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Excitation of the Roper resonance*

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Abstract. We study a model of the Roper resonance in which the two-pion decay proceeds via intermediate hadrons, the $\Delta(1232)$ isobar and the σ meson. We derive the coupled channel formalism for the K-matrix and show that the coupling of the σ meson to the N(1440) and N(1710) resonances is responsible for the peculiar behavior of the inelasticity in the P11 channel.

1 Introduction

Among the low-lying nucleon excitations, the Roper resonance N(1440) plays a very special role due to its relatively low energy as well as a rather peculiar behavior of the scattering and electro-excitation amplitudes. Its low energy can be explained in models in which the quarks are strongly coupled to chiral mesons, e.g. in the framework of the Constituent Quark Model [1]. Yet, the form of the scattering amplitudes which is far from the familiar Breit-Wigner shape, and in particular the unusual behavior of the inelasticity in the P11 channel, indicate that the structure of the state can not be explained by a simple excitation of the quark core (like most of the other low lying states) and that other degrees of freedom have to be included (see [2] and references therein).

In this work we shall concentrate on the decays of the Roper resonance rather than on the problem of its low energy. We shall show that the behavior of the scattering amplitude can be explained in a simple model in which the chiral partner of the pion, the σ meson, is included together with the quark and pion degrees of freedom. The model assumes that the two-pion decay proceeds only through intermediate hadrons, either the $\Delta(1232)$, the σ or the ρ meson. Since the decay into the ρ meson and the nucleon is relatively weak, we keep only the first two intermediate hadrons.

In our previous work [3,4] we have introduced an approach to calculate the K-matrix for pion scattering and electro-production in quark models with chiral mesons. We have successfully applied it to the calculation of the phase shift and electro-production amplitudes in the P33 channel. We have also presented a method how to include the simplest two pion decay, namely the decay into the intermediate Δ and the pion.

* Talk delivered by B. Golli

2 K matrix in chiral quark models

Chew and Low [5] have shown that in models in which the mesons are coupled linearly to the source, it is possible to find the exact expression for the T matrix without explicitly specifying the form of asymptotic states. In [3,4] we have found that the expression for the K matrix in this case and write down an integral equation which can be used to calculate the K matrix for a particular model.

To describe the core to which the mesons are coupled we consider quark models in which the quarks emit/absorb a meson by flipping the spin and isospin, and through the excitation to a higher radial state. The part of the Hamiltonian referring to the p-wave pions can be written as

$$H_\pi = \int dk \sum_{mt} \left\{ \omega_k \, a^\dagger_{mt}(k) a_{mt}(k) + \left[V_{mt}(k) a_{mt}(k) + V^\dagger_{mt}(k)\, a^\dagger_{mt}(k) \right] \right\} , \quad (1)$$

where $a^\dagger_{mt}(k)$ is the creation operator for a pion with the third components of spin m and isospin t, and $V_{mt}(k) = -v(k) \sum_{i=1}^{3} \sigma^i_m \tau^i_t$ is the general form of the pion source, with the quark operator, $v(k)$, depending on the model. It includes also the possibility that the quarks change their radial function which is specified by the reduced matrix elements $V_{BB'} = \langle B \| V(k) \| B' \rangle$, where B are the bare baryon states (e.g. the bare nucleon, Δ, Roper, ...)

The coupling of the σ meson to the quark core is explicitly present in the linear sigma model, however, due to the meson self-interaction potential it is no longer possible to write down the meson part of the Hamiltonian in the form (1) which would permit the use of the exact expressions for the T and the K matrix. In non-linear versions of different models with chiral mesons the σ meson represents two strongly correlated pions in a relative s-state. The σ meson has been included at purely phenomenological level in several multichannel analyzes of πN reactions (see [6] and references therein).

In our approach we consider the s-wave σ mesons as independent degrees of freedom linearly coupled to the quark core, so that we can use the same formalism as in the case of the pion. We assume the one-σ meson states are labeled by the momentum k and by the σ meson rest mass μ equivalent to the two-pion invariant mass. The effective σ Hamiltonian is taken in the form

$$H_\sigma = \int d\mu \int dk \, \omega_{\mu k} \, b^\dagger_\mu(k) b_\mu(k) + \bar{V}^\dagger_\mu(k) b^\dagger_\mu(k) + \bar{V}_\mu(k) b_\mu(k) \, , \quad (2)$$

where

$$\omega^2_{\mu k} = k^2 + \mu^2 \, . \quad (3)$$

The operators b_μ and $b^\dagger_\mu$ are the annihilation and creation operators for the s-wave σ mesons with the invariant mass μ, $2m_\pi < \mu < \infty$. The quark-sigma interaction is taken in the form:

$$\bar{V}_\mu(k) = \kappa \frac{k}{\sqrt{2\omega_{\mu k}}} \, w_\sigma(\mu) \, . \quad (4)$$

Here $w_\sigma(\mu)$ is a weight function centered around the experimental value of the σ meson mass ($\sim$ 600 MeV) and normalized as $\int_{2m_\pi}^{\infty} d\mu\, w_\sigma^2(\mu) = 1$. The (dimensionless) coupling parameter κ is taken as a free parameter.

In the basis with good total angular momentum J and isospin T, in which the K and T matrices are diagonal, it is possible to express the K matrix for the elastic channel in the form [4]

$$K_{NN}(k, k_0) = -\pi\sqrt{\frac{\omega_k}{k}}\langle\Psi^N(W)||V(k)||\Phi_N\rangle\,. \tag{5}$$

Here Φ_N is the ground state of the system, and Ψ^N the principal-value state [7] obeying

$$|\Psi^N(W)\rangle = \sqrt{\frac{\omega_0}{k_0}}\left\{\left[a^\dagger(k_0)|\Phi_N\rangle\right]^{JT} - \frac{\mathcal{P}}{H-W}\,[V(k_0)|\Phi_N\rangle]^{JT}\right\}, \tag{6}$$

where $[\,]^{JT}$ denotes coupling to good J and T, k_0 and ω_0 are the pion momentum and energy:

$$k_0 = \sqrt{\omega_0^2 - m_\pi^2}\,, \qquad \omega_0 = \frac{W^2 - m_N^2 + m_\pi^2}{2W}\,, \tag{7}$$

m_N is the nucleon rest mass and W the invariant energy of the system ($W = \sqrt{s}$). The K matrices for the inelastic processes $\pi + N \to \pi + \Delta(m)$ where m is the invariant Δ mass can be written as

$$K_{N\Delta}(k, k_0) = -\pi\sqrt{\frac{\omega_k}{k}}\langle\Psi^N(W)||V(k)||\Psi_\Delta(m)\rangle\,. \tag{8}$$

Here $\Psi_\Delta(m)$ is the principal value state corresponding to the πN scattering in the P33 channel as determined in [4] except that it is now normalized to $\delta(m - m')$ rather than to $(1 + K_\Delta(m)^2)\delta(m - m')$. For the process $\pi + N \to \sigma(\mu) + N$ we have

$$K_{N\sigma}(k_\mu, k_0) = -\pi\sqrt{\frac{\omega_{\mu k}}{k_\mu}}\langle\Psi^N(W)|\bar{V}_\mu(k_\mu)|N\rangle\,. \tag{9}$$

3 Coupled channels

The K matrix is related to the T matrix through the Heitler equation:

$$T = -\frac{K}{(1 - iK)} \quad \text{or} \quad T = -K + iKT\,. \tag{10}$$

Since the elements of the K matrix corresponding to inelastic channels depend on the invariant masses m and μ, the above matrix equation becomes a set of coupled integral equations for the T matrix, valid for each W:

$$\begin{aligned}
T_{NN} &= -K_{NN} + i\left[K_{NN}T_{NN} + \int_{m_N+m_\pi}^{W-m_\pi} dm\, K_{N\Delta}(m)T_{\Delta N}(m)\right. \\
&\quad \left. + \int_{2m_\pi}^{W-m_N} d\mu\, K_{N\sigma}(\mu)T_{\sigma N}(\mu)\right], \qquad (11)\\
T_{\Delta N}(m) &= -K_{\Delta N}(m) + i\left[K_{\Delta N}(m)T_{NN} + \int_{m_N+m_\pi}^{W-m_\pi} dm'\, K_{\Delta\Delta}(m,m')T_{\Delta N}(m')\right. \\
&\quad \left. + \int_{2m_\pi}^{W-m_N} d\mu\, K_{\Delta\sigma}(m,\mu)T_{\sigma N}(\mu)\right], \qquad (12)\\
T_{\sigma N}(\mu) &= -K_{\sigma N}(\mu) + i\left[K_{\sigma N}(\mu)T_{NN} + \int_{m_N+m_\pi}^{W-m_\pi} dm\, K_{\sigma\Delta}(\mu,m)T_{\Delta N}(m)\right. \\
&\quad \left. + \int_{2m_\pi}^{W-m_N} d\mu'\, K_{\sigma\sigma}(\mu,\mu')T_{\sigma N}(\mu')\right]. \qquad (13)
\end{aligned}$$

The equations involve only the on-shell K matrix elements*. Apart of the K matrix elements corresponding to the processes with the nucleon and the pion in the initial state we have to include the processes with the pion and the Δ, as well as the σ meson and the nucleon in the initial and in the final state. The pertinent on-shell matrix elements are defined as

$$\begin{aligned}
K_{\Delta N}(W,m) &= -\pi\sqrt{\frac{\omega_m}{k_m}}\,\langle\Psi_\Delta(m)||V^\dagger(k_m)||\Psi^N(W)\rangle\,, \\
K_{\Delta\Delta}(W,m,m') &= -\pi\sqrt{\frac{\omega_m}{k_m}}\,\langle\Psi_\Delta(m')||V^\dagger(k_{m'})||\Psi^\Delta(W,m)\rangle\,, \\
K_{\sigma N}(W,\mu) &= -\pi\sqrt{\frac{\omega_0}{k_0}}\,\langle\Phi_N||V^\dagger(k_0)||\Psi^\sigma(W,\mu)\rangle\,, \\
K_{\Delta\sigma}(W,m,\mu) &= -\pi\sqrt{\frac{\omega_\mu}{k_\mu}}\,\langle\Phi_N|\bar{V}^{\mu\dagger}(k_\mu)|\Psi^\Delta(W,m)\rangle\,, \\
K_{\sigma\Delta}(W,\mu,m) &= -\pi\sqrt{\frac{\omega_m}{k_m}}\,\langle\Psi_\Delta(m)||V^\dagger(k_m)||\Psi^\sigma(W,\mu)\rangle\,, \\
K_{\sigma\sigma}(W,\mu,\mu') &= -\pi\sqrt{\frac{\omega_{\mu'}}{k_{\mu'}}}\,\langle\Phi_N|\bar{V}^{\mu'\dagger}(k_{\mu'})|\Psi^\sigma(W,\mu)\rangle\,. \qquad (14)
\end{aligned}$$

Here $\Psi^\Delta(W,m)$ is the principal value state corresponding to the pion scattering on the Δ state of invariant mass m in the P11 channel, and $\Psi^\sigma(W,\mu)$ the state corresponding to the scattering of the σ meson of invariant mass μ on the nucleon. These states obey similar relations as the principal value state for πN scattering (see eq. 6):

$$|\Psi^\Delta(W,m)\rangle = \sqrt{\frac{\omega_m}{k_m}}\left\{\left[a^\dagger(k_m)|\Psi_\Delta(m)\rangle\right]^{JT} - \frac{\mathcal{P}}{H-W}\,\left[V(k_m)|\Psi_\Delta(m)\rangle\rangle\right]^{JT}\right\}, \qquad (15)$$

* To label the on shell matrix elements we prefer to use the total invariant energy of the system, W, (which we sometimes drop) as well as the invariant masses m and μ instead of pion momenta.

where

$$\omega_m = \frac{W^2 - m^2 + m_\pi^2}{2W}\,, \qquad k_m = \sqrt{\omega_m^2 - m_\pi^2}\,. \tag{16}$$

For scattering of the σ meson on the nucleon we have

$$|\Psi^\sigma(W,\mu)\rangle = \sqrt{\frac{\omega_\mu}{k_\mu}}\left\{b_\mu^\dagger(k_\mu)|\Phi_N\rangle - \frac{\mathcal{P}}{H-W}\,\bar{V}^\mu(k_\mu)|\Phi_N\rangle\rangle\right\}, \tag{17}$$

where

$$\omega_\mu = \frac{W^2 - m_N^2 + \mu^2}{2W}\,, \qquad k_\mu = \sqrt{\omega_\mu^2 - \mu^2}\,. \tag{18}$$

To preserve unitarity, the K matrix has to be real and symmetric, i.e.: $K_{HH'} = K_{H'H}$.

Let us mention that the set of coupled equations similar to (13) has been used in several analysis of experimental data for the pion scattering (see e.g. Ref. [8] and [9] and references therein). In these approaches the K matrix is taken at the tree level with meson-baryon form-factors as well as the masses of the hadrons considered as free parameters.

4 Integral equations for the scattering amplitudes

The equations (6), (15) and(17) are too difficult to treat in their general form and we rather use a suitable *ansatz* for the state Ψ^H, $\{H = N, \Delta \text{ or } \sigma\}$, valid in the low energy regime. Let us note that the second term in the above equations generates configurations with different recoupling of the quark spins and isospins as well as excitations to higher radial states. In addition, the quark core gets dressed by a cloud of pions and σ mesons. If we allow asymptotic states with only one pion and one σ meson, the ansatz takes the form

$$\begin{aligned}|\Psi^H\rangle = \sqrt{\frac{\omega_H}{k_H}}&\left\{|\Psi_0^H\rangle + c_R^H|\Phi_R\rangle + \int dk\,\frac{\chi^{NH}(k,k_H,k_0)}{\omega_k - \omega_0}[a^\dagger(k)|\Phi_N\rangle]^{\frac12\frac12}\right.\\ &+\int dk\int dm'\,\frac{\chi^{\Delta H}(k,k_H,k_{m'})}{\omega_k - \omega_{m'}}[a^\dagger(k)|\widehat{\Psi}_\Delta(m')\rangle]^{\frac12\frac12}\\ &\left.+\int dk\int d\mu'\,\frac{\chi^{\sigma H}(k,k_H,k_{\mu'})}{\omega_{\mu'k} - \omega_{\mu'}}b_{\mu'}^\dagger(k)|\Phi_N\rangle + c_N|\Phi_N\rangle\right\}\end{aligned} \tag{19}$$

Here Ψ_0^H is the first term on the RHS of (6), (15) and (17) respectively, the state Φ_R is a resonant state with the excited quark core with the nucleon quantum numbers, (it corresponds to the Roper state as obtained in a calculation with the bound state boundary conditions). The next three terms represent one-pion states on top of the nucleon and Δ and one-σ meson state on top of the nucleon, respectively, with scattering boundary conditions (i.e. the irregular waves). The last term ensures the orthogonality of the scattering state with respect to the ground state Φ_N, and is responsible for the proper behavior of the scattering amplitudes at the nucleon pole. The states denoted by Φ may contain the meson cloud which however vanishes asymptotically; among such states, only the ground state Φ_N is the eigenstate of the Hamiltonian.

From (5), (8), and (9), we immediately obtain the relations between the matrix elements of the K matrix and the pion amplitudes, χ, in the above ansatz. For the on-shell matrix elements we have

$$\begin{aligned} K_{NH} &= \pi\sqrt{\frac{\omega_0\omega_H}{k_0 k_H}}\,\chi^{NH}(k_0, k_H, k_0)\,, \\ K_{\Delta H} &= \pi\sqrt{\frac{\omega_{m'}\omega_H}{k_{m'} k_H}}\,\chi^{\Delta H}(k_{m'}, k_H, k_{m'})\,, \\ K_{\Delta H} &= \pi\sqrt{\frac{\omega_{\mu'}\omega_H}{k_{\mu'} k_H}}\,\chi^{\sigma H}(k_{\mu'}, k_H, k_{\mu'})\,, \end{aligned} \tag{20}$$

Here $k_H = k_0$ for $H = N$, $k_H = k_m$ for $H = \Delta$ and $k_H = k_\mu$ for $H = \sigma$.

Using the ansatz (19) and the equations for the principal value state (6), (15) and (17)), we obtain a set of integral equations for the scattering amplitudes $\chi^{HH'}$ of the form

$$\begin{aligned} \chi^{HH'}(k, k_{H'}, k_H) = -c_R^{H'} V_{RH}(k) - c_N^{H'} V_{NH}(k) + \mathcal{K}^{HH'}(k, k_{H'}, k_H) \\ + \sum_{H''}\int dk'\,\frac{\mathcal{K}^{HH''}(k,k')\chi^{H''H'}(k', k_{H''}, k_{H'})}{\omega'_k - \omega_{H''}} \end{aligned} \tag{21}$$

where the sum over H'' implies also the integration over the corresponding invariant mass m'' or μ'' in the $\pi\Delta$ and σN case, respectively, and $\omega_{H''}$ is either ω_0, $\omega_{m''}$ or $\omega_{\mu''}$, respectively. The matrix elements V_{RH} are $V_{RN} = \langle\Phi_R||V(k)||\Phi_N\rangle$, $V_{R\Delta} = \langle\Phi_R||V(k)||\Psi_\Delta(m)\rangle$, and $V_{R\sigma} = \langle\Phi_R|\bar{V}^\mu(k_\mu)|\Phi_N\rangle$; the V_{NH} have analogous structure with Φ_R replaced by Φ_R. The coefficients c_R and c_N obey the following equations

$$(\omega_0 - \varepsilon_R^0)c_R^H = V_{RH}(k_H) + \sum_{H'}\int dk\,V_{RH'}(k)\,\frac{\chi^{H',H}(k, k_H, k_{H'})}{\omega_k - \omega_{H'}} \tag{22}$$

$$(\omega_0 - \varepsilon_N)c_N^H = V_{NH}(k_H) + \sum_{H'}\int dk\,V_{NH'}(k)\,\frac{\chi^{H'H}(k, k_H, k_{H'})}{\omega_k - \omega_{H'}} \tag{23}$$

Here $\varepsilon_R^0 = ({m_R^0}^2 - m_N^2)/2W$, $\varepsilon_N = m_\pi^2/2W$, and m_R^0 is the rest energy of the state Φ_R. The kernel $\mathcal{K}^{HH''}$ has in general a very complicated structure. It can be considerably simplified by making the following assumptions: (i) in the ansatz (19) the ground state Φ_N and the state corresponding to the incoming and outgoing Δ, $\Psi_\Delta(m)$, is not modified in the presence of the scattering mesons, and (ii) the integral over the invariant masses is substituted by the integrand evaluated at $m = m_B$, i.e. at the position of the resonance. The first assumption yields the usual approximation made in this type of calculation:

$$\frac{1}{\omega_k + \omega'_k - \omega} \approx \frac{\omega}{\omega_k\omega'_k}$$

which makes the kernel *separable*. The second assumption requires that the resonances are sufficiently narrow, so that the main contribution to the integral comes from values of m close to the position of the resonance (i.e. the pole of the corresponding K matrix). This assumption is justified in the case of the Δ resonance but less valid in the case of higher resonances. In the P11 channel this approximation does not have a large effect since the contribution of the Δ resonance dominates over the contribution of higher resonances. Under these two assumptions the kernel takes the form:

$$\mathcal{K}^{HH'}(k,k') = \sum_{H''} g_{HH'H''} \frac{(\omega_H + \varepsilon_{H''} - \varepsilon_H - \varepsilon_{H'})\, V_{HH''}(k)\, V_{H'H''}(k')}{(\omega_k + \varepsilon_{H''} - \varepsilon_H)(\omega'_k + \varepsilon_{H''} - \varepsilon_{H'})} . \quad (24)$$

Here $g_{HH'H''}$ are spin-isospin recoupling coefficients; in the static approximation $\varepsilon_H = m_H - m_N$; taking into account the recoil, we use approximate u-channel denominators averaged over the directions of the meson momenta (see e.g. [10]).

The important point in the above derivation is that due to the separable kernels the set of integral equations reduces to a set of algebraic equations which immediately leads to the exact solution for χ and c. Furthermore, it can be explicitly shown that the approximations preserve the symmetry of the K matrix which in turn *ensures the unitarity* of the S matrix.

Neglecting the integrals in (21)–(23) the problem reduces to the tree level and is the usual starting point in analyzing the experimental data using the K matrix approach mentioned above.

The solution of the system (21)–(23) can be written in a similar form as the expression at the tree level:

$$\chi^{HH'}(k,k_{H'},k_H) = -c_R^{H'}\mathcal{V}_{RH}(k) - c_N^{H'}\mathcal{V}_{NH}(k) + \mathcal{D}^{HH'}(k,k_{H'},k_H)\,, \quad (25)$$

$$c_R^H = \frac{\mathcal{V}_{RH}(k_H) - \mathcal{V}_{NH}(k_H)n_{RN}}{Z_R(W)(W-m_R)}\,, \quad (26)$$

$$c_N^H = \frac{\mathcal{V}_{NH}(k_H)}{Z_N(W)(W-m_N)} + n_{RN}c_R^H\,. \quad (27)$$

Here $\mathcal{V}_{HH'}$ can be interpreted as a renormalized vertex and $Z_H(W)$ as the wave function renormalization of the state. In addition, the quasi bound Roper state Φ_R acquires an admixture of the ground state due to the requirement that the ground state is orthogonal to the full scattering state rather than to Φ_R itself. Furthermore, it can be easily seen that at the nucleon pole (i.e. $W = m_N$) the residuum involves only the pion-nucleon interaction vertex, so that the behavior of the phase shift at low energies is governed by the πNN coupling constant alone.

5 Results for the Roper in the Cloudy Bag Model

We illustrate the method by calculating scattering amplitudes in the P11 channel. Though the expressions derived in the previous sections are general and can be applied to any model in which mesons linearly couple to the quark core, we choose here the Cloudy Bag Model, primarily because of its simplicity. The pion part of the Hamiltonian of the model has the form (1) with

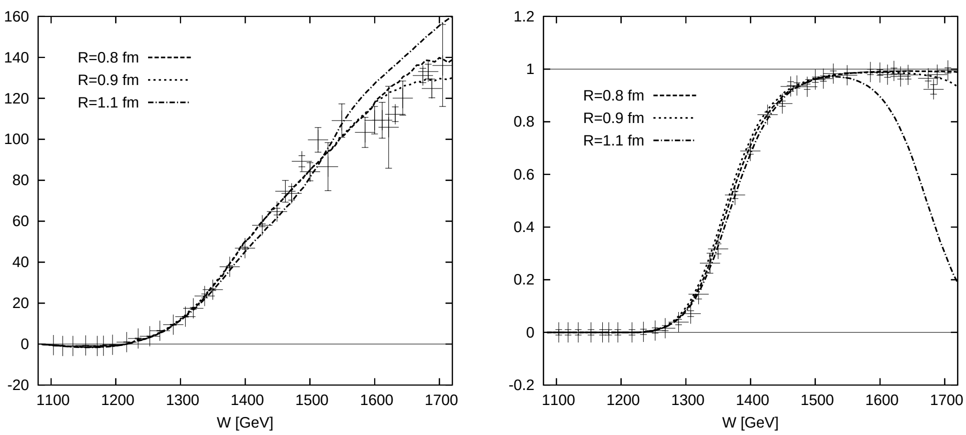
$$v(k) = \frac{1}{2f_\pi} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \frac{\omega^0_{\rm MIT}}{\omega^0_{\rm MIT}-1} \frac{j_1(kR)}{kR}\,, \quad (28)$$

when no radial excitation of the core takes place, while

$$v^*(k) = r_\omega v(k)\,, \qquad r_\omega = \frac{1}{\sqrt{3}}\left[\frac{\omega^1_{\mathrm{MIT}}(\omega^0_{\mathrm{MIT}}-1)}{\omega^0_{\mathrm{MIT}}(\omega^1_{\mathrm{MIT}}-1)}\right]^{1/2}, \tag{29}$$

when one quark is excited from the 1s state to the 2s state. Here $\omega^0_{\mathrm{MIT}} = 2.04$ and $\omega^1_{\mathrm{MIT}} = 5.40$. The free parameter is the bag radius R. Though the bare values of different 3-quark configurations are in principle calculable in the model, the model lacks a mechanism that would account for large hyperfine splitting between certain states, e.g. the nucleon and the Δ. For each R we therefore adjust the splitting between the bare states such that the experimental position of the resonance is reproduced. Furthermore, using the experimental value of f_π in (28) leads to a too small πNN coupling constant irrespectively of the bag radius; in our calculation we have therefore decreased this value by 10 %.

We include also the excited state of the Δ, the $\Delta(1600)$ isobar assuming the same radial structure as for the N(1440). In order to see the effect of other higher positive-parity nucleon excitation we have included the N(1710) isobar.



(a) (b)

Fig. 1. The phase shift (a) and the inelasticity (b) normalized such that the unitarity limit is 1, as a function of the invariant mass for three choices of the bag radius. Beside the $\Delta(1232)$ and N(1440), the $\Delta(1600) \equiv \Delta^*$ and $\mathrm{N}(1710) \equiv \mathrm{R}^*$ are included in the calculation. The pole in the K matrix is chosen to be at 1480 MeV for N(1440), 1700 MeV for $\Delta(1600)$ and 1900 MeV for N(1710). Depending on the bag radius, the strength of the $\pi\mathrm{N}\Delta$ coupling is 45 % – 55 % larger, while that of πNR 3 % – 15 % smaller than the corresponding bare quark values. The mass of the σ meson is 550 MeV and its width 600 MeV; the effective σNR coupling parameter κ (see (4)) is between 0.7 and 0.6, depending on the bag radius. The admissible $\pi\mathrm{NR}^*$ is in the range 0 % – 20 % of the πNN coupling constant, while the couplings $\sigma\mathrm{NR}^*$ and σNR are comparable. The data points are from [11].

The coupling of the σ meson to the quark core is not explicitly present in the model. It is interpreted as the coupling of two correlated pions through non-

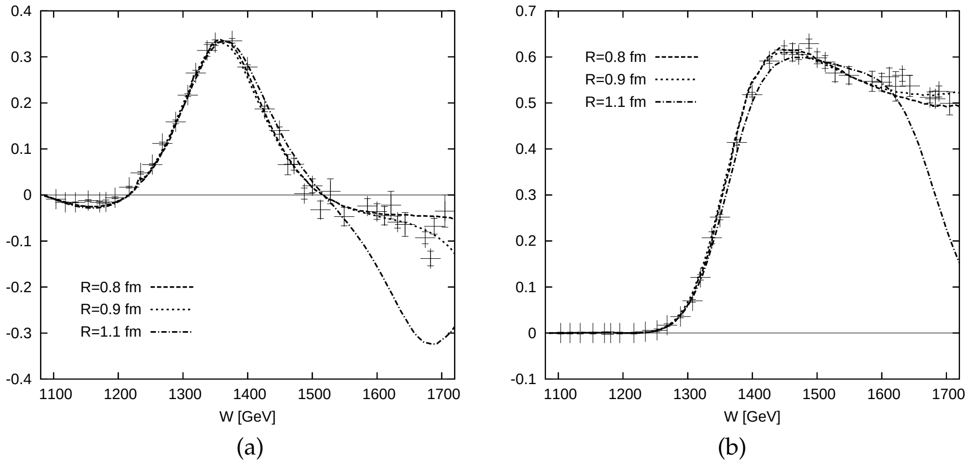


Fig. 2. The real (a) and the imaginary (b) parts of the T matrix as a function of the invariant mass for three choices of the bag radius. For the explanation of different curves see Figure 1.

linear term in the expansion of the pion field [12]. In our approach we simply include this coupling at the phenomenological level and consider its strength as an adjustable parameter.

At low energies the phase shift (Fig. 1) is dominated by the nucleon pole term, and the crossed (u-channel) term with the $\Delta(1232)$ and $\Delta(1600)$ as the intermediate states. Here the $\pi N\Delta$ coupling strength has to be increased with respect to its bare value by some 40 % to 50 % in accordance with our results in the P33 channel [4]. At higher energies around the resonance, the amplitude is governed by the πNR coupling; its strength is enhanced as compared to the bare quark value by a factor 1.4 – 1.8 through the vertex and wave function renormalization such that the bare value has to be decreased up to 15 % in order to obtain reasonable agreement with the experiment.

The presence of the σN channel is most clearly manifested in the inelasticity, $-(4\mathrm{Im}T_{NN} + |T_{NN}|^2)$. It becomes important already at energies slightly above the two pion threshold (Fig. 1). This effect is a clear consequence of the s-wave meson coupling to the quark core and can not be obtained in the competitive process in which the two pions are produced through the intermediate Δ, since in this case the p-wave pions contribute only at relatively high energies. The results are sensitive mostly to the σNR coupling and much less to the σNN; the latter can be even put to 0. This can be understood since in the static limit the s-channel and the u-channel contributions cancel each other in the case of the nucleon intermediate state. At higher energies ($W > 1600$ MeV) the role of the N(1710) becomes more important; we treat the corresponding meson couplings as adjustable parameters. We do not include other isobars such as the negative-parity excitations so the results in this energy range may be somewhat inconclusive. Nonetheless, there is a rather clear indication that N(1710) more strongly couples to the σN channel rather than to the elastic channel and supports our conjecture about the nature of this excited state [2]. For the function $w_\sigma(\mu)$ we have assumed a Breit-Wigner shape; the results favor the σ meson mass in the range from 500 MeV to 600 MeV and a relatively large width of 600 MeV or even higher, though the results are rather insensitive to the width provided we readjust the strength of the parameter κ in (4). Choosing a larger width can to some extend compensate the fall-off of the inelasticity at higher energies for larger bag radii.

In conclusion, we emphasize two most important results of our calculation: (i) though the quark models – including the CBM – predict relatively weak πNR coupling which would result in a much too small width of the resonance, we have shown that through the dressing of pions and other isobars the coupling becomes considerably stronger and produces the correct behavior of the scattering amplitudes in the vicinity of the resonance (Fig. 2); (ii) by including the σ meson we have been able to explain the unusual behavior of the inelasticity as well as the scattering amplitude from the two-pion threshold up to energies well above the resonance.

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