UPORABA TEOREMA KORESPONDENČNIH STANJ PRI IZRAČUNU ZGORNJIH VREDNOSTI PASIVNIH ZEMELJSKIH TLAKOV

BORUT MACUH in STANISLAV ŠKRABL

o avtorjih

Borut Macuh
Univerza v Mariboru,
Fakulteta za gradbeništvo
Smetanova ulica 17, 2000 Maribor, Slovenija
E-pošta: borut.macuh@uni-mb.si

Stanislav Škrabl
Univerza v Mariboru,
Fakulteta za gradbeništvo
Smetanova ulica 17, 2000 Maribor, Slovenija
E-pošta: stanislav.skrabl@uni-mb.si

izvleček

Rešitve nekaterih mejnih stanj (nosilnost temeljnih tal, aktivnih in pasivnih zemeljskih pritiskov itd.) za čiste-trenjske zemljine so za večji del praktičnih primerov enostavnejše v primerjavi z rešitvami enakih primerov za kohezijsko-trenjske zemljine. Teorem of Corresponding States nam v nekaterih primerih omogoča doseganje rešitev pripadajočih mejnih stanj za kohezijsko trenjske materiale s transformacijo že poznanih rešitev robnih elasto-plastičnih rešitev mejnih stanj za trenjske nekohezijske zemljine. Veljavnost oz. uporabnost osnovnega transformacijskega teorema (Caquot 1934) je omejena in velja le za enostavnejše primere mejnih stanj, kjer so napetostni vektorji pravokotni na robne površine ter kadar se pri transformaciji ohranjajo smeri trajektorij glavnih napetosti (Michalowski 2001). Pri aplikaciji kinematičnega pristopa mejne analize na osnovi teorema zgornje vrednosti je za primer določanja pasivnih zemeljskih pritiskov dokazano, da je za izbrani kinematični model tudi v splošnih primerih robnih napetosti dopustna posredna uporaba teorema korespondenčnih stanj v spremenjeni obliki. Rezultati opravljenih analiz pasivnih zemeljskih pritiskov kažejo, da je v splošnejših primerih nekritična uporaba teorema korespondenčnih stanj v osnovni obliki nedopustna, ker so dobljeni rezultati lahko pravilni le naključno ter v odvisnosti od robnih pogojev lahko pomenijo precenjene ali podcenjene vrednosti pasivnih zemeljskih pritiskov v geotehnični praksi.

ključne besede

korespondenčno stanje, zemeljski tlak, pasivni tlak, mejna analiza, zgornja vrednost

PASSIVE EARTH PRESSURE DETERMINATION: APPLICATION OF THE CORRESPONDING STATE THEOREM FOR CALCULATING UPPER-BOUND VALUES

BORUT MACUH and STANISLAV ŠKRABL

About the authors

Borut Macuh
Univerza v Mariboru,
Fakulteta za gradbeništvo
Smetanova ulica 17, 2000 Maribor, Slovenija
E-mail: borut.macuh@uni-mb.si

Stanislav Škrabl
Univerza v Mariboru,
Fakulteta za gradbeništvo
Smetanova ulica 17, 2000 Maribor, Slovenija
E-mail: stanislav.skrabl@uni-mb.si

Abstract

The validity of some limit state solutions, when strictly applied to the basic corresponding state theorem (Caquot, 1934), is limited and valid only for simpler limit states, where stress vectors are either perpendicular to the boundary surfaces or when the direction of stress eigenvalue trajectories in transformation are preserved (Michalowski, 2001). The theorem of corresponding states allows us, in some cases, to attain solutions belonging to the limit states for cohesive-friction materials with the transformation of the known boundary of elasto-plastic solutions of limit states for pure friction materials. We demonstrated that for the selected kinematically admissible model, in general cases of boundary stresses, the indirect application of the corresponding states theorem in modified form is permitted. To determine this, we applied the kinematic approach of limit state analysis and used the upper-bound theorem for determining passive earth pressures. The results of our analyses show that incautious application of the corresponding state theorem in its basic form and for general cases is inadmissible because the results obtained can be correct only coincidentally, depending on the boundary conditions.

Keywords

corresponding state, earth pressure, passive pressure, limit analysis, upper-bound

1 INTRODUCTION

The corresponding state theorem (Caquot, 1934) is based on the fact that for a considered boundary problem the stress state of cohesive-friction soils in a limit state is similar to the sum of the stress states of the same boundary problem for non-cohesive soils and hydrostatic pressure $p = c/\tan\phi$. The solutions of equal elasto-plastic boundary problems at limit states of cohesive-frictional and pure frictional material are undeniably similar. However, for more general and more complex boundary problems it is necessary to apply more exacting transformation relations to obtain solutions of limit states for cohesive-frictional materials, such as an inclined back fill or boundary conditions that require the transformation of limit state solutions for non-cohesive soils. The solutions of these limit states (bearing capacity of foundation ground, active and passive pressures, etc.) for pure friction soils are simpler for most practical examples compared with solutions of the same examples for cohesive-friction soils.

With the advancement of mathematical knowledge and numerical methods the practical significance of the corresponding state theorem has been reduced. However, it can frequently be found useful in the field of limit states, in investigating active and passive earth pressures and ground bearing capacities. Many authors, including Caquot (1934), Michalowski (2001) and Silvestri (2006), have suggested that there are limitations to applying the theorem in its basic form. In the past the corresponding state theorem was typically applied uncritically or unacceptably: Caquot and Kérisel (1948), Soubra and Regenass (2000), Škrabl and Macuh (2005), Vrecl-Kojc and Škrabl (2007) and many other authors.

The most practical use of the corresponding state theorem in limit state analysis using the upper-bound theorem most frequently occurs in three-dimensional cases where the transformation of known solutions compensate extensive integrations along individual discontinuity surfaces of deformation velocities. In the analyses of two-dimensional cases of limit-state analysis, it is most successfully applied to control the results of mathematical analyses.

This article describes the procedure of determining the limit values of passive earth pressures for two-dimensional cases using the kinematic model of limit states with the upper-bound theorem. A comparison of several results of passive earth pressure coefficients, determined using the procedure of Kérisel and Absi (1990), shows that applying the corresponding state theorem, in its original form, to more general situations is not admissible.

2 KINEMATIC FAILURE MECHANISM

Figure 1 describes a general two-dimensional example of a rigid inclined wall having inclination *α*, height *h* with inclined backfill *β*. The kinematical failure mechanism comprises *n* triangular rigid blocks. As presented in Figure 1b, the kinematically admissible deformation velocities of individual blocks act in directions that enclose angle *ϕ* with individual discontinuity lines d_i (*i*=1,2,…*n*). The velocities of individual rigid blocks are uniformly defined by the condition that relative velocity directions between individual rigid blocks should enclose angle *ϕ* with lateral contact surfaces l_i (*i*=1,2,…*n*). The hodograph of individual rigid blocks is shown in Figure 1c.

The velocities of the whole failure mechanism can be uniformly determined from the chosen value of the deformation velocity of the first rigid block:

$$\dot{V}_1 = 1 \quad \dot{V}_{i+1} = \dot{V}_i \frac{\sin(\beta_{i,i+1}+\alpha_i)}{\sin(\beta_{i,i+1}+\alpha_{i+1})}\ , \qquad \dot{V}_{i,i+1} = \dot{V}_i \frac{\sin(\alpha_{i+1}-\alpha_i)}{\sin(\beta_{i,i+1}+\alpha_{i+1})} \qquad (1)$$

The resultant value of passive earth pressures (P_p) is defined by equation two:

$$P_p = K_{p\gamma}\gamma\frac{h^2}{2} + K_{pc}ch + K_{pq}qh \qquad (2)$$

where $K_{p\gamma}$ denotes the coefficient of passive earth pressures due to soil self-weight, *γ* denotes the soil unit weight, K_{pc} denotes the coefficient of passive earth pressures due to cohesion (*c*) and K_{pq} is the coefficient of passive earth pressures due to the surcharge *q*. The passive pressure distribution along a wall height for a part that belongs to soil self-weight is triangular, while the part that belongs to cohesion and surcharge is rectangular or constant along the wall height.

This paper assumes that the backfill soil fulfills the Mohr-Coulomb yield criterion with the associative plastic flow rule (normality principle). The change of energy dissipation per volume unit of backfill soil can be evaluated by (Michalowski, 2001):

$$\dot{D} = -\dot{\varepsilon}_v\, c\cos\phi = -(\dot{\varepsilon}_1 + \dot{\varepsilon}_3)c\cot\phi \qquad (3)$$

where $\dot{\varepsilon}_1$ and $\dot{\varepsilon}_3$ denote major and minor eigenvalues of strain rate; $\dot{\varepsilon}_v$ rate of volumetric strain deformation, and *c* and *ϕ* represent the cohesion and angle of inner friction of backfill soil.

3 WORKING EQUATION

For soils that follow the associative flow rule, the change of inner energy dissipation is never lower than the change of work of outer forces for an arbitrary kinematically admissible failure mechanism (Fig. 1):

$$\int_V \dot{D}(\dot{\varepsilon}_{ij})dV = \frac{c}{\tan\phi}\Big[\sin(\alpha_n-\beta)\frac{l}{\cos\beta}\dot{V}_n - \cos(\alpha+\alpha_1)\frac{h}{\cos\alpha}\dot{V}_1\ \Big] \geq \gamma\frac{h^2}{2}K_{p\gamma}\big[\cos\delta\cos(\alpha_1+\alpha) - \sin\delta\sin(\alpha_1+\alpha)\ \big]\dot{V}_1 + chK_{pc}\big[\cos\delta\cos(\alpha_1+\alpha) - \sin\delta\sin(\alpha_1+\alpha)\ \big]\dot{V}_1 + qhK_{pq}\big[\cos\delta\cos(\alpha_1+\alpha) - \sin\delta\sin(\alpha_1+\alpha)\ \big]\dot{V}_1 - \sum_{i=1}^{n} G_i\sin\alpha_i\dot{V}_i - q\frac{l}{\cos\beta}\sin\alpha_n\dot{V}_n \qquad (4)$$

where *V* denotes the total volume of the failure mechanism. Provided that deformation velocity $\dot{V}_1 = 1$ and the generalized wall height *h** = 1 equation 4 leads to:

$$\frac{c^*}{\tan\phi}\Big[\sin(\alpha_n-\beta)\frac{l^*}{\cos\beta}\dot{V}_n - \cos(\alpha+\alpha_1)\frac{1}{\cos\alpha}\ \Big] \geq \frac{K_{p\gamma}}{2}\cos(\delta+\alpha_1+\alpha) + c^*K_{pc}\cos(\delta+\alpha_1+\alpha) + q^*K_{pq}\cos(\delta+\alpha_1+\alpha) - \sum_{i=1}^{n}G_i^*\sin\alpha_i\dot{V}_i - q^*\frac{l^*}{\cos\beta}\sin\alpha_n\dot{V}_n \qquad (5)$$

where $c^* = \frac{c}{\gamma h}$ and $q^* = \frac{q}{\gamma h}$ denote normalized cohesion and normalized surcharge; $G_i^* = \frac{G_i}{\gamma h^2} = \gamma^* V_i^*$ and $l^* = \frac{l}{h}$ normalized weight of the individual triangular block of backfill soil and the normalized length of the failure line (surface); $\gamma^* = 1$, $h^* = 1$ and V_i^* denote the generalized unit weight and unit height of the wall and the appurtenant volume of individual soil blocks.

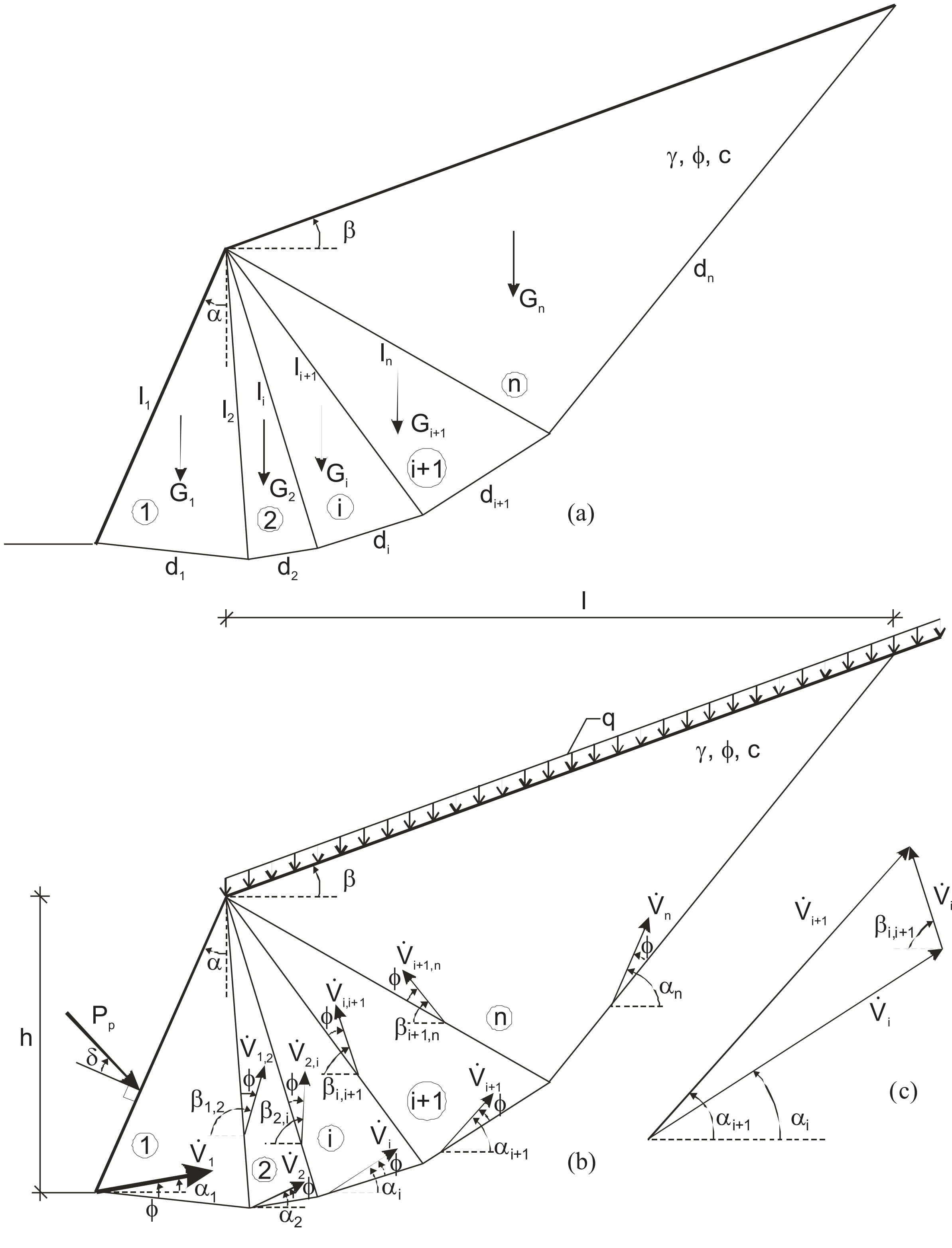

Figure 1. Translational failure mechanism; (a) geometry, (b) absolute and relative velocities of individual rigid blocks and (c) hodograph.

4 NUMERICAL ANALYSES AND RESULTS

The original failure mechanism is completely defined by *n* coordinates that define the individual blocks (Fig. 1). They have to be selected in a way that ensures that the original failure mechanism is kinematically admissible. In numerical analyses using the process of mathematical optimization, the critical kinematical admissible failure mechanism is obtained by minimizing equation 6:

$$f=\frac{K_{p\gamma}}{2}+c^{*}K_{pc}+q^{*}K_{pq}=\sum_{i=1}^{n}\frac{G_i^{*}\sin\alpha_i}{\cos(\delta+\alpha_1+\alpha)}\dot{V}_i+ +q^{*}\frac{l^{*}}{\cos\beta\cos(\delta+\alpha_1+\alpha)}\sin\alpha_n\dot{V}_n+ \frac{c^{*}}{\tan\phi}[\sin(\alpha_n-\beta)\frac{l^{*}}{\cos\beta\cos(\delta+\alpha_1+\alpha)}\dot{V}_n- \cos(\alpha+\alpha_1)\frac{1}{\cos\alpha\cos(\delta+\alpha_1+\alpha)}] \quad (6)$$

Where *f* represents the objective function of the optimization problem. The coefficients of passive earth pressures are defined by equations 7, 8 and 9.

$$K_{p\gamma}=\frac{2}{\cos(\delta+\alpha_1+\alpha)}\sum_{i=1}^{n}G_i^{*}\sin\alpha_i\dot{V}_i \quad (7)$$

$$K_{pq}=\frac{l^{*}}{\cos\beta\cos(\delta+\alpha_1+\alpha)}\sin\alpha_n\dot{V}_n \quad (8)$$

$$K_{pc}=\frac{1}{\tan\phi\cos(\delta+\alpha_1+\alpha)}[\sin(\alpha_n-\beta)\frac{l^{*}}{\cos\beta}\dot{V}_n- \cos(\alpha+\alpha_1)\frac{1}{\cos\alpha}] \quad (9)$$

Using equation 8, the coefficient of passive earth pressures, due to cohesion, can be given in the following form:

$$K_{pc}=\frac{1}{\tan\phi}\{K_{pq}(\cos\beta-\frac{\sin\beta}{\tan\alpha_n})- \frac{1}{\cos\alpha[\cos\delta-\tan(\alpha_1+\alpha)\sin\delta]}\} \quad (10)$$

Equation 10 represents the transformation rule for determining the coefficient of passive earth pressures for cohesive-frictional soils K_{pc} from the known and, as a rule, easier solutions for pure friction soils K_{pq}. We can establish that equation 10 is valid only for the selected failure mechanism and differs from the original transformation theorem (Caquot, 1934) that is used for passive pressure states given in equation 11.

$$K_{pc}=\frac{1}{\tan\phi}[K_{pq}-\frac{1}{\cos\delta}] \quad (11)$$

A comparison of equations 10 and 11 shows that the original transformation of equation 11 is applicable only for the simplest cases of passive earth pressure on vertical walls. Such cases do not consider the friction between the wall, backfill soil and horizontal backfill.

We numerically analyzed the kinematical admissible failure mechanisms using n = 30 triangular soil blocks (Fig. 1).

Equation 6 shows that for different ratios of generalized unit weights of soil blocks γ^*, the surcharge intensities q^* and soil cohesions c^*, we were able to obtain different geometries of the critical failure mechanism through the process of mathematical optimization. This enabled us to determine the lowest total value of passive pressures P_p.

Table 1 represents a comparison of passive earth pressure with coefficients K_{pc} calculated using equations 9 and 10 with the original transformation in equation 11. This was done in accordance with the procedure of Kérisel and Absi (1990). In the procedure of mathematical optimization, we first analyzed the equal conditions $\gamma^* = c^* = 0$ and $q^* > 0$ that were considered in the method used by Kérisel and Absi (1990). For backfill soil analysis, we considered the inner friction angle ϕ = 35°, $\delta = \phi/2$, α = 0, and β = 30° to 35° in increments of 5°.

Furthermore, we used the same set of coefficients of passive earth pressures and applied them to the cohesion for three different combinations of influences on soil unit weight, cohesion and surcharge (Table 1).

The calculations of coefficient K_{pc} using equations 9 and 10 give exactly the same results for all kinematical admissible failure mechanisms.

5 CONCLUSIONS

The results of our numerical analyses show that it is not admissible to determine the coefficient of passive earth pressures K_{pc} to cases of friction between wall and soil and inclined backfills when applying the original transformations according to the corresponding state theorem (Caquot, 1934). The results of the original transformations usage can indicate overestimated or underestimated values of passive earth pressures in geotechnical practice.

Table 1. Comparison of passive pressure coefficients K_{pq} and K_{pc} obtained with the results of calculations using the method of Kérisel and Absi (1990) for $\phi = 35°$, $\delta = \phi/2$ and $\alpha = 0$.

Backfill inclination	Kinematical model n=30			Kérisel and Absi		Kinematical model n=30		
	$\gamma^* = c^*$=0 and $q^* > 0$					$\gamma^*=q^*$=0 and c^*>0	$c^*=q^*$=0 and γ^*>0	$c^*=q^*$=0.2 and γ^*=1
β (°)	K_{pq} (8)	K_{pc} (11)	K_{pc} (9)	K_{pq}	K_{pc} (11)	K_{pc} (9)	K_{pc} (9)	K_{pc} (9)
-30	1.610	0.803	3.532	1.505	0.652	2.301	3.532	2.604
-25	2.153	1.577	3.376	2.092	1.490	2.764	3.371	2.975
-20	2.757	2.439	3.690	2.739	2.415	3.332	3.687	3.479
-15	3.474	3.464	4.255	3.448	3.427	4.038	4.222	4.125
-10	4.282	4.617	4.979	4.264	4.541	4.841	4.915	4.877
-5	5.179	5.898	5.824	5.155	5.864	5.747	5.786	5.760
0	6.167	7.310	6.824	6.116	7.237	6.769	6.839	6.791
5	7.246	8.851	7.985	7.220	8.814	7.920	8.203	7.986
10	8.412	10.516	9.338	8.368	10.454	9.221	9.630	9.375
15	9.657	12.294	10.923	9.615	12.235	10.688	11.413	10.986
20	10.962	14.158	12.801	10.929	14.112	12.346	13.605	12.853
25	12.298	16.067	15.068	12.270	16.025	14.215	16.277	15.053
30	13.620	17.956	17.955	13.605	17.933	16.331	19.736	17.594
35	14.823	19.673	21.829	14.706	19.505	18.718	23.489	20.556

We estimate that similar deviations and miscalculations will also appear in analyzing the limit states of ground bearing capacities for horizontally loaded shallow foundations, foundations near slopes and foundations with an inclined foundation base.

The largest deviations appeared in the limit states of passive earth pressures for inclined backfills, where the negative inclination approached the value of the soil's inner angle of friction. In such cases the coefficients of passive earth pressures, obtained from equation 11, are essentially lower from the actual deviations, which can reach up to 300% of the lowest values. The overestimated values of coefficient K_{pc} using the transformation expression (equation 11) also appear for horizontal backfills and backfills with a moderate inclination. The values come to 12% of the lowest value, determined according to the limit state method using the upper-bound theorem.

It is therefore false and unacceptable to calculate passive pressures for cohesive-friction material from solutions of pure friction material using the known procedure for calculating passive pressures described in Kérisel and Absi (1990). Slopes with decreasing inclinations are very frequent in geotechnical practice. They are characteristic of embedded regions of embedded retaining structures on slopes (pile walls, sheet pile walls etc.). Such situations require detailed and systematic approaches of passive earth pressure. The results of our analyses also show that different geometries of the failure mechanism are critical for determining the different influences of soil self-weight, cohesion and surcharge (Table 1). We obtained the lowest expected values of coefficients K_{pc} when analyzing $\gamma^* = q^* = 0$ and $c^* > 0$. In our opinion, these values are generally applicable because the passive pressure coefficients, in practice, are a bit higher due to the cohesion that occurs in practice.

The transformation expression (equation 11), first proposed by Caquot (1934) and uncritically used in the procedure of Kérisel and Absi (1990), is not generally applicable. It should be replaced with the expression defined by equation 10 for determining passive pressures with the limit state method using the upper-bound theorem. This procedure is also applicable to three-dimensional limit state analyses where similar failure mechanisms are in accordance with the upper-bound theorem.

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