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Resonances and strength functions of few-body systems

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Abstract. A resonance offers a testing ground for few-body dynamics. Two types of resonances are discussed in detail. One is very narrow Hoyle resonance in ^{12}C that plays a crucial role in producing that element in stars. The other includes broad high-lying negative-parity resonances in $A = 4$ nuclei, ^{4}H, ^{4}He, ^{4}Li. The former is dominated by the Coulomb force of three-α particles at large distances, while the latter are by short-ranged nuclear forces. The structure of these resonances is described by different approaches, adiabatic hyperspherical method and correlated Gaussians used for strength function calculations. The localization of the resonance is successfully realized by a complex absorbing potential and a complex scaling method, respectively.

1 Hoyle resonance

The synthesis of ^{12}C is essential to ^{12}C-based life and its process at low temperatures is sequential via a narrow resonance of ^{8}Be:

$$\alpha + \alpha \to \ {}^8\mathrm{Be}, \quad \alpha + \ {}^8\mathrm{Be} \to \ {}^{12}\mathrm{C} + \gamma. \tag{1}$$

As predicted by Hoyle, however, an existence of a very narrow resonance at around E_x =7.7 MeV is vital to explain the abundance of ^{12}C element. The resonance is found to be just 0.38 MeV above 3α threshold with its width of 8.5 eV.

Since nobody has ever succeeded in reproducing the Hoyle resonance width, we have undertaken to tackle this problem in the adiabatic hyperspherical method [1, 2]. This study has further been motivated by the fact that there exists huge discrepancy in the rate of triple-α reactions, $\alpha + \alpha + \alpha \to \ {}^{12}\mathrm{C} + \gamma$, calculated by several authors [3–5].

In contrast to two-body resonances, the Hoyle resonance is characterized by the followings: (1) 3α particles interact via long-ranged Coulomb force even at large distances. (2) no asymptotic wave function is known. (3) 2α subsystem forms a sharp resonance, which causes successive avoided crossings with three-particle continuum states.

The detail of our approach is given in Refs. [1, 2]. The three-body system is completely specified by six coordinates excluding the center-of-mass coordinate. Among six coordinates one is chosen to be the hyperradius of length dimension, and other five coordinates are hyperangles. Among the five angle coordinates three are Euler angles and two are used to specify the geometry of the three body system. By changing the geometry as much as possible, we can study the adiabatic potential curve of the three-body system as a function of the hyperradius. A resonance can be confined by introducing a complex absorbing potential [6] at large distances of the hyperradius. This method works excellently for quantitatively reproducing the very narrow width of the Hoyle resonance as well as predicting the triple-α reaction rate at low temperatures without relying on any ambiguous ansatz.

2 Resonances in $A = 4$ nuclei

^{4}He nucleus is doubly magic and its 0^+ ground state is strongly bound. The first excited state of ^{4}He is not a negative-party but again 0^+. The negative-parity excited states appear above the ^{3}He+p threshold. Seven negative-parity states are known and some of them have very broad widths. There exist isobar resonances in ^{4}H and ^{4}Li that are also very broad. Most of these resonances are identified by R-matrix phenomenology.

These resonances offer typical four-body resonances governed by the nuclear force. The decay channels include not only two-body but three-body systems. To describe the resonance we have employed correlated Gaussians [7,8] that provide us with efficient and accurate performance as few-body basis functions. A general form of the correlated Gaussians is

$$[\theta_L \times \chi_S]_{JM} \exp\Big[-\sum_{i<j} a_{ij}(\mathbf{r}_i - \mathbf{r}_j)^2\Big]\eta_{TM_T}, \tag{2}$$

where θ_L, χ_S, η_T stand for the functions of orbital angular momentum, spin, isospin parts. a_{ij} are variational parameters that control the spatial configuration of the system. See also Ref. [9] for recent review on the correlated Gaussians.

The negative-parity resonances may be studied by analyzing strength functions for electromagnetic excitations from the ground state of ^{4}He. Actually we have considered the spin-dipole operator specified by type p and $\lambda\mu$ tensor (λ=0,1,2)

$$\mathcal{O}^p_{\lambda\mu} = \sum_{i=1}^{4} [(\mathbf{r}_i - \mathbf{R}) \times \boldsymbol{\sigma}_i]_{\lambda\mu} T^p_i \tag{3}$$

where the center-of-mass coordinate of $A = 4$ nucleus, $\mathbf{R}$, is subtracted from the position coordinate $\mathbf{r}_i$ to make sure excitations of intrinsic motion only and T^p_i distinguishes different types of isospin operators (t_x, t_y, t_z)

$$T^p_i = \begin{cases} 1 & \text{Isoscalar} \\ 2t_z(i) & \text{Isovector} \\ t_x(i) \pm it_y(i) & \text{Charge} - \text{exchange} \end{cases} \tag{4}$$

We calculate the strength function $S^p_\lambda(E)$ corresponding to the response of the ^{4}He ground state Ψ_0 induced by $\mathcal{O}^p_{\lambda\mu}$

$$S^p_\lambda(E) = \mathcal{S}_{\mu f}|\langle\Psi_f|\mathcal{O}^p_{\lambda\mu}|\Psi_0\rangle|^2\delta(E_f - E_0 - E), \tag{5}$$

where $\mathcal{S}_{\mu f}$ denotes a sum over all possible final states This strength function can be computed by using the complex scaling method. The important thing for accurate evaluation of $S^p_\lambda(E)$ is to span possible final configurations as much as possible.

We have studied three negative-parity states with isospin 0 in ^{4}He and four negative-parity states with isospin1 in ^{4}He, ^{4}H, ^{4}Li [10–12]. Some of the resonance widths are very broad, and thus it is hard to identify their resonance parameters on the complex plane. However, the strength functions calculated above clearly indicate peaks near the resonance energies. We have confirmed that even the broad resonance can be identified with this calculation.

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