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The spectrum of charmonium in the Resonance-Spectrum Expansion*

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Abstract. We argue that the resonance-like structures Y(4260) [1,2], Y(4360), Y(4660) [3] and Y(4635) [4], which were recently reported to have been observed in experiment, are non-resonant manifestations of the Regge zeros that appear in the production amplitude of the Resonance-Spectrum Expansion. Charmonium $c\bar{c}$ states are visible on the slopes of these enhancements.

In the Resonance-Spectrum Expansion (RSE) [5], which is based on the model of Ref. [6], the meson-meson scattering amplitude is given by an expression of the form (here restricted to the one-channel case)

$$T(E) = \left\{ -2\lambda^2 \mu p j_\ell^2 \left(p r_0 \right) \sum_{n=0}^{\infty} \frac{\left| g_{nL(\ell)} \right|^2}{E - E_{nL(\ell)}} \right\} \Pi(E) \quad , \tag{1}$$

where p is the center-of-mass (CM) linear momentum, $E = E(p)$ is the total invariant two-meson mass, j_ℓ and $h_\ell^{(1)}$ are the spherical Bessel function and Hankel function of the first kind, respectively, μ is the reduced two-meson mass, and r_0 is a parameter with dimension mass^{-1}, which can be interpreted as the average string-breaking distance. The coupling constants g_{NL}, as well as the relation between ℓ and $L = L(\ell)$, were determined in Ref. [7]. The overall coupling constant λ, which can be formulated in a flavor-independent manner, represents the probability of quark-pair creation. The dressed partial-wave RSE propagator for strong interactions is given by

$$\Pi_\ell(E) = \left\{ 1 - 2i\lambda^2 \mu p j_\ell \left(p r_0 \right) h_\ell^{(1)} \left(p r_0 \right) \sum_{n=0}^{\infty} \frac{\left| g_{NL} \right|^2}{E - E_{NL}} \right\}^{-1} \quad . \tag{2}$$

* Talk delivered by Eef van Beveren

This propagator has the very intriguing property that it vanishes for $E \to E_{NL}$. We will show in the following that this phenomenon can be, and has indeed been, observed in experiment, but not in scattering processes, as one easily verifies that the RSE amplitude for strong scattering (1) does not vanish in the limit $E \to E_{NL}$. However, for strong production processes, we derived in Ref. [8], following a procedure similar to the one by Roca, Palomar, Oset, and Chiang [9], a relation between the production amplitude $\mathbf{P}$ and the scattering amplitude $\mathbf{T}$, reading

$$\mathbf{P}_\ell = j_\ell\,(pr_0) + i\,\mathbf{T}_\ell h_\ell^{(1)}\,(pr_0) \quad , \tag{3}$$

which, using Eqs. (2) and (1), can also be written as

$$\mathbf{P}_\ell = j_\ell\,(pr_0)\,\mathbf{\Pi}_\ell(E) \quad . \tag{4}$$

From this expression we find, by the use of Eq. (2), that the production amplitude tends to zero when $E \to E_{NL}$. This effect must be visible in experimental strong production cross sections.

Actually, in Ref. [8] we found, for the complete production amplitude in the case of multi-channel processes, that Eq. (4) represents the leading term, and that the remainder is expressed in terms of the inelastic components of the scattering amplitude. The latter terms do not vanish in the limit $E \to E_{NL}$, as we have seen above. Hence, the production amplitude only vanishes *approximately* in this limit, in case inelasticity is suppressed.

The reaction of electron-positron annihilation into multi-hadron final states takes basically place via one photon, hence with $J^{PC} = 1^{--}$ quantum numbers. Consequently, when the photon materializes into a pair of current quarks, which couple via the $q\bar{q}$ propagator to the final multi-hadron state, we may assume that the intermediate propagator carries the quantum numbers of the photon. Moreover, alternative processes are suppressed.

We may thus conclude that, if we want to discover whether the propagator really vanishes at $E \to E_{NL}$, then the ideal touchstone is e^+e^- annihilation into multi-hadron states. Furthermore, there also exist predictions for the values of E_{NL}, with $L = 0$ or $L = 2$, given by the parameter set of Ref. [10]. For $c\bar{c}$ one finds in the latter paper $E_{0,0} = 3.409$ GeV and $\omega = 0.19$ GeV, which results for the higher $c\bar{c}$ confinement states in the spectrum $E_{1,0} = E_{0,2} = 3.789$ GeV, $E_{2,0} = E_{1,2} = 4.169$ GeV, $E_{3,0} = E_{2,2} = 4.549$ GeV, $\dots$.

The latter two levels of the $c\bar{c}$ confinement spectrum can indeed be clearly observed in experiment. For example, the non-resonant signal in $e^+e^- \to \pi^+\pi^-\psi(2S)$ (see Fig. 5 of Ref. [3]) is divided into two substructures [11–13], since the full $c\bar{c}$ propagator (2), dressed with meson loops, vanishes at $E_3 = 4.55$ GeV [10]. In the same set of data, one may observe a lower-lying zero at $E_2 = 4.17$ GeV [10], more distinctly visible in the data on $e^+e^- \to \pi^+\pi^- J/\psi$ (see Fig. 3 of Ref. [2]). The true $c\bar{c}$ resonances can be found on the slopes of the above-mentioned non-resonant structures, unfortunately with little statistical significance, if any.

In fact, in Ref. [1], where the BaBar Collaboration announced the observation of the $Y(4260)$ structure in $e^+e^- \to \pi^+\pi^- J/\psi$, one reads: "*no other structures are evident at the masses of the quantum number $J^{PC} = 1^{--}$ charmonium states, i.e.,* *the* $\psi(4040)$, $\psi(4160)$, *and* $\psi(4415)$". However, in Ref. [14], we demonstrated that the BaBar data at about 4.15 GeV are consistent with the mass and width of the $\psi(4160)$. Here, we will show that also the $\psi(4415)$ is clearly visible in the BaBar data, possibly even with enough statistical significance.

So we indeed observe minima in production processes, which confirm vanishing $q\bar{q}$ propagators. Moreover, the $q\bar{q}$ confinement spectrum predicted 25 years ago in Ref. [10] seems to agree well with experimental observations for vector mesons. Accordingly, we expect vector-meson $q\bar{q}$ resonances associated with each of the Regge states: one ground state, dominantly in a $q\bar{q}$ S-wave, and two resonances for each of the higher excited Regge states, viz. one dominantly in an S-wave, and the other mostly in a D-wave. In Ref. [15], we reported on indications for four, possibly five, new $c\bar{c}$ vector states, in the $e^+e^- \to \Lambda_c^+\Lambda_c^-$ amplitudes of the Belle Collaboration [4]. Here, we will just concentrate on the $\psi(4S)$.

A full description of the $\pi^+\pi^- J/\psi$ involves a three-body calculation. In the present work, however, we will limit us to an effective two-body calculation for $(\pi^+\pi^-)\, J/\psi$, assuming for the $\pi^+\pi^-$ effective mass just a fraction of the available phase space. Furthermore, we assume an S-wave for the relative orbital angular momentum of the $\pi^+\pi^-$-J/ψ system. Under these assumptions, we obtain for the amplitude the result depicted in Fig. 1a, for the case that the propagator of Eq. (2) is substituted by a structureless vertex.

As expected, we observe no resonances in the amplitude of Fig. 1a. Next, we suppress the effect of the resonance poles in the propagator of Eq. (2), such that the zeros at $E = E_{NL}$ dominate production (see Eq. (4)). The resulting amplitudes are shown in Figs. 1b and 1c. We observe that now our theoretical amplitude is in rather good agreement with the data. There is an excess of data for energies below 4.0 GeV, stemming from the tail of the $\psi(3685)$ resonance, which dominates the amplitude at lower masses and which is not accounted for in our amplitude. Furthermore, in Ref. [14] we discussed the $\psi(4160)$ resonance, which, since not accounted for here, leads to an overestimate of the BaBar data by our theoretical amplitude.

However, there is a rather large overestimate visible in Fig. 1c at the mass of the $\psi(4415)$. In Fig. 1d, we show that the difference between data and our non-resonant amplitude can be perfectly explained by accounting for a $c\bar{c}$ resonance with mass and width consistent with the $\psi(4415)$. Moreover, the experimental error bars indicate that sufficient statistics is available to include this resonance in a data analysis for the non-resonant Y structures.

Summarizing, we have shown that the $c\bar{c}$ confinement spectrum, which underlies scattering and production of multi-meson systems containing charmonium $q\bar{q}$ pairs, can be observed in production amplitudes. Moremore, we have shown that the $c\bar{c}$ resonance poles are present in the $e^-e^+ \to J/\psi\pi^+\pi^-$ amplitude.

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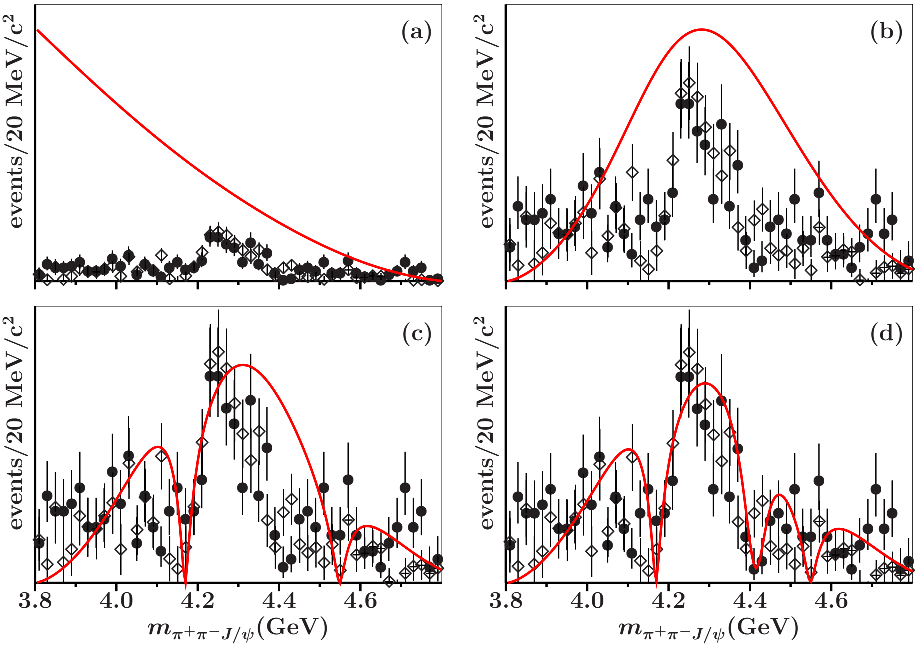

Fig.1. The $J/\psi(\pi^+\pi^-)$ invariant-mass distributions for the reaction $e^-e^+ \to J/\psi\pi^+\pi^-$. Data are taken from Ref. [1] (•) and Ref. [2] (⋄). The theoretical results (solid lines) are also discussed in the text: (a) shows the distribution for a non-resonant structureless $c\bar{c}$ propagator; (b) and (c) show the effect of Regge zeros in the $c\bar{c}$ vector propagator, thereby suppressing the contributions of its $c\bar{c}$ poles; (b) for just the Regge zero at 3.79 GeV; (c) for the zeros at 4.17 and 4.55 GeV as well; (d) shows the additional effect of the $c\bar{c}$ resonance pole in the propagator at $4.415 - i0.036$ GeV [16].

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