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INFLUENCE OF BODY MASS ON PERFORMANCE IN DOWNHILL SKIING

VPLIV TELESNE MASE SMUČARJA NA NJEGOV REZULTAT PRI SMUKU

Abstract

A physical model of downhill skiing has been constructed with the aim of determining the influence of a skier's body mass on their performance. The present article investigates the relationship between a skier's body mass and the shortest time needed for a downhill skier to complete the distance.
The accuracy of the physical model used in the study is essentially determined by the accuracy of the input parameters, but their errors do not noticeably affect the reliability of this comparative simulation study. The computer simulations show that the total time needed to complete the whole distance decreases as the skier's body mass increases. Slightly better total times have been simulated for higher skier's body masses. When a skier's body mass is 80 kg instead of 75 kg, the total time is about 0.4% shorter, i.e. approximately 0.6 s for the profile used in these simulations. The only factor that is causing this is the drag force influencing the skier, which has more effect on skiers with lower body mass.

Key words: downhill skiing, physical model, body mass, performance, computer simulation

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Izvleček

Fizikalen model smuka je bil izdelan s ciljem določiti vpliv telesne mase smučarja na njegov rezultat. Članek preučuje razmerje med telesno maso smučarja in najboljšim časom, ki ga je smučar potreboval, da je presmučal določeno razdaljo.
Natančnost fizikalnega modela, ki je bil uporabljen v raziskavi, v osnovi določajo vhodni podatki, vendar napake ne vplivajo bistveno na zanesljivost te primerjalne simulacijske raziskave. Računalniške simulacije kažejo, da je skupen čas, potreben za premagovanje razdalje, tem krajši, čim vecja je telesna masa smučarja. Pri smučarjih z večjo telesno maso je bil v simulaciji dosežen nekoliko boljši skupen čas. Če je telesna masa smučarja 80 kg namesto 75 kg, se skupen čas skrajša za približno 0.4 %, kar pomeni okoli 0.6 s za profil, ki je bil uporabljen v omenjenih simulacijah. Edini dejavnik, ki je za to odgovoren, je sila vleke, ki deluje na smučarja in ki ima večji vpliv na smučarje z manjšo telesno maso.

Ključne besede: smuk, fizikalen model, telesna masa, rezultat, računalniška simulacija

INTRODUCTION

Extremely low body mass is becoming widespread in many sports, such as bicycle racing, horse racing, gymnastics, long-distance running and ski-jumping, but there are some sports in which higher body mass gives athletes some advantages. One of them is downhill skiing, which is a very complex skill.

In the last several decades, biomechanics of skiing has demonstrated considerable growth, evolving from an exercise in the filming of human movement to an applied science with a vast array of measurements and modelling techniques. The earliest analytical model of skiing was made by Straumann (1927), who used a model for the wind tunnel measurements of drag forces. Since then, many approaches to studying the physics of skiing have been adopted, but there are only a few reliable studies of the effects of an athlete's body mass on their performance.

Using a physical model, the present article investigates the relationship between a skier's body mass and the total time needed to complete the whole distance.

METHODS

The model

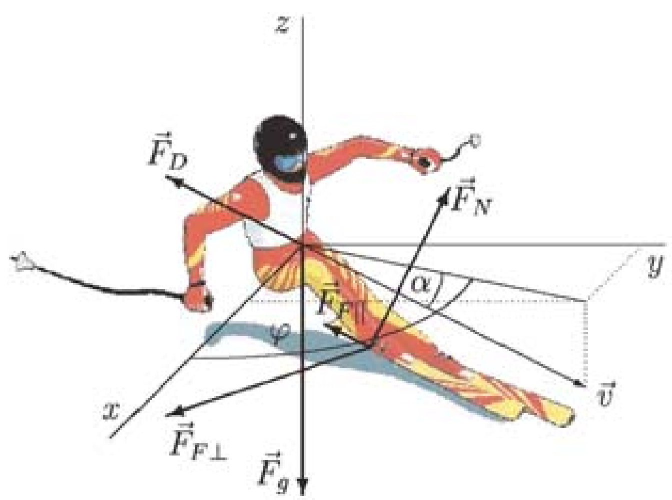

Figure 1: The forces that exert on the downhill skier

The gravitational force $\vec{F}_g$, the normal contact force $\vec{F}_N$, the drag component $\vec{F}_D$ of the dynamic fluid force, and the components of the friction force (longitudinal $\vec{F}_{F\parallel}$ and transversal $\vec{F}_{F\perp}$), exerted on a downhill skier. The x- and y-axes of the coordinate system are in the horizontal plane.

When a skier descends, as shown in Figure 1, the gravitational force $\vec{F}_g$, the normal contact force $\vec{F}_N$, the longitudinal $\vec{F}_{F\parallel}$ and transversal $\vec{F}_{F\perp}$ components of the friction force, and the drag component $\vec{F}_D$ of the dynamic fluid force are exerted on him. The magnitude of the drag force is proportional to air density ρ, the surface area A of the skier in the air, and the square of the skier's velocity $\vec{v}$ relative to air (Halliday, Resnick, & Walker, 2001; McGinnis, 1999).

$$F_D = \frac{1}{2} C_D \rho A v^2, \tag{1}$$

where C_D is the coefficient of drag. The drag force $\vec{F}_D$ acts in opposition to the relative motion of the skier with respect to air and that force tends to decrease the magnitude of the relative velocity. It is produced by two different factors: surface drag and form drag. Surface drag is the sum of the friction forces acting between air molecules and the surface of the skier. Form drag is the sum of the impact forces resulting from the collisions between the air and the skier.

The friction force $\vec{F}_F$ is a contact force that acts between and is parallel to the surfaces in contact, i.e. skis and snow. The magnitude of the friction force is proportional to the normal contact force pushing the two surfaces together

$$F_F = \mu F_N, \tag{2}$$

where μ is the coefficient of friction and F_N is the magnitude of the normal contact force. The friction force has been divided into two components: longitudinal $F_{F\parallel} = \mu_\parallel F_N$ and transversal $F_{F\perp} = \mu_\perp F_N$, where $\mu_\parallel$ and $\mu_\perp$, respectively, are the longitudinal and transversal coefficients of friction. The longitudinal component of the friction force acts in opposition to the skier's motion with respect to snow and that force tends to decrease the magnitude of the velocity. The transversal component of the friction force acts as centripetal force parallel to the contact surfaces and perpendicular to the skier's velocity, and its effect results in a change of the direction of the skier's motion.

The horizontal components F_x and F_y of the resultant force that acts on the skier are

$$\begin{aligned} F_x &= -\left(\frac{1}{2} C_D \rho v^2 A + \mu_\parallel F_N\right)\cos(\alpha)\cos(\varphi) \\ &\quad + \mu_\perp F_N \cos(\alpha)\cos(\varphi) + F_N \sin(\alpha), \\ F_y &= -\left(\frac{1}{2} C_D \rho v^2 A + \mu_\parallel F_N\right)\sin(\varphi) \\ &\quad - \mu_\perp F_N \cos(\varphi), \end{aligned} \tag{3}$$

where φ is the angle of the horizontal component of the velocity at that instant with respect to the x-axis, and α is the angle of the velocity with respect to the horizontal plane (see Figure 1). The vertical component F_z of the resultant force is

$$\begin{aligned} F_z &= -mg + \left(\frac{1}{2} C_D \rho v^2 A + \mu_\parallel F_N\right)\sin(\alpha)\cos(\varphi) \\ &\quad + \mu_\perp F_N \sin(\alpha)\sin(\varphi) + F_N \cos(\alpha), \end{aligned} \tag{4}$$

where m is the skier's body mass and $g = 9.8\ \mathrm{ms}^{-2}$ is the acceleration caused by the Earth's gravitational force.

Using the horizontal F_x, F_y and vertical F_z components of the resultant force, we have calculated the components of the skier's velocities (v_x, v_y, v_z) and positions (x, y, z) at that instant using the following differential equations:

$$\frac{dv_x}{dt} = \frac{F_x}{m}, \qquad \frac{dv_y}{dt} = \frac{F_y}{m}, \qquad \frac{dv_z}{dt} = \frac{F_z}{m}$$
$$\frac{dx}{dt} = v_x, \qquad \frac{dy}{dt} = v_y, \qquad \frac{dz}{dt} = v_z. \tag{5}$$

In addition to the resultant force exerted on the skier, skier's velocity and position change during downhill skiing as well.

Simulation

We have used differential equations (5) together with the expressions (3) and (4) for the horizontal and vertical components of the resultant force that acts on a downhill skier. In the computer simulation series, for a skier's body mass of 75 kg, for the minimal and maximal values of the frontal skier's surfaces we have applied A_{min} = 0.12 m^2, A_{max} = 1.0 m^2. We have used minimal surface when the skier tends to increase the magnitude of the velocity crouching in an "egg position" (at the beginning and after each curve), and maximal surface when he tends to decrease the velocity (before each curve). Surfaces of other body masses used in computer simulations have been adjusted in line with the skier's body mass (A ~ m$^{2/3}$). The air density ρ, according to the International Civil Aviation Organisation (Dubs, 1987), in normal atmosphere conditions equals 1.225 kgm^{-3} at mean sea level, 1.15 kgm^{-3} at an elevation of 650 m, and 1.0 kgm^{-3} at 2000 m. Since the elevation of skiing hills is about 1000 to 2500 m, the following exponential function was used to describe the air density as a function of elevation

$$\rho = \rho_0 e^{-kz}, \tag{6}$$

where ρ_o = 1.225 kgm^{-3} is the air density at mean sea level and $k = 1.015 \cdot 10^{-4}$ m^{-1} is the constant. For the longitudinal and transversal coefficients between the surface of the skis and the snow we have chosen $\mu_{\|} = 0.07$ and $\mu_{\perp} = 1.1$. The transversal friction force acts perpendicularly to skis, allowing skiing in curves. The profile of a typical skiing hill (see Table 1) with an average gradient of α = 13.5^{o} (α_{min} = 6^{o}, α_{max} = 42^{o}) has been applied ($\Delta x = 4\,270$ m, $\Delta z = 1\,028$ m). For the drag coefficient we have used C_D = 1 as in (Schmölzer, & Müler, 2002). Fixed time step integration of $\Delta t = 0.0001$ s has been used.

Table 1: The profile of the skiing hill used in the simulations (the total length of the course is 4400 m)

Elevation (m)	2315-2155 m	2155-2145 m	2145-2000 m	2000-1980 m
Gradient (^{0})	12^0	6^0	16^0	9^0
Elevation, cont. (m)	1980-1955 m	1955-1850 m	1850-1810 m	1810-1770 m
Gradient, cont. (^{0})	41^0	16^0	9^0	10^0
Elevation, cont. (m)	1770-1675 m	1675-1600 m	1600-1500 m	1500-1450 m
Gradient, cont. (^{0})	15^0	14^0	22^0	8^0
Elevation, cont. (m)	1450-1410 m	1410-1330 m	1330-1310 m	1310-1287 m
Gradient, cont. (^{0})	20^0	10^0	42^0	13^0

RESULTS

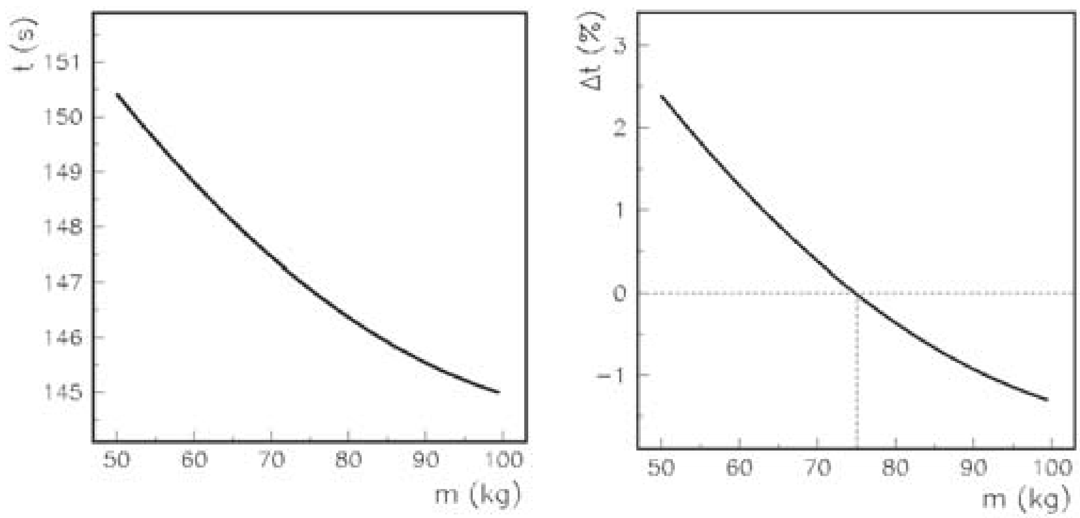

Figure 2: Left: the total time needed to complete the whole distance as a function of the skier's body mass. Right: relative difference of the total time.

For various body masses the total time needed to complete the whole distance has been calculated and displayed in Figure 2 left. Figure 2 right shows the relative difference of the total time as a function of the skier's body mass. Shorter times have been obtained in the case of higher body masses. For example, when a skier's body mass is 80 kg instead of 75 kg, the computer simulations show that the total time will be about 0.4% shorter, i.e. approximately 0.6 s for the hill profile used in the simulations. The drag force is the only factor that causes this difference in the total time, which tends to decrease the magnitude of the velocity and is more important for skiers with lower body masses, as indicated in Figures 3 and 4.

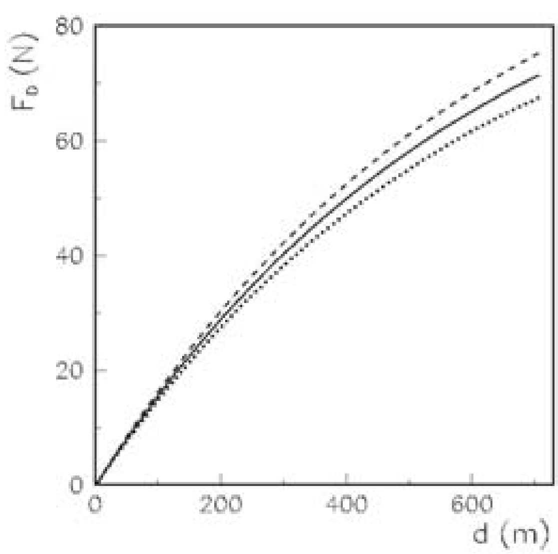

Figure 3: The drag force for three different body masses (70 kg - dotted line, 75 kg - full line, and 80 kg - dashed line), which acts on a skier during the first 700 m of skiing

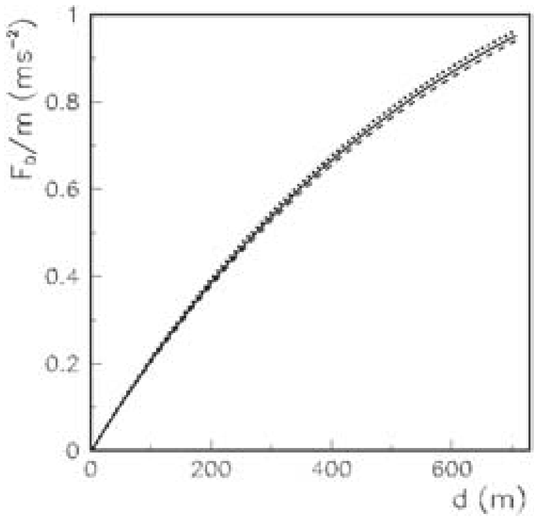

Figure 4: The drag force divided by the skier's body mass (70 kg - dotted line, 75 kg - full line, and 80 kg - dashed line)

Figure 3 shows the drag force for three different body masses (70 kg - dotted line, 75 kg - full line, and 80 kg - dashed line), which acts on the skier during the first 700 m of downhill skiing. Figure 4 shows the drag force divided by the skier's body mass. Obviously, the magnitude of the drag force is higher in the case of skiers with higher body mass, but, as shown in the Figure, the drag force divided by body mass has slightly lower magnitude in the case of higher body masses, thus causing higher speeds of skiers with higher body masses.

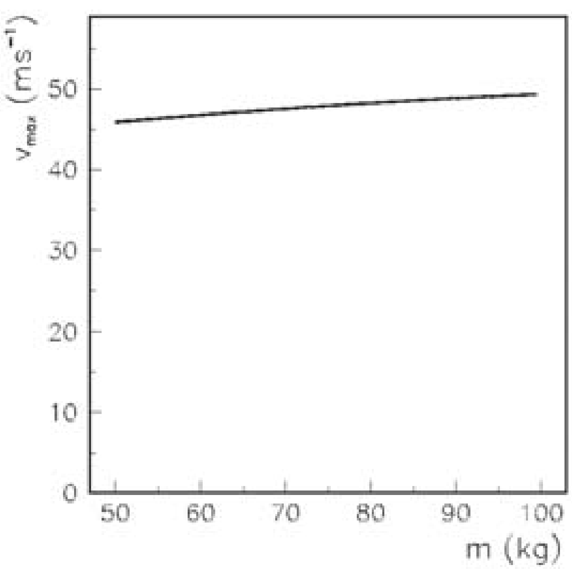

Figure 5: The maximal magnitude of velocity reached during downhill skiing as a function of a skier's body mass

Figure 5 shows the maximal magnitude of the velocity reached during downhill skiing as a function of a skier's body mass. Higher values were obtained in the case of higher body masses. The possible hill-profile dependence of the results obtained in the simulations requires detailed studies, which we plan to carry out in the near future.

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