3, \psi_4, 0, 0), \\ \tilde{\tau}^-(\psi_1, \psi_2, \psi_3, \psi_4) &= (0, 0, \psi_1, \psi_2). \end{aligned} \quad (4.29)$$

This is indeed what the two SU(2) operators in the *spin-charge-family* theory do. The gauge scalar fields of these operators determine, together with the corresponding coupling constants, the off diagonal and diagonal matrix elements on the tree level. In addition to these two kinds of SU(2) scalars there are three U(1) scalars, which distinguish among the family members, contributing on the tree level the same diagonal matrix elements for all the families. In loop corrections in all orders the symmetry of mass matrices remains unchanged, while the three U(1) scalars,

⁵ It is the term $\gamma^0 \gamma^s \phi_s^{\Lambda i}$, where $\phi_s^{\Lambda i}$, with $s = (7, 8)$ denotes any of the scalar fields, which transforms the right handed fermions into the corresponding left handed partner [3,4,13]. This mass term originates in $\bar{\psi} \gamma^a p_{0a} \psi$ of the action Eq.(4.27), with $a = s = (7, 8)$ and $p_{0s} = f_s^\sigma (p_\sigma - \frac{1}{2} \zeta^{ab} \hat{\omega}_{ab\sigma} - \frac{1}{2} S^{st} \omega_{st\sigma})$.

⁶ It can be seen that all the loop corrections keep the starting symmetry of the mass matrices unchanged. We have also started [3,14] with the evaluation of the loop corrections to the tree level values. This estimation has been done so far [14] only up to the first order and partly to the second order.

contributing coherently with the two kinds of $SU(2)$ scalars and all the massive fields to all the matrix elements, manifest in off diagonal elements as well. All the scalars are doublets with respect to the weak charge, contributing to the weak and the hypercharge of the fermions so that they transform the right handed members into the left handed ones.

With the above (Eq. (4.29) presented choices of phases of the left and the right handed basic states in the massless basis the mass matrices of all the family members manifest the symmetry, presented in Eq. (4.1). One easily checks that a change of the phases of the left and the right handed members, there are $(2n - 1)$ possibilities, causes changes in phases of matrix elements in Eq. (4.1).

4.6 APPENDIX: Properties of non Hermitian mass matrices

This pedagogic presentation of well known properties of non Hermitian matrices can be found in many textbooks, for example [18]. We repeat this topic here only to make our discussions transparent.

Let us take a non Hermitian mass matrix M^α as it follows from the *spin-charge-family* theory, α denotes a family member (index \pm used in the main text is dropped).

We always can diagonalize a non Hermitian M^α with two unitary matrices, $S^\alpha (S^{\alpha\dagger} S^\alpha = I)$ and $T^\alpha (T^{\alpha\dagger} T^\alpha = I)$

$$S^{\alpha\dagger} M^\alpha T^\alpha = \mathbf{M}_d^\alpha = (m_1^\alpha \dots m_i^\alpha \dots m_n^\alpha). \quad (4.30)$$

The proof is added below.

Changing phases of the basic states, those of the left handed one and those of the right handed one, the new unitary matrices $S'^\alpha = S^\alpha F_{\alpha S}$ and $T'^\alpha = T^\alpha F_{\alpha T}$ change the phase of the elements of diagonalized mass matrices \mathbf{M}_d^α

$$\begin{aligned} S'^{\alpha\dagger} M^\alpha T'^\alpha &= F_{\alpha S}^\dagger \mathbf{M}_d^\alpha F_{\alpha T} = \\ &\text{diag}(m_1^\alpha e^{i(\phi_1^{\alpha S} - \phi_1^{\alpha T})} \dots m_i^\alpha e^{i(\phi_i^{\alpha S} - \phi_i^{\alpha T})}, \dots m_n^\alpha e^{i(\phi_n^{\alpha S} - \phi_n^{\alpha T})}), \\ F_{\alpha S} &= \text{diag}(e^{-i\phi_1^{\alpha S}}, \dots, e^{-i\phi_i^{\alpha S}}, \dots, e^{-i\phi_n^{\alpha S}}), \\ F_{\alpha T} &= \text{diag}(e^{-i\phi_1^{\alpha T}}, \dots, e^{-i\phi_i^{\alpha T}}, \dots, e^{-i\phi_n^{\alpha T}}). \end{aligned} \quad (4.31)$$

In the case that the mass matrix is Hermitian T^α can be replaced by S^α , but only up to phases originating in the phases of the two basis, the left handed one and the right handed one, since they remain independent.

One can diagonalize the non Hermitian mass matrices in two ways, that is either one diagonalizes $M^\alpha M^{\alpha\dagger}$ or $M^{\alpha\dagger} M^\alpha$

$$\begin{aligned} (S^{\alpha\dagger} M^\alpha T^\alpha)(S^{\alpha\dagger} M^\alpha T^\alpha)^\dagger &= S^{\alpha\dagger} M^\alpha M^{\alpha\dagger} S^\alpha = \mathbf{M}_{dS}^{\alpha 2}, \\ (S^{\alpha\dagger} M^\alpha T^\alpha)^\dagger (S^{\alpha\dagger} M^\alpha T^\alpha) &= T^{\alpha\dagger} M^{\alpha\dagger} M^\alpha T^\alpha = \mathbf{M}_{dT}^{\alpha 2}, \\ \mathbf{M}_{dS}^{\alpha\dagger} &= \mathbf{M}_{dS}^\alpha, \quad \mathbf{M}_{dT}^{\alpha\dagger} = \mathbf{M}_{dT}^\alpha. \end{aligned} \quad (4.32)$$

One can prove that $\mathbf{M}_{dS}^\alpha = \mathbf{M}_{dT}^\alpha$. The proof proceeds as follows. Let us define two Hermitian (H_S^α, H_T^α) and two unitary matrices (U_S^α, H_T^α)

$$\begin{aligned} H_S^\alpha &= S^\alpha \mathbf{M}_{dS}^\alpha S^{\alpha\dagger}, & H_T^\alpha &= T^\alpha \mathbf{M}_{dT}^{\alpha\dagger} T^{\alpha\dagger}, \\ U_S^\alpha &= H_S^{\alpha-1} M^\alpha, & U_T^\alpha &= H_T^{\alpha-1} M^{\alpha\dagger}, \end{aligned} \quad (4.33)$$

It is easy to show that $H_S^{\alpha\dagger} = H_S^\alpha, H_T^{\alpha\dagger} = H_T^\alpha, U_S^\alpha U_S^{\alpha\dagger} = I$ and $U_T^\alpha U_T^{\alpha\dagger} = I$. Then it follows

$$\begin{aligned} S^{\alpha\dagger} H_S^\alpha S^\alpha &= \mathbf{M}_{dS}^\alpha = \mathbf{M}_{dT}^{\alpha\dagger} = S^{\alpha\dagger} M^\alpha U_S^{\alpha-1} S^\alpha = S^{\alpha\dagger} M^\alpha T^\alpha, \\ T^{\alpha\dagger} H_T^\alpha T^\alpha &= \mathbf{M}_{dT}^{\alpha\dagger} = \mathbf{M}_{dS}^\alpha = T^{\alpha\dagger} M^{\alpha\dagger} U_T^{\alpha-1} T^\alpha = T^{\alpha\dagger} M^{\alpha\dagger} S^\alpha, \end{aligned} \quad (4.34)$$

where we recognized $U_S^{\alpha-1} S^\alpha = T^\alpha$ and $U_T^{\alpha-1} T^\alpha = S^\alpha$. Taking into account Eq. (4.31) the starting basis can be chosen so, that all diagonal masses are real and positive.

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