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On regular and equivelar Leonardo polyhedra

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Abstract

A Leonardo polyhedron is a 2-manifold without boundary, embedded in Euclidean 3-space $\mathbf{E}^3$, built up of convex polygons and with the geometric symmetry (or rotation) group of a Platonic solid and of genus $g \geq 2$. The polyhedra are named in honour of Leonardo's famous illustrations in [1] (cf. also [2]). Only six combinatorially regular Leonardo polyhedra are known: Coxeter's four regular skew polyhedra, and the polyhedral realizations of the regular maps by Klein of genus 3 and by Fricke and Klein of genus 5. In this paper we construct infinite series of equivelar (i.e. locally regular) Leonardo polyhedra, which share some properties with the regular ones, namely the same Schläfli symbols and related topological structure. So the weaker condition of local regularity allows a much greater variety of symmetric polyhedra.

[1] L. Pacioli, *De Divina Proportione (Disegni di Leonardo da Vinci 1500-1503)*, Faksimile Dominiani, Como, 1967.

[2] D. Huylebrouk, Lost in triangulation: Leonardo da Vinci's mathematical slip-up, *Scientific American*, March 29, 2011.

Keywords: Equivelar polyhedron, Leonardo polyhedron, regular polyhedron, genus, Schläfli symbol, symmetry group.

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O regularnih in ekvivelarnih Leonardovih poliedrih

Povzetek

Leonardov polieder je 2-razsežna mnogoterost brez roba, vložena v 3-razsežen evklidski prostor $\mathbf{E}^3$, zgrajena iz konveksnih poligonov in z geometrijsko (oz. rotacijsko) grupo simetrij platonskega telesa ter rodu $g \geq 2$. Ti poliedri so dobili ime na čast Leonardovim slavnim ilustracijam v [1] (glej tudi [2]). Znanih je le šest kombinatorično regularnih Leonardovih poliedrov: štirje Coxeterjevi regularni poševni poliedri in poliedrski realizaciji regularnega Kleinovega zemljevida rodu 3 ter Fricke- Kleinovega rodu 5. V članku konstruiramo neskončno zaporedje lokalno regularnih Leonardovih poliedrov, ki imajo nekatere skupne lastnosti z regularnimi poliedri: iste Schläflijeve simbole ter sorodno topološko strukturo. Torej šibkejši pogoj lokalne regularnosti omogoča precej večjo raznolikost simetričnih poliedrov.

[1] L. Pacioli, *De Divina Proportione (Disegni di Leonardo da Vinci 1500-1503)*, Faksimile Dominiani, Como, 1967.

[2] D. Huylebrouk, Lost in triangulation: Leonardo da Vinci's mathematical slip-up, *Scientific American*, March 29, 2011.

Ključne besede: Ekvivelarni polieder, Leonardov polieder, regularni polieder, rod, Schläflijev simbol, simetrijska grupa.