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Zarlino's Harpsichord: A Contribution to the (Pre)History of Equal Temperament

Zarlinov čembalo: prispevek k (pred)zgodovini enakomerne uglasitve

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Accepted: 11th December 2017**Ključne besede:** Gioseffo Zarlino, glasbena teorija, renesansa, vprašanje o uglasitvi**Keywords:** Gioseffo Zarlino, music theory, Renaissance, tuning question

IZVLEČEK

ABSTRACT

Prispevek obravnava vprašanje o uglasitvi, kot se kaže v osrednjem glasbenoteoretskem spisu Gioseffa Zarlina *Istitutioni harmoniche* (1558). Predstavljen je sistem uglasitve, ki ga Zarlino utemeljuje v vokalni glasbi (sintonični diatonični sistem), prikazane so njegove omejitve in slednjič njegova prilagoditev (temperacija) za rabo v instrumentalni glasbi.

This paper deals with the tuning question as it is discussed in Gioseffo Zarlino's principal musical-theoretical treatise, *Istitutioni harmoniche* (1558). First, Zarlino's tuning system in vocal music (the syntonic diatonic) is presented; then, its shortcomings are discussed; and finally, its adjustment (temperament) for the use in instrumental music is explained.

In 1548, Domenico da Pesaro constructed a *gravecembalo* for Gioseffo Zarlino, the description and sketch of which were later included in the theorist's famous treatise, *Istitutioni harmoniche* (1558).¹ The instrument immediately draws a modern reader's

1 Gioseffo Zarlino, *Istitutioni armoniche* (Treviso: Diastema, 2011), 293–294.

attention. It is quite unconventional, with many more keys on the keyboard than one would expect to find on a modern piano: Not one but two chromatic keys are placed between diatonic keys framing a whole tone, and there is even an additional chromatic key between those diatonic keys that frame a semitone. Therefore, questions may be posed about why the instrument was constructed this way and what the function of so many keys on its keyboard could be.

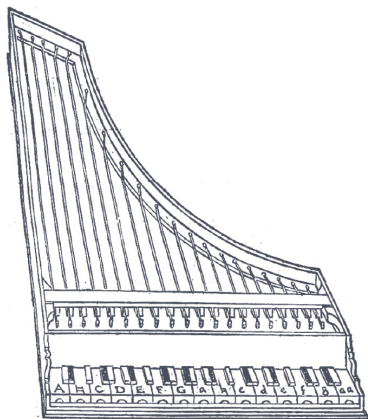


Figure 1: The sketch of the instrument that Domenico da Pesaro constructed for Gioseffo Zarlino in 1548 (Gioseffo Zarlino, *Istitutioni harmoniche* (Venezia, 1558), 141).

The answers to the posed questions are closely connected to the tuning of Zarlino's harpsichord. The tuning question (the theoretical search for an acoustic system within which practical music evolves) was one of the key questions discussed by the renaissance music theorists in their treatises. For centuries, the Pythagorean system, founded on perfect consonances whose ratios can be described by the first four numbers,² prevailed. However, with the rise of equal-voice polyphony, besides the Pythagorean perfect consonances, the pleasant-sounding thirds and sixths were becoming more and more important, and the framework of the Pythagorean system soon became too narrow: The Pythagorean tuning had to be adjusted (tempered) to the contemporary musical reality, and in *Istitutioni harmoniche*, Zarlino presented his proposal for the system.

The ancient system as the basis for Zarlino's discussion of the tuning question

Like many other renaissance theorists, Zarlino bases his discussion of the tuning system in *Istitutioni harmoniche* on several important ancient music theorists' writings, above all Boethius' and Ptolemy's. Consequently, the model for his system is a

2 Double octave (4:1), octave plus fifth (3:1), octave (2:1), fifth (3:2), and fourth (4:3). For a more detailed description of the Pythagorean system, see André Barbera, "Pythagoras", and Mark Lindley, "Pythagorean Intonation".

double-octave scale of 16 tones in which the five tetrachords of the ancient Greater and Lesser Perfect Systems are joined.³



A – hypaton

- 2 hypate hypaton
- 3 parhypate hypaton
- 4 lychanos hypaton
- 5 hypate meson

B – meson

- 5 hypate meson
- 6 parhypate meson
- 7 lychanos meson
- 8 mese

C – diezeugmenon

- 10 paramese
- 11 trite diezeugmenon
- 12 paranete diezeugmenon
- 13 nete diezeugmenon

D – hyperboleon

- 13 nete diezeugmenon
- 14 trite hyperboleon
- 15 paranete hyperboleon
- 16 nete hyperboleon

E – synemmenon

- 1 proslambanomenos
- 8 mese
- 9 trite synemmenon
- 11 paranete synemmenon
- 12 nete synemmenon

Figure 2: The Greater Perfect System with tetrachord synemmenon in diatonic genre.

When he explains the construction of his system, Zarlino also gives a detailed account of some of the species of all three tetrachord genera.⁴ At the beginning of this presentation, the theorist informs us that he will not deal with Aristoxenus', Archytas', Didymus', or Eratosthenes' divisions of the tetrachord, as they were rejected by both Boethius and Ptolemy and are therefore useless.⁵ He will only discuss the divisions that according to Ptolemy were accepted as the best by the musicians of his time. In Zarlino's opinion, these are the most rational (*rationali*), and the ear hears them as the

3 Zarlino dedicates the major part of the *Istitutioni*'s second book to the construction of the double-octave system in which the ancient Greater and Lesser Perfect Systems are joined (see above all Zarlino, *Istituzioni armoniche*, 215–242). Zarlino's discussion is based mostly on Chapter 20 from the first book of Boethius' *Fundamentals of Music* (Anicius Manlius Severinus Boethius, *Fundamentals of Music*, trans. Calvin M. Bower (New Haven: Yale University Press, 1989), 29–39).

4 As is commonly known, in the core of the ancient Greek tonal system was the tetrachord. It was based on the interval, which the Greek music theorists understood as the smallest consonance: Each tetrachord was composed of four tones, the outer two always being in the ratio of the fourth. On the other hand, the inner two were movable and could divide the fourth in many different ways, thus forming many different variants of the tetrachord, some of them more similar to each other than others. Consequently, it was already Aristoxenus (fourth century BC) who systematized three different genera of music, the *diatonic* (in a modern approximation composed of the interval sequence semitone – tone – tone), the *chromatic* (semitone – semitone – minor third), and the *enharmonic* (quartertone – quartertone – major third). The tetrachords systematized into one of the given genera became their species. Aristoxenus discussed the genera of music in the first and second books of *Harmonica Stoicheia* (Andrew Barker, *Greek Musical Writings II* (Cambridge: Cambridge University Press, 1989), 139–144, 159–161).

5 Boethius deals with the divisions of the tetrachord in Chapters 15, 16, 17, 18, and 19 of Book 5 (Boethius, *Fundamentals of Music*, 174–179); however, he almost literally follows Ptolemy, who discusses the subject in Chapters 12, 13, 14, 15, and 16 of Book 1 of *Harmonika* (Barker, *Greek Musical Writings II*, 301–314).

most consonant (*più consonanti al udito*).⁶ The tetrachord species Zarlino discusses in *Istituzioni* are listed in the table below. To make things clearer, a presentation of the modern approximations of the ancient tetrachord genera has been added to it (figure 3), and the values of interval ratios (R) have been calculated in cents (C).

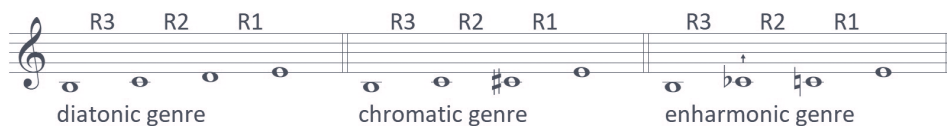


Figure 3: Modern approximations of the ancient tetrachord genera.

Species of tetrachord	R 1	C 1	R 2	C 2	R 3	C 3
ditonic diatonic (<i>diatonico diatono</i>)	9:8	203.89	9:8	203.89	256:243	90.22 ⁷
soft diatonic (<i>diatonico molle</i>)	8:7	231.16	10:9	182.39	21:20	84.46
syntonic or tense diatonic or Natural (<i>diatonico sintono overo incitato, ò Naturale</i>)	10:9	182.39	9:8	203.89	16:15	111.72
tonic diatonic (<i>diatonico toniaco</i>)	9:8	203.89	8:7	231.16	28:27	62.96
equal diatonic (<i>diatonico equale</i>)	10:9	182.39	11:10	164.99	12:11	150.62
old chromatic (<i>chromatico antico</i>)	19:16	297.49	81:76	110.30	256:243	90.22
soft chromatic (<i>chromatico molle</i>)	6:5	315.61	15:14	119.43	28:27	62.96
tense chromatic (<i>chromatico incitato</i>)	7:6	266.85	12:11	150.62	22:21	80.53
old enharmonic (<i>enharmonico antico</i>)	81:64	407.79	499:486	45.70	512:499	44.52
Ptolemy's enharmonic (<i>enharmonico di Tolomeo</i>)	5:4	386.28	24:23	73.67	46:45	38.05

Table 1: Species of tetrachord discussed by Zarlino in *Istituzioni*.⁷

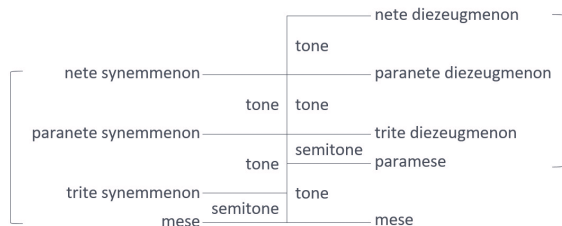
In connection with the just-listed species of tetrachord, it should be noted that the above-presented double-octave system of 16 tones can only be the result of the construction based on the tetrachord species used by Zarlino (following Boethius): It has to be constructed with the first species of the diatonic tetrachord (ditonic diatonic).⁸ If

6 Zarlino, *Istituzioni armoniche*, 186-187.

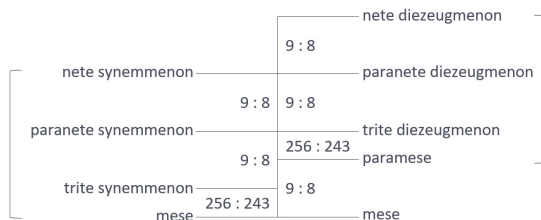
7 The sum of the cents in a single table row has to be 498 - it is the size of the fourth in the ratio 4:3 (which at the same time is the sum of all single table row ratios).

8 As can be seen in the above Table 1, the ditonic diatonic tetrachord is composed of a diatonic semitone in the ratio of 256:243 and two equal whole tones in the ratio of 9:8. It is on this tetrachord that the so-called Pythagorean tuning system, founded on ratios that were supposed to have been discovered in audible music already by Pythagoras, was built. The whole tone in the ratio of 9:8 can actually be expressed as a difference between the fifth (3:2) and fourth (4:3), $\frac{3}{2} : \frac{4}{3} = \frac{3}{2} \cdot \frac{3}{4} = (9:8)$. Moreover, the ratio of the diatonic semitone 256:243 can be expressed as a difference between the fourth (composed of two whole tones and a semitone) and two whole tones: $\frac{4}{3} : \frac{2}{3} \cdot \frac{2}{8} = \frac{4}{3} \cdot \frac{2}{8} \cdot \frac{2}{8} = (256:243)$. Also constructed on the ditonic diatonic tetrachord is the system in Boethius' *Fundamentals of Music*.

the system were built on any other tetrachord species, the result could be different. The differences would indeed occur within tetrachords *synemmenon* and *diezeugmenon*, namely between *paranete synemmenon* and *trite diezeugmenon* and between *nete synemmenon* and *paranete diezeugmenon*. If we assumed that all semitones in the system were of the same size (for example 100 cents, as it is common in modern tuning), then the named tones would sound in unison:



Since in a ditonic diatonic tetrachord the semitone in the ratio of 256:243 is followed by two equal tones in the ratio of 9:8, *paranete synemmenon* and *trite diezeugmenon* and *nete synemmenon* and *paranete diezeugmenon* are of the same relative pitch:



Quite to the contrary, the differences between the tones in question would emerge if the system were constructed with any of the other diatonic tetrachords. In the first of the tables below, the ratios of all five species of diatonic genre are compared. For a clearer image, the second table shows the summarized ratios between *mese* and *paranete synemmenon* and *mese* and *trite diezeugmenon*, whereas in the third table, the summarized ratios between *mese* and *nete synemmenon*⁹ and *mese* and *paranete diezeugmenon* are listed.

					nete diez.					
					1 2 3 4 5					
					tone	9 : 8	8 : 7	10 : 9	9 : 8	10 : 9
					paran. diez.					
					tone	9 : 8	10 : 9	9 : 8	8 : 7	11 : 10
					trite diez.					
					tone	256 : 243	21 : 20	16 : 15	28 : 27	12 : 11
					paramese					
					tone	9 : 8	9 : 8	9 : 8	9 : 8	9 : 8
					mese					
					semitone					
					mese					

1	2	3	4	5
9 : 8	8 : 7	10 : 9	9 : 8	10 : 9
9 : 8	10 : 9	9 : 8	8 : 7	11 : 10
256 : 243	21 : 20	16 : 15	28 : 27	12 : 11

9 Since the whole tetrachord *synemmenon* is in question here, the ratio is always 4:3.

1	2	3	4	5	paran. syn.	trite diez.	1	2	3	4	5
32 : 27	7 : 6	6 : 5	32 : 27	6 : 5	tone	semitone	32 : 27	189 : 160	6 : 5	7 : 6	27 : 22
					trite syn.	tone					
					mese	semitone					
						mese					

1–5	nete syn.	paran. diez.	1	2	3	4	5
		tone	4 : 3	21 : 16	54 : 40	4 : 3	27 : 20
	paran. syn.	tone					
		trite diez.					
	4 : 3	semitone					
		tone					
	trite syn.	paramese					
		tone					
	mese	semitone					
		mese					

Diatonic tetrachord

1 ditonic 3 syntonic or tense 5 equal
 2 soft 4 tonic

It can be observed that *paranete synemmenon* and *trite diezeugmenon* differ in pitch in soft, tonic, and equal diatonic tetrachords and that *nete synemmenon* and *paranete diezeugmenon* are different in soft, syntonic, and equal diatonic tetrachords. Thus, the system will only include 16 tones if built with the ditonic diatonic tetrachord. If it were constructed with the syntonic or tonic diatonic tetrachord, the number would increase to 17, and if the soft or equal diatonic tetrachord were taken as its foundation, there would be 18 different tones in the system. Moreover, the use of different chromatic or enharmonic species or even a merging of all three genera in a unified system (which is actually done in *Istituzioni*¹⁰) would result in many more variants; in this case, the system could include even more than 30 different tones.

Zarlino's syntonic diatonic tuning in vocal music

After the construction and exploration of the system in all of the presented species of all three musical genera, the next obvious step for Zarlino is to find the one used in contemporary music or – in other words – to determine the contemporary tuning. He maintains that up to his time, everyone was convinced that in playing and singing, the first species of diatonic genre, namely the ditonic diatonic, was used; it was advocated by both ancient and modern theorists.¹¹ Zarlino does not agree with them: He is convinced that in contemporary music, the syntonic diatonic tuning is in use.

The syntonic diatonic system is constructed on the syntonic diatonic tetrachord, in which a diatonic semitone in the ratio of 16:15 is followed by two different tones in the ratios of 9:8 and 10:9.

10 See Zarlino, *Istituzioni armoniche*, 251–254.

11 Zarlino, *Istituzioni armoniche*, 190.



Figure 4: The syntonic diatonic tetrachord.

The syntonic diatonic system is one of those combinations mentioned above that would include 17 tones, as in it, *nete synemmenon* and *paranete diezeugmenon* differ in pitch. In this case, the difference between them is the difference between the major tone in the ratio of 9:8 (*trite diezeugmenon* – *paranete diezeugmenon*) and the minor tone in the ratio of 10:9 (*paranete synemmenon* – *nete synemmenon*); it is in the ratio of 81:80.¹²

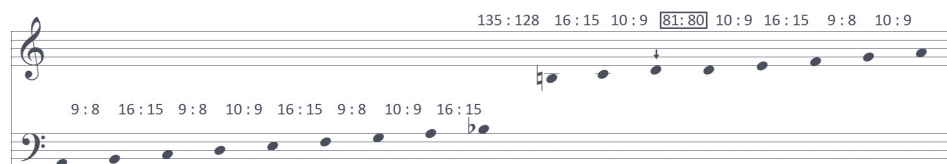


Figure 5: System in the syntonic diatonic genre.

Zarlino describes the interval that occurs as the difference between the major and minor tones as the smallest interval (*minimo intervallo*) and names it *comma*.¹³ Even if it is small and cannot be used on its own, it doesn't mean it's entirely useless, Zarlino explains. Only with the help of this interval can the fifth between d and a¹⁴ and the minor third between d and f¹⁵ be obtained. If in practice, in singing and playing instruments, the comma were used on its own, it wouldn't please the ear. However, nature has seen to it that its effect is dispersed through the voices and does not affect the hearing, Zarlino concludes.¹⁶

Zarlino is convinced that intervals, as they were created by nature, can only be found within the ratios of the syntonic diatonic system: According to *senario*,¹⁷ only

12 $\frac{9}{8} : \frac{10}{9} = \frac{9}{8} \cdot \frac{9}{10} = \frac{81}{80} = (81:80)$.

13 Zarlino, *Istituzioni armoniche*, 262. Since the comma in question originates in the difference between the major and minor tones of the syntonic species of diatonic genera, it is called syntonic comma; its size is 21.5 cents.

14 Between d and a there is a diminished fifth in the ratio of 40:27: $\frac{10}{9} : \frac{9}{8} = \frac{10}{9} \cdot \frac{8}{9} = \frac{80}{81}$. $\frac{10}{9} = \frac{14400}{9720} = \frac{40}{27}$ (40:27). If the comma (81:80) is added to it, the result is the fifth in its true ratio 3:2: $\frac{40}{27} \cdot \frac{81}{80} = \frac{3240}{2160} = \frac{3}{2} = (3:2)$

15 Between d and f there is a diminished minor third in the ratio of 32:27: $\frac{16}{15} : \frac{10}{9} = \frac{160}{135} = \frac{32}{27}$ (32:27). If the comma (81:80) is added to it, the result is the minor third in its true ratio 6:5: $\frac{32}{27} \cdot \frac{81}{80} = \frac{2592}{2160} = \frac{6}{5} = (6:5)$.

16 Zarlino, *Istituzioni armoniche*, 262.

17 As is well known, in *Istituzioni*, Zarlino rejects the Pythagorean system founded on the first four numbers, from which the ratios of perfect consonances can be derived. Besides the Pythagorean consonances, he also defines thirds and sixths as consonant intervals (although imperfect). However, the thirds and sixths did not sound good in the Pythagorean system since their ratios are quite complex (32:27, 81:64, 128:81, and 27:16). Therefore, instead of number four (*numero quaternario*), Zarlino advocates the importance of number six (*numero senario*). In addition to the ratios of perfect consonances, from the first six numbers, the simpler ratios of better-sounding major (5:4) and minor (6:5) third and major sixth (5:3) can be derived. Since the ratio of minor sixth (8:5), which was also considered to be consonant, remains outside the first six numbers, along with the number six, Zarlino also advocates the importance of number eight. For a more detailed description of *senario*, see Zarlino, *Istituzioni armoniche*, 61–70, 130–133.

the thirds in the ratios of 5:4 (major) and 6:5 (minor) and sixths in the ratios of 5:3 (major) and 8:5 (minor) are consonant, and these are only present in the syntonic species of diatonic genre.¹⁸



Figure 6: The ratios of thirds in the syntonic diatonic genre.

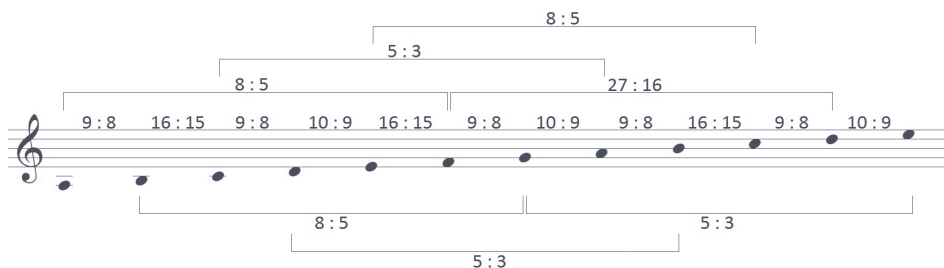


Figure 7: The ratios of sixths in the syntonic diatonic genre.

After discovering that the consonant thirds and sixths are only present within the syntonic diatonic genre, Zarlino considers all the other diatonic species imperfect: There is no perfect harmony within them.¹⁹ And if there is no perfect harmony within the other diatonic species, how could it ever be present within the chromatic or enharmonic genre? Their species are not only without imperfect consonances but, in many cases, also without the perfect ones, Zarlino concludes.²⁰

Since in syntonic tuning only untempered intervals in their basic (natural) ratios are used, it is also referred to as *just intonation* or *natural tuning*.²¹

The weaknesses of syntonic diatonic system

Although Zarlino advocates it as the only perfect system, there are some discrepancies to be found in the syntonic diatonic tuning as well. From the above figures 6 and 7, it is clear that not all of the thirds and sixths are in the ratios defined as natural by Zarlino after all: Between d and f, there is the already mentioned third in the ratio

¹⁸ Zarlino, *Istituzioni armoniche*, 231–233.

¹⁹ Zarlino, *Istituzioni armoniche*, 231.

²⁰ Zarlino, *Istituzioni armoniche*, 245, 250.

²¹ For a more detailed description see Bruno Ravnikar, *Osnove glasbene akustike in informatike* (Ljubljana: DZS, 1999), 38–39, and Ross W. Duffin, *How Equal Temperament Ruined Harmony (and Why You Should Care)* (New York: W. W. Norton & Company, 2007), 33.

of 32:27, which is a comma smaller than minor third in the regular ratio of 6:5, and between f and d, there is a sixth in the ratio of 27:16, which is a comma larger than the major sixth in the regular ratio of 5:3.²² Some discrepancies can also be found among the intervals of fourth, fifth, and seventh.

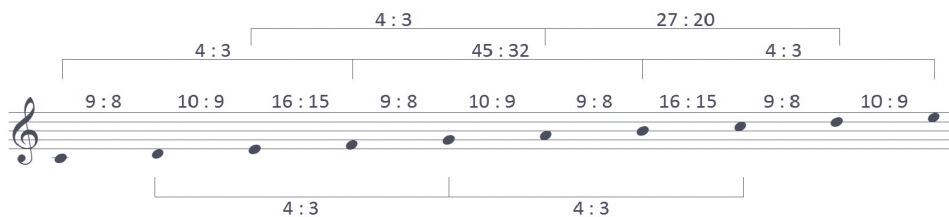


Figure 8: The ratios of fourths in syntonic diatonic tuning.



Figure 9: The ratios of fifths in syntonic diatonic tuning.

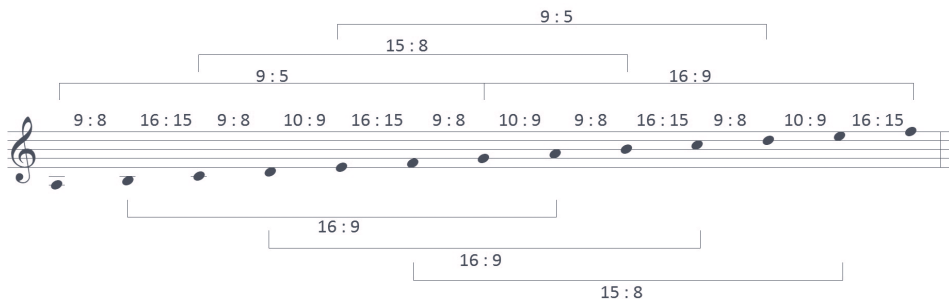


Figure 10: The ratios of sevenths in syntonic diatonic tuning.

Besides the fourth (4:3) and the tritone (45:32), there is also an augmented fourth in the ratio of 27:20 (a comma larger than pure²³) present in the system. Besides the fifth in the ratio of 3:2 and the semidiapente in the ratio of 64:45, there is the fifth in the ratio

22 $\frac{5}{3} \cdot \frac{81}{80} = \frac{405}{320} = \frac{27}{16} = (27:16)$.
 23 $\frac{5}{3} \cdot \frac{81}{80} = \frac{27}{20} = (27:20)$.

of 40:27 (a comma smaller than pure²⁴). And finally, besides the major (15:8) and minor (9:5) sevenths, there's also the one in the ratio of 16:9 (a comma smaller than minor²⁵). In addition to the discrepancies just listed, it is the two different tones (9:8 and 10:9) that represent a special and perhaps even bigger problem in the system. For better understanding, in the following table, a comparison between the intervals of syntonic diatonic and modern (equally tempered) tuning is given.

Interval	Syntonic tuning		Modern tuning		Comparison (cents)
	ratio	cents	no. of semitones	cents	
comma	81:80	21.5	-	-	-
semitone	16:15	111.72	1	100	+ 11.72
minor tone	10:9	182.39	2	200	- 17.61
major tone	9:8	203.89	2	200	+ 3.89
minor third	6:5	315.62	3	300	+ 15.62
major third	5:4	386.28	4	400	- 13.72
fourth	4:3	498.00	5	500	- 2.00
augmented fourth	45:32	590.18	6	600	- 9.82
diminished fifth	64:45	609.72	6	600	+ 9.72
fifth	3:2	701.90	7	700	+ 1.90
minor sixth	8:5	813.62	8	800	+ 13.62
major sixth	5:3	884.29	9	900	- 15.71
minor seventh	9:5	1017.52	10	1000	+ 17.52
major seventh	15:8	1088.18	11	1100	- 11.82
octave	2:1	1199.91	12	1200	- 0.09

Table 2: The comparison between the intervals of syntonic diatonic and modern tuning.

As can be seen, the differences are significant, especially if one considers them in the context of 16th-century polyphony. While in the case of equally tempered tuning (based on an absolute mathematical division), the various combinations of intervals in a composition are not a problem, considerable discrepancies leading to distunement could occur in the case of syntonic diatonic tuning (based on the intervals in their pure ratios). Let's imagine a 16th-century polyphonic composition of equal voices that intertwine with all of the listed untempered syntonic intervals: Both between consequent chords and within individual chords, discrepancies could quickly occur. With a careful composer and singers' ability to adjust their intonation while singing, in vocal music, these discrepancies could mostly be avoided.²⁶ On the other hand, to make all the intervals in instrumental music simultaneously consonant, above all those on keyboard instruments, represented a serious problem: If, for example, the thirds were

24 $\frac{3}{2} \cdot \frac{81}{80} = \frac{3}{2} \cdot \frac{81}{80} = \frac{243}{160} = \frac{40}{27} = (40:27)$.

25 $\frac{9}{5} \cdot \frac{81}{80} = \frac{9}{5} \cdot \frac{81}{80} = \frac{729}{400} = \frac{16}{9} = (16:9)$.

26 The singers' adjustment of pitch in the case of syntonic intervals whose ratios differ from their pure forms (for example, the diminished fifth in the ratio of 40:27) is also confirmed by Zarlino (Zarlino, *Istituzioni armoniche*, 262). In a way, this means that the syntonic diatonic tuning was not consistently employed.

consonant, the octaves and fifths were not in their true ratios and vice versa. The just said can be illustrated with the juxtaposition of the series of four fifths and the series of two octaves plus a major third.

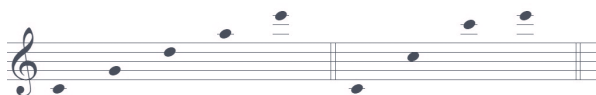


Figure 11: The juxtaposition of four fifths and two octaves plus a major third.

In the case of modern equal temperament (in which the size of all semitones is 100 cents), the last tone in the series of four fifths (fifth = 7 semitones = 700 cents; $4 \cdot 700 = 2800$ cents) and the last tone in the series of two octaves plus a major third (octave = 12 semitones = 1200 cents, major third = 4 semitones = 400 cents; $2 \cdot 1200 + 400 = 2800$ cents) will sound in unison. However, the result would be different if the syntonic intervals were used. In this case, the size of four fifths would be 2807.6 cents, whereas the size of two octaves with a major third would be 2786.09. The difference (a syntonic comma of 21.5 cents) is significant, and when thirds and sixths were already completely established as important intervals in compositions, it could have posed a serious problem.

Zarlino's temperament of the syntonic diatonic system in instrumental music

Zarlino was aware of the presented problems and of the fact that it is impossible to solve them within the framework of syntonic diatonic tuning. In *Istituzioni*, he clearly states that none of the presented species of diatonic genre is (and none can be) used in the tuning of instruments.²⁷ In order to achieve concordance between various intervals, they must, in fact, be slightly changed, tempered. The temperament, however, has to be small enough that the thirds still sound consonant and please the ear (even if they aren't in their natural ratios). The starting point for instrumental tuning advocated in *Istituzioni* is therefore the temperament of the syntonic diatonic system. The latter can be performed in three different ways:²⁸ (1) All of the intervals become irrational.²⁹ (2) Major third and minor sixth remain rational and perfect, whereas all other intervals become irrational. (3) Minor third and major sixth remain rational and perfect, whereas all other intervals become irrational. In all three cases, some sort of median system between the Pythagorean and syntonic will emerge: In the equality of tones, it will resemble the first, and in the consonance of all other intervals, the second.

In modern theorists' opinion, the temperament in question is the product of a long and thorough study, says Zarlino, who doesn't agree with them at all: He is certain that

²⁷ Zarlino, *Istituzioni armoniche*, 264.

²⁸ Zarlino, *Istituzioni armoniche*, 264-265.

²⁹ It means that these intervals cannot be expressed by fractions, and consequently, their ratios cannot be determined.

musicians only discovered this temperament by coincidence, when one of them incidentally tuned his instrument this way. Since both thirds and sixths were not treated as consonances in the past, the theorists certainly didn't make the effort of including them in the system.³⁰ Thus in Zarlino's opinion (regardless of its accuracy), the instrumental tuning was discovered by coincidence in musical practice and was not the result of theoretical deliberations – just as thirds and sixths are consonant because (unlike in the past and especially in antiquity) they are used as such in the contemporary practice.³¹

A detailed presentation of the temperament of the syntonic diatonic system, made in accordance with the first of the three given possibilities, follows. The starting point for its realization is the division of the syntonic comma found between *nete synemmenon* and *paranete diezeugmenon*.³² As a result, both tones will sound in unison, and in number of tones, the system will become equal to ditonic diatonic, since it will only include 16 tones instead of 17.³³ The distribution of the comma has to be made in such a way that it will alter the order and the form of the intervals as little as possible and the hearing won't be offended, Zarlino continues to explain. With that in mind, the temperament of the individual intervals is as follows:³⁴

- (1) Each fifth has to be diminished by $2/7$ of a comma, and each fourth has to be augmented by the same amount; since together they form the octave (which is unchangeable), the amount taken from the first has to be added to the other.
- (2) The major third is diminished by $1/7$ of a comma, and the minor third is diminished by the same amount; since these two consonances together form the fifth that was diminished by $2/7$ of the comma, they must both share an equal part of that temperament and become equally imperfect.
- (3) The major tone is diminished by $4/7$ of a comma, and the minor tone is augmented by $3/7$ of a comma; this way, together they will be equally imperfect as their whole, which is the major third diminished by $1/7$ of a comma.
- (4) The major semitone is augmented by $3/7$ of a comma; since with the major tone it forms the minor third, this way their sum will equal it.
- (5) Both sixths are augmented by $1/7$ of a comma, the major sixth being formed from the fourth and the major third, and the minor sixth from the fourth and the minor third.

30 Zarlino, *Istituzioni armoniche*, 265.

31 As can be concluded from Zarlino's explanation, in *Istituzioni*, he actually gives a theoretical description of a system that had already been in use among practicing musicians for quite some time. Therefore, the system is by no means his *invention* as Mark Lindley maintains (see Mark Lindley, "Zarlino's 2/7-comma meantone temperament," in *Music in performance and society: Essays in honor of Roland Jackson*, ed. Malcolm S. Cole and John Koegel (Detroit: Harmonie Park Press, 1997): 179–194, in particular, 181, 183); in the best case, Zarlino was the first to theoretically describe it.

32 Zarlino, *Istituzioni armoniche*, 267. As was already mentioned, Zarlino defines the comma as the smallest interval, which is, however, at this point still further divided for the purpose of temperament of the syntonic diatonic system. As it will be shown further on, the smallest interval used by Zarlino is therefore actually a $1/7$ comma, which, however, is irrational since it cannot be expressed with a fraction. Zarlino's division of the comma and the use of its parts for the temperament of the already-constructed system is indeed an important theoretical concept, since intervals smaller than the comma were not considered by theorists in this way before.

33 Zarlino refers to the above-presented system of united tetrachords in the diatonic genre together with tetrachord *synemmenon*. As was already clarified, since in the ditonic diatonic genre there is only one sort of tone in the ratio of 9:8, *nete synemmenon* and *paranete diezeugmenon* will be of the same relative pitch.

34 Zarlino, *Istituzioni armoniche*, 267–268.

If the temperament is made according to the given instructions, all intervals (except the octave) will be outside their true ratios. Since they will only be changed by small amounts, however, the hearing will not be too affected. This shouldn't surprise us, since it is commonly known that the senses are not perfectly accurate and are often not able to observe small differences, Zarlino maintains, and he goes on to justify the temperament with the comparison to other disciplines as well: Imperfection may be found in every art and in every other creation too.³⁵ The resulting intervals will all be irrational, Zarlino explains: The parts of a comma that are added to or taken from them are all irrational and unintelligible (*irrationali e incognite*), since they cannot be described by definite numbers.³⁶

As we could imagine, the main difficulty of the described procedure was the exact search for the needed parts of a comma. Since it cannot be divided rationally, Zarlino recommends the use of a special geometrical tool, *mesolabe*, for its division – he, however, does not describe the exact procedure.³⁷ It is difficult to imagine that, with the use of this geometrical method, an interval as small as a comma could be accurately divided into seven parts, which would then have to be even more accurately added to or taken from the already-existing intervals. Let us thus try to test Zarlino's temperament instructions with the help of individual interval values, calculated into cents, wherein 1/7 of a comma equals approximately 3.07 cents. In the following table, the comparison (in cents) of syntonic, Zarlino's tempered syntonic, and modern equally tempered tuning is given.

Interval	Syntonic tuning	Zarlino's temp. tuning	Comparison	Modern equally temp. tuning	Zarlino's temp. tuning	Comparison
semitone	111.72	120.93	+ 9.21	100	120.93	+ 20.93
minor tone	182.39	191.60	+ 9.21	200	191.60	- 8.40
major tone	203.89	191.61	- 12.28	200	191.61	- 8.39
minor third	315.62	312.55	- 3.07	300	312.55	+ 12.55
major third	386.28	383.21	- 3.07	400	383.21	- 16.79
fourth	498.00	504.14	+ 6.14	500	504.14	+ 4.14
fifth	701.90	695.76	- 6.14	700	695.76	- 4.24
minor sixth	813.62	816.69	+ 3.07	800	816.69	+ 16.69
major sixth	884.29	887.30	+ 3.07	900	887.30	+ 12.70

Table 3: The comparison between the intervals of syntonic, Zarlino's tempered syntonic, and modern equally tempered tuning.

Let us now return to the above juxtaposition of four fifths and two octaves plus a major third and examine it with the values of Zarlino's tempered intervals.

³⁵ Zarlino, *Istituzioni armoniche*, 268–270.

³⁶ Zarlino, *Istituzioni armoniche*, 271.

³⁷ Zarlino, *Istituzioni armoniche*, 272. Nevertheless, Zarlino gives a detailed description of the *mesolabe* and explains its use at another place in *Istituzioni* (see Zarlino, *Istituzioni armoniche*, 210–211).

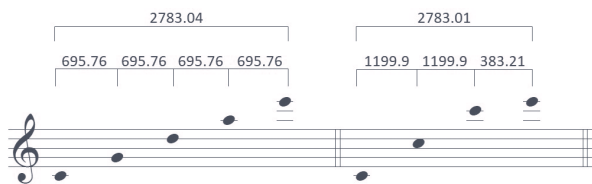


Figure 12: The juxtaposition of four fifths and two octaves with a major third in Zarlino's tempered intervals.

As can be seen, Zarlino's temperament works: The difference between the values of the series of four fifths and that of two octaves with a major third is negligible (0.03 cents). Furthermore, let us check the effect of Zarlino's temperament on the above-presented discrepancies within the syntonic diatonic tuning.



Figure 13: Values of tones and semitones in Zarlino's tempered system (in cents).

As it was presented, the discrepancies between the intervals were those of the third d-f, the fourth a-d, the fifth d-a, the sixth f-d, and the sevenths b-a, f-e, and g-f; they all differed from the corresponding intervals in their true ratios by a comma. It is precisely in the case of discrepant intervals that the effect of Zarlino's distribution of the comma among all intervals can most clearly be observed: Even if after the temperament none of them is in its true ratio, they are nevertheless all of the same value. The minor third between d and f ($191.60 + 120.93 = 312.53$) is thus equal to the minor third between e and g ($120.93 + 191.60 = 312.53$) and to all other minor thirds; the same can be said for all of the other intervals in question as well. Therefore, the temperament results in equal intervals, wherein the equal tones seem to be of major importance: It is after them that this kind of temperament has been named *meantone temperament* (the tempered tone is in between the minor and major syntonic tones).³⁸

Even if the system was constructed and tempered in the just-presented way, some discrepancies remained. The main problem was connected to the manner in which the instruments were tuned: Since the instruments were not tuned in tones but in fifths (following the fifth circle),³⁹ the discrepancy of the tuning with tempered fifths was even bigger than that of the tuning with the twelve pure fifths (3:2). The difference may be illustrated with the juxtaposition of 12 fifths and seven octaves (the latter are in the unchangeable ratio of 2:1 in both cases).⁴⁰

38 For a more detailed account, see Ravnikar, *Osnove glasbene akustike*, 39–40, and Duffin, *How Equal Temperament Ruined Harmony*, 34–35.

39 The tuning in fifths is also mentioned by Zarlino (Zarlino, *Istituzioni armoniche*, 268).

40 As was demonstrated in the above Table 2, the value of the octave in modern equal temperament (1200 cents) is quite close to the value of the octave in the ratio of 2:1 (1199.91 cents); the difference between them (0.09 cents) is negligible.



Figure 14: The juxtaposition of 12 fifths and 7 octaves.

If the fifths in their pure ratios were used, the difference between the last notes in the series of 12 fifths and seven octaves would be 23.43 cents.⁴¹ If the tempered fifths were used, the difference would be considerably larger, namely, 50.25 cents.⁴² As a consequence, the last (twelfth) fifth was by this amount larger than the others. It was called the *wolf fifth* and it was placed in a way that it could most easily be avoided in compositions, usually between *c#* and *g#* (*ab*) or between *g#* and *d#* (*eb*).⁴³

After the temperament instructions are given in *Istituzioni*, the division of the monochord in the tempered system is explained (and thus the practical, audible test of the system is performed).⁴⁴ Zarlino hopes that the detailed presentation given will be of use to everyone who wishes to understand the true intervals of the tempered tuning and above all to instrument makers.⁴⁵ From the mathematical-acoustical point of view, the major problem of the presented division of the monochord in the tempered syntonic system is its accuracy: Zarlino looks for the parts of the comma with the already-mentioned geometrical tool *mesolabe* and transfers them to the monochord's string (that is, to the line drawn underneath it) with a pair of compasses.

The intensity and versatility of Zarlino's study of the tuning question may also be confirmed by the fact that he not only tried out the theoretically determined ratios on the monochord, but on Pesaro's harpsichord as well. As described in *Istituzioni*,⁴⁶ the instrument was tuned in Zarlino's tempered syntonic tuning, expanded with the tones of the chromatic and enharmonic genera. Thus, besides the diatonic keys (larger white keys), the chromatic keys (smaller black keys) and enharmonic keys (smaller white keys) were installed on its keyboard as well. The chromatic (minor) semitone was the difference between the tempered tone (191.60 cents) and the diatonic (major) semitone (120.93 cents): The difference is 70.76 cents, which equals the ratio of 25:24.⁴⁷

41 12 untempered fifths equal 8422.8 cents, while seven octaves equal 8399.4 cents. The difference, 23.4 cents, is also referred to as the *Pythagorean comma*; it is in the ratio of 531,411:524,288.

42 12 tempered fifths equal 8349.12 cents, while seven octaves equal 8399.37 cents; the difference is 50.25 cents.

43 Duffin, *How Equal Temperament Ruined Harmony*, 35. Theoretically, in the circle of fifths, *c - g - d - a - e - b - f# - c# - g# - d# - a# - e# - b#*, any fifth can serve as a starting point for the tuning of an instrument. This also means that, theoretically, any of the fifths could be the wolf fifth.

44 For a detailed discussion, see Zarlino, *Istituzioni armoniche*, 275–281.

45 Zarlino, *Istituzioni armoniche*, 275.

46 Zarlino, *Istituzioni armoniche*, 293–294.

47 For a detailed explanation, see Zarlino, *Istituzioni armoniche*, 286–290.

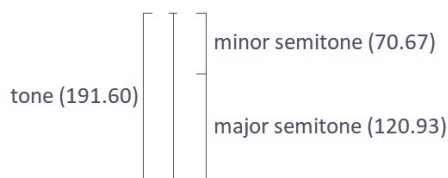


Figure 15: Division of the tempered tone on Zarlino's harpsichord into diatonic (major) and chromatic (minor) semitones.

The major semitone was further divided enharmonically into two *dieses* (quarternotes). The lower of these intervals was larger and equal to the minor semitone (25:24), whereas the upper was smaller, in the ratio of 128:125 augmented by 3/7 of a comma. Together, they formed the major semitone in the ratio 16:15 augmented by 3/7 of a comma.⁴⁸



Figure 16: Division of tempered major semitone on Zarlino's harpsichord into major and minor dieses.

With the presented division of the tone, Zarlino actually made the chromatic raising and lowering of all diatonic notes possible. Since the signs for sharp and flat notes in fact denoted the raising or lowering of the note by a chromatic (25:24) and not a diatonic semitone,⁴⁹ the tones $g\sharp$ and $a\flat$, for example, did not sound in unison but differed by a minor diesis in the ratio of 128:125 (in the case of syntonic or pure intonation) or by the same interval, augmented by 3/7 of a comma (in the case of tempered syntonic system or meantone temperament).

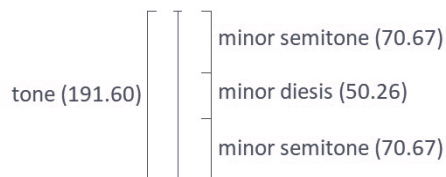


Figure 17: Division of tempered tone on Zarlino's harpsichord into two minor semitones and minor diesis.

Because the whole tone and the diatonic semitone were both divided into smaller units, the octave on Zarlino's harpsichord was divided into no less than 20 different tones.

⁴⁸ For a detailed explanation, see Zarlino, *Istituzioni armoniche*, 290–293.

⁴⁹ Zarlino, *Istituzioni armoniche*, 289–290, 355.

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POVZETEK

Vprašanje o uglasitvi (utemeljevanje akustičnega sistema, znotraj katerega poteka glasba) je bilo eno osrednjih vprašanj, ki so jih v svojih traktatih obravnavali renesančni glasbeni teoretiki; v svojem osrednjem glasbenoteoretskem spisu *Istitutioni harmoniche* (1588) se mu podrobno posveča tudi Gioseffo Zarlino.

V razpravi o sistemu uglasitve je Zarlino izhajal iz premislekov nekaterih antičnih piscev o glasbi, predvsem Boetija in Ptolemaja. Po postavitvi in temeljiti preučitvi sistema v nekaterih vrstah vseh treh antičnih rodov glasbe (diatonični, kromatični, enharmonski) želi Zarlino določiti tistega, ki se rabi v glasbi njegovega časa: prepričan je, da se v sodobni glasbi uporablja sintonična diatonična uglasitev, saj so le med razmerji sintoničnega diatoničnega sistema prisotni intervali, kot jih je ustvarila narava. Iz *senaria* namreč izhaja, da so konsonančne le terce v razmerjih 5 : 4 (velike) in 6 : 5 (male) ter sekste v razmerjih 5 : 3 (velike) in 8 : 5 (male), te pa so prisotne le v sintonični vrsti diatoničnega rodu.

Čeprav ga Zarlino zagovarja kot edinega popolnega, so tudi v sintoničnem diatoničnem sistemu določena odstopanja, ki so v polifonih kompozicijah 16. stoletja lahko povzročila veliko težav. Tem se je v vokalni glasbi ob pazljivosti skladatelja in ob možnosti pevcev, da po potrebi prilagodijo in-

tonacijo, v veliki meri dalo izogniti. Posebno težavo pa je predstavljala zelena hkratna konsonančnost vseh intervalov v uglasitvi instrumentov, posebno tistih s tipkami. Da so nakazane težave znotraj sistema sintonične diatonične uglasitve neresljive, se je zavedal tudi Zarlino, ki je v *Istitutioni* razložil, da morajo biti za uporabo v instrumentalni glasbi posamezni intervali nekoliko spremenjeni, temperirani. Prilagoditev sintonične diatonične uglasitve je nato podrobno pojasnjena.

Kako poglobljeno se je Zarlino ukvarjal z vprašanjem o uglasitvi, kaže tudi dejstvo, da o sistemu ni le teoretično razpravljal, temveč ga je tudi praktično preizkusil, tako na monokordu, kot na posebnem čembalu, ki ga je posebej zanj izdelal znani italijanski izdelovalec instrumentov Domenico da Pesaro. Kot je opisan v *Istitutioni*, je bil instrument uglašen v Zarlinovi temperirani sintonični uglasitvi, razširjeni s toni kromatičnega in enharmonskega rodu, kar pomeni, da so bile ob diatoničnih nanj umeščene še kromatične in enharmonske tipke.

Z zgodovinskega stališča se Zarlinova razprava o uglasitvi kaže kot eden prvih korakov k enakomerno temperirani uglasitvi, kot je v rabi danes, čeprav je pot do nje še dolga. S tega gledišča je temperacija sistema (katere rezultat je kot končni produkt zgodovinskega razvoja tudi sodobna uglasitev) pravzaprav njegova razglasitev.