APPROXIMATE ANALYTICAL MODELING OF SILICON INTERNAL INDUCTANCE OF VLSI INTERCONNECTS

H. Ymeri[a, *], B. Nauwelaers[a], K. Maex[a,b], D. De Roest[b], M. Stucchi[b]

[a]Katholieke Universiteit Leuven, Department of Electrical Engineering (ESAT), Div. ESAT-TELEMIC, Leuven-Heverlee, Belgium
[b]The Interuniversity Microelectronics Center (IMEC), Leuven, Belgium

Key words: semiconductors, microelectronics, IC, Integrated Circuits, VLSI circuits, Very Large Scale of Integration circuits, interconnects, internal inductance, lossy silicon substrates, analytical modeling, approximate modeling, GREEN'S unctions, electric current, current distribution, eddy current, substrate resistivity

Abstract: In previous work, a closed form expressions for the series line impedance per unit length of a single and multiple coupled interconnects on lossy silicon substrate were presented. In this paper, the induced current density distribution inside silicon substrate and quasi-static Green's function approach were used to derive the frequency-dependent closed-form expression for the internal inductance per unit length accociated with the silicon substrate. With this expression, we can quantitatively determine what percentage of the total inductance per unit length is associated with the internal silicon substrate inductance for a given frequency. The effect of the substrate resistivity on the inductance of on-chip interconnects was examined. It was shown that depending on the frequency, large changes in the inductance can occur due to this effect.

Približno analitično modeliranje notranje induktivnosti VLSI povezav na siliciju

Ključne besede: polprevodniki, mikroelektronika, IC vezja integrirana, VLSI vezja integracije zelo visoke stopnje, povezave medsebojne, induktivnost notranja, substrati silicijevi izgubni, modeliranje analitično, modeliranje približno, GREEN funkcije, tok električni, porazdelitev toka, tok vrtinčni, upornost substrata

Izvleček: V enem od prejšnjih prispevkov smo predstavili zaključen izraz za frekvenčno odvisno vzajemno impedanco povezav na izgubni silicijevi tabletki. Tokrat s pomočjo porazdelitve inducirane tokovne gostote znotraj silicija in s pristopom, ki uporablja kvazi statične Greenove funkcije izpeljemo zaključen izraz za frekvenčno odvisno notranjo induktivnost na enoto dolžine povezano s silicijevim substratom. S pomočjo dobljenega izraza lahko kvantitativno določimo, kakšen del celotne induktivnosti na enoto dolžine odpade na silicijev substrat pri dani frekvenci. Ravno tako smo preučili vpliv substratne upornosti na induktivnost povezav na tabletki. Pokazali smo, da zaradi tega efekta lahko pride do velikih sprememb induktivnosti v odvisnosti od frekvence.

1. Introduction

As the sub-nano second transition time of a signal becomes comparable to its propagation delay on interconnection lines, the transmission line effects on the IC interconnects become extremely important. The IC interconnect transmission line parameters are inherently frequency-dependent because of the silicon substrate effect, the metal skin effect and the proximity effect in the current return paths /1/. In order to accomplish this, it is necessary to analyze and model the broad-band characteristics /2 - 5, 8 - 10/ of the silicon IC interconnects since the signals tend to exhibit both the short rising and falling times. Since the conductive silicon substrate does not act as a perfect ground plane, significant amount of return currents comes back through it. The internal inductance of the silicon semiconducting substrate is governed by the skin effect (i.e. the skin effect associated with the silicon substrate). As a results, it is known qualitatively that at very high frequencies, the total inductance is dominated by the external inductance and the internal inductance (silicon substrate) can be neglected. However, for a given frequency there has not been a way of quantitatively determining how large or small the contribution of the internal silicon inductance is compared to the total inductance of IC interconnects.

In this paper, for the first time, we introduced a closed-form expression for the internal inductance per unit length of the silicon associated with VLSI interconnects. This expression can be used to determine the internal silicon substrate inductance for any given frequency and substrate resistivity. The results presented here illustrate that the resistivity, geometry of the interconnect lines as well as the frequency play a strong role in determining the relative importance of the internal silicon inductance.

2. New closed-form expression for the internal silicon inductance

The new modeling approach is described for a microstrip line on a lossy silicon substrate with conductivity σ, as illustrated in Fig. 1. The silicon substrate and silicon oxide have the thickness h and d, respectively, and the the microstrip conductor is infinitely thin with width w.

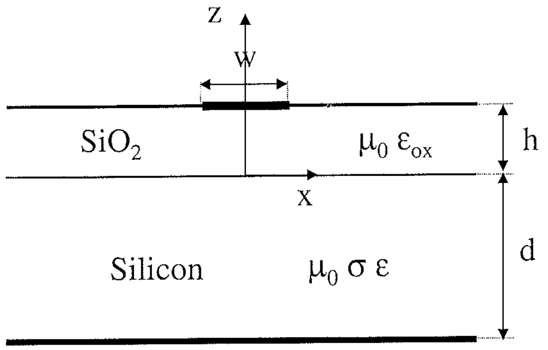

Fig. 1. Cross section of a microstrip interconnect on an oxide-semiconductor substrate.

The internal inductance of the silicon semiconducting substrate is given by

$$L_{si} = \frac{1}{I_{si}^2} \int_V \mathbf{B} \cdot \mathbf{H} dv \tag{1}$$

where I_{si} is the total current flowing in the silicon substrate and **B** and **H** are magnetic flux density and magnetic field intensity inside the silicon substrate, respectively. For infinitely long interconnect conductor, the fields are uniform in the y direction and the problem is two-dimesional. In general, the magnetic fields inside the lossy silicon have both an x and z component. However, in /6/ it was shown that inside a highly conducting region (VLSI interconnects in CMOS technology) the tangential components of the fields (the x component in this case) dominates the normal component (the z component), and to zero-order, $H_z \approx 0$. (Little generality is lost by considering this quasi-TEM case). By utilizing this fact and assuming that the materilas are nonmagnetic, (1) reduces to

$$L_{si} = \frac{\mu_0}{I_{si}^2} \int_{-d}^{0} \int_{-\infty}^{\infty} H_x^2 dxdz. \tag{2}$$

It was shown in /6/ that the fields exhibit an exponential decay of $e^{z/\delta}$ (where $\delta = \sqrt{2/(\omega\mu\sigma)}$ is the skin depth) away from the surface of a conducting region. Therefore, by assuming that the magnetic field inside silicon substrate is governed by this same skin depth behaviour, we get

$$H_x = H_{0x}(x) e^{z/\delta} \tag{3}$$

where H_{0x} is the value of the magnetic field on the surface of the silicon substrate and is function of x. By using the boundary condition for the tangential component of the magnetic field, $\mathbf{a}_n \times \mathbf{H}\ |_{z=0} = J_{si}$, the H_{0x} can be approximated by

$$H_{0x}(x) = J_{si}(x) \tag{4}$$

where $J_{si}(x)$ is the surface current density of the silicon substrate. Upon substituting (3) and (4) into (2), the z integration can be done explicitly and the expression for internal inductance become

$$L_{si} = \frac{\mu_0 \delta}{2}(1 - e^{-2d/\delta}) \int_{-\infty}^{\infty} [J_{si}(x) / I_{si}]^2 dx. \tag{5}$$

The surface current density on a semiconducting silicon substrate was derived via a quasistatic Green's function approach /7, 9, 10/ and was shown to be

$$J_{si}(x) = (I_{si} / w\pi)[\tan^{-1}((w - 2x)/2h) + \tan^{-1}((w + 2x)/2h)] \tag{6}$$

where w and h are defined in Fig. 1. Substituting the surface current density (6) into (5), we get

$$L_{si} = (\mu_0 \delta / 2(w\pi)^2)(1 - e^{-2d/\delta}) \int_{-\infty}^{\infty} [\tan^{-1}((w - 2x)/2h) + \tan^{-1}((w + 2x)/2h)]^2 dx. \tag{7}$$

Using the concepts of complex analysis /11/(the Cauchy Integral Theorem and proper complex contour of integration), the x integration can be evaluated in closed form (the details we leave out) and the internal inductance per unit length of the silicon substrate is given by

$$L_{si} = \frac{\mu_0 \delta}{w\pi}(1 - e^{-2d/\delta}) \left\{ \tan^{-1}\left(\frac{w}{2h}\right) - \frac{h}{w}\log\left[1 + \left(\frac{w}{2h}\right)^2\right] \right\} \tag{8}$$

3. Results and discussions

In this section, two sets of resistivities (low and high resistivity substrate) used commonly in VLSI intergated circuits are analyzed. Fig.2 illustrates the significance of the frequency dependent internal inductance of silicon for single interconnects on a heavily doped CMOS 300 mm substrate (resistivity ρ_{si} =0.01 Ωcm) with a 3 μm oxide layer. The width of zero-thickness conductor is 2 μm. Fig. 2a shows the results of the total inductance ($L_t = L_{ex} + L_{si}$), the external inductance L_{ex}, and the internal silicon substrate

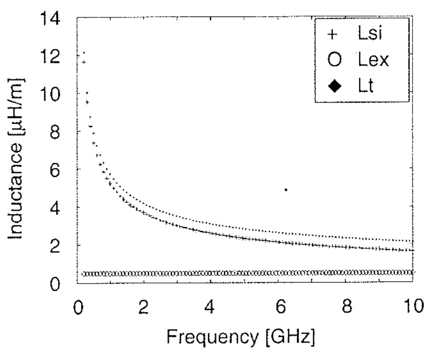

Fig. 2. (a) Total, external, and internal silicon substrate inductance for single interconnects used in silicon-based IC circuits,

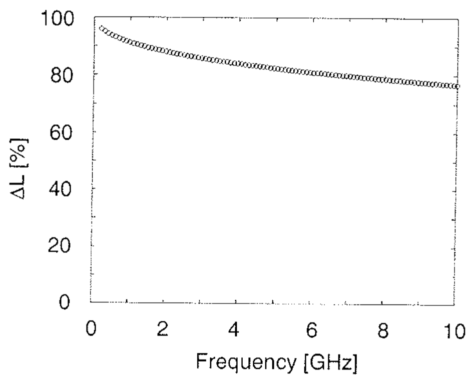

Fig. 2. *(b) The percent of internal silicon substrate inductance compared to total inductance for single interconnects.*

inductance L_{si}. (For computation of total and external inductances of on-chip interconnects we used the proposed methodology in /9, 10/) The external inductance is very small compared to the internal inductance of silicon substrate at low frequencies, but at very high frequencies the differences becomes smaller. This is further illustrated in Fig. 2b, where the percent ΔL of silicon substrate inductance compared to the total inductance L_t is plotted as a function of frequency. For low, medium and high frequencies up to 20GHz internal inductance of silicon substrate have a strong effect on the propagation properties of IC interconnects.

For the case of a microstrip interconnect on a 500-μm high resistivity silicon substrate (resistivity ρ_{si} = 10 Ω-cm) with a 2-μm oxide layer, the internal inductance of silicon as a function of frequency is shown in Fig. 3. The width of microstrip is 4μm. For high resistivity silicon substrate, the total, external and internal silicon inductances becomes frequency-independent (see Fig. 3) and the variation of the magnetic flux penetration into silicon substrate becomes frequency-independent, too. The internal inductance of silicon is comparable to the external inductance, as illustrated Fig. 3. The numerical results presented in this chapter show that the influence of the resistivity of the semiconducting substrate on the frequency-dependent and

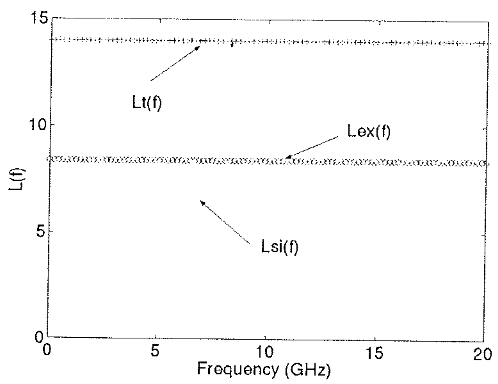

Fig. 3. *Frequency dependence of total, external and internal silicon inductances as a function of frequency for high resistivity silicon substrate.*

transient behaviour of on-chip interconnects is significant and can not be neglected.

4. Conclusion

In this paper, we introduced a closed-form expression for the frequency-dependent internal inductance of silicon of on-chip interconnects. We have investigated the sensitivity of the frequency-dependent inductance per unit length of microstrip interconnect to the substrate resistivity. The results presented here illustrate that the resistivity of the silicon as well as frequency play strong role in determining the relative importance of the silicon substrate inductance (substrate skin effect).

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H. Ymeri, B. Nauwelaers, K. Maex,
Katholieke Universiteit Leuven, Department of Electrical Engineering (ESAT), Div. ESAT-TELEMIC, Kasteelpark Arenberg 10, B-3001 Leuven-Heverlee, Belgium

D. De Roest, M. Stucchi
The Interuniversity Microelectronics Center (IMEC), Kapeldreef 75, B-3001 Leuven, Belgium

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