BLED WORKSHOPS
IN PHYSICS
VOL. 10, NO. 1
p. 48

Proceedings of the Mini-Workshop
Problems in Multi-Quark States
Bled, Slovenia, June 29 - July 6, 2009

Meson-Baryon Interaction Vertices*

T. Melde[a], L. Canton[b], W. Plessas[a]

[a]Theoretical Physics, Institute of Physics, University of Graz, Universitätsplatz 5, A-8010 Graz, Austria
[b]Istituto Nazionale di Fisica Nucleare, Sezione di Padova, Via F. Marzolo 8, I-35131 Padova, Italy

We discuss predictions of the relativistic constituent-quark model (RCQM) for the structure of πNN as well as $\pi N\Delta$ strong interaction vertices. The results are put into perspective with strong meson-baryon form factors from lattice quantum chromodynamics (QCD) and phenomenological models.

Notions on the structure of meson-baryon interaction vertices are important in many areas of particle and nuclear physics. Often the corresponding strong form factors have been parametrized phenomenologically, especially in meson-baryon and baryon-baryon interaction models. Certainly, it is desirable to understand the structure of the hadronic interaction vertices on a microscopic level.

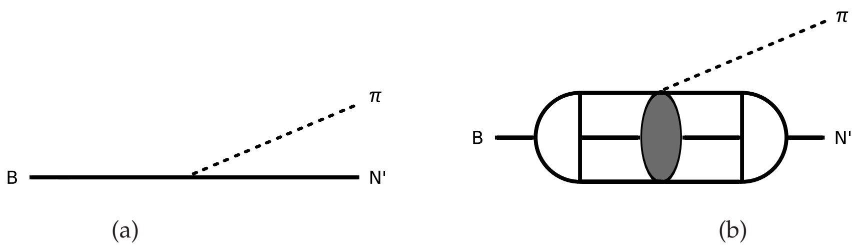

Fig. 1. Graphical representation of the meson-baryon vertex (a) and the corresponding amplitude in the RCQM (b).

We have recently performed a covariant study of the πNN and $\pi N\Delta$ interaction vertices within a relativistic constituent-quark model (RCQM) by considering the process of Fig. 1(a) resolved in the way as shown in Fig. 1(b) [1]. Predictions of the form factor dependences on the relativistic four-momentum transfer Q^2 have been obtained directly from the RCQM without introducing any fit parameters. The transition amplitudes from initial $|i\rangle$ to final $\langle f|$ states

$$F_{i\to f} = (2\pi)^4 \langle f| \mathcal{L}_I(0) |i\rangle \qquad (1)$$

* Talk delivered by W. Plessas

with the πNN and $\pi N\Delta$ interaction Lagrangian densities

$$\mathcal{L}_I^N = -\frac{f_{\pi NN}}{m_\pi}\bar{\Psi}(x)\gamma_5\gamma^\mu T\Psi(x)\,\partial_\mu\Phi(x)\ , \quad (2)$$

$$\mathcal{L}_I^\Delta = -\frac{f_{\pi N\Delta}}{m_\pi}\bar{\Psi}(x)\,T\Psi^\mu(x)\,\partial_\mu\Phi(x) + \text{h.c.}\,, \quad (3)$$

where in obvious notation T represents the transition operator for the emission of the pion Φ from a nucleon Ψ or a delta Ψ^μ with couplings $f_{\pi NN}$ and $f_{\pi N\Delta}$, respectively, are thus identified with the matrix elements

$$F_{i\to f}^{RCQM} = \langle V', M', J', \Sigma'|\,\hat{D}_{rd}^{\pi}\,|V, M, J, \Sigma\rangle\ , \quad (4)$$

where the baryon states $|V, M, J, \Sigma\rangle$ are eigenstates of the RCQM invariant mass operator characterized by the four-velocity V, the invariant-mass eigenvalue M, and the intrinsic spin J with z-component Σ, and analogously for $\langle V', M', J', \Sigma'|$. These matrix elements are calculated within point-form (PF) relativistic quantum mechanics

$$\begin{aligned}
\langle V', M', J', \Sigma'|\hat{D}_{rd}^{m}|V, M, J, \Sigma\rangle &= \frac{2}{MM'}\sum_{\sigma_i\sigma_i'}\sum_{\mu_i\mu_i'}\int d^3\mathbf{k}_2 d^3\mathbf{k}_3 d^3\mathbf{k}_2' d^3\mathbf{k}_3' \\
&\times\sqrt{\frac{\left(\sum_i \omega_i'\right)^3}{\prod_i 2\omega_i'}}\Psi^\star_{M'J'M_{J'}T'M_{T'}}\left(\mathbf{k}_1', \mathbf{k}_2', \mathbf{k}_3'; \mu_1', \mu_2', \mu_3'\right)\prod_{\sigma_i'} D^{\star\frac{1}{2}}_{\sigma_i'\mu_i'}\{R_W[k_i'; B(V')]\} \\
&\times\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'|\,\hat{D}_{rd}^{m}\,|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle \\
&\times\prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i\mu_i}\{R_W[k_i; B(V)]\}\sqrt{\frac{\left(\sum_i \omega_i\right)^3}{\prod_i 2\omega_i}}\Psi_{MJM_JTM_T}\left(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3; \mu_1, \mu_2, \mu_3\right), \quad (5)
\end{aligned}$$

where the matrix element of the reduced transition operator $\hat{D}_{rd}^{\pi}$ between free three-quark states $|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle$ is taken according to the point-form spectator model (PFSM) [2]

$$\begin{aligned}
&\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'|\,\hat{D}_{rd}^{\pi}\,|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle = \\
&\quad 3\mathcal{N}_S\frac{ig_{qqm}}{2m_1(2\pi)^{\frac{3}{2}}}\bar{u}(p_1', \sigma_1')\gamma_5\gamma_\mu\lambda_m u(p_1, \sigma_1)\,\tilde{q}^\mu \\
&\quad\times 2p_{20}\,\delta(\mathbf{p}_2 - \mathbf{p}_2')\,2p_{30}\,\delta(\mathbf{p}_3 - \mathbf{p}_3')\,\delta_{\sigma_2\sigma_2'}\delta_{\sigma_3\sigma_3'}\,. \quad (6)
\end{aligned}$$

Here, the individual quark four-momenta k_i (k_i') and p_i (p_i') are connected through the boost transformations of the incoming and (outgoing) states, namely, $p_i = B(V)k_i$ (and analogously $p_i' = B(V')k_i'$). The normalization factor $\mathcal{N}_S$ as well as the momentum transfer $\tilde{q}^\mu = p_1^\mu - p_1'^\mu$ are specific for the PFSM and explicitly given in ref. [2], where also other details of the formalism/notation can be found. While there is a freedom in the choice of the normalization factor, which can cause minor influences on the results (cf. ref. [2]), it should be emphasized that $\tilde{q}^\mu$ is uniquely defined through the overall momentum conservation and the two spectator conditions. The off-shell extrapolation of the transition amplitude is made by keeping all hadrons and quarks on their respective mass shells. Obviously it implies energy non-conservation in the transition process. By virtue of the pseudovector-pseudoscalar equivalence the above construction also guarantees that the pseudovector and pseudoscalar quark-meson couplings lead to the same transition amplitude.

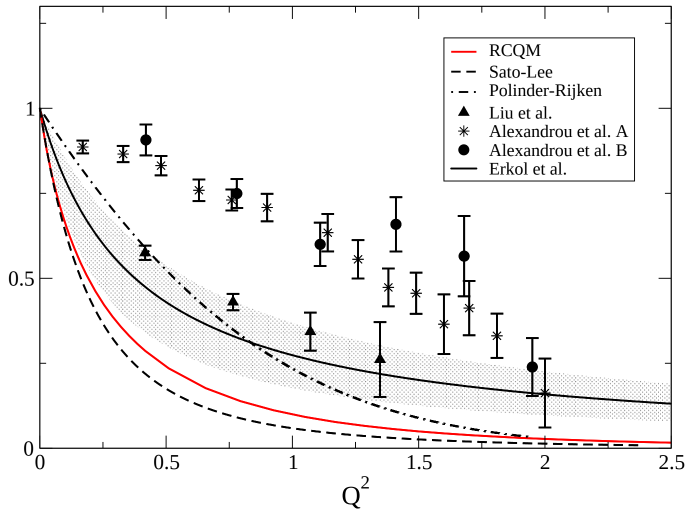

Fig. 2. Prediction of the strong form factor $G_{\pi NN}$, normalized to 1 at $Q^2 = 0$, by the GBE RCQM (solid/red line) in comparison to parametrizations from the dynamical meson-baryon models of Sato-Lee [5] and Polinder-Rijken [6,7] as well as results from three lattice QCD calculations [8–11] (cf. the legend); the shaded area around the result by Erkol *et al.* gives their theoretical error band.

The strong πNN and $\pi N\Delta$ form factors as dependent on the space-like momentum transfer $Q^2 = -q^2 > 0$ are then given by

$$G_{\pi NN}\left(Q^2\right) = \frac{1}{f_{\pi NN}} \frac{m_\pi \sqrt{2\pi}}{\sqrt{2M_N}} \frac{\sqrt{E'_N + M'_N}}{E'_N + M'_N + \omega} \frac{F^{RCQM}_{i\to f}}{Q_z}, \tag{7}$$

$$G_{\pi N\Delta}\left(Q^2\right) = -\frac{1}{f_{\pi N\Delta}} \frac{3\sqrt{2\pi}}{2} \frac{m_\pi}{\sqrt{E'_N + M'_N}\sqrt{2M_\Delta}} \frac{F^{RCQM}_{i\to f}}{Q_z}, \tag{8}$$

where the momentum transfer is taken into the *z*-direction. The results for the Goldstone-boson-exchange (GBE) RCQM [3,4] are shown in Figs. 2 and 3, where also a comparison is given to corresponding results from dynamical meson-baryon models and various lattice-QCD calculations. It is interesting to observe that the Q^2 dependence of both the $G_{\pi NN}$ and $G_{\pi N\Delta}$ form factors resulting directly and in a parameter-free manner from the RCQM qualitatively agrees with the parametrizations of the meson-baryon vertices in the Sato-Lee model [5]. In the case of $G_{\pi N\Delta}$ the RCQM result is also close to the Polinder-Rijken meson-baryon model [6,7]. On the other hand, the strong form factors from the lattice calculations show a (sometimes much) slower fall off with increasing Q^2. Even for the smaller differences between our results (as well as the form factors of Sato-Lee) and the data sets by Liu *et al.* and Erkol *et al.* it remains to be seen if dressing effects can account for these differences. Regarding all of the lattice data by Alexandrou *et al.* one has also to keep in mind that they correspond to relatively large pion masses with no extrapolations applied.

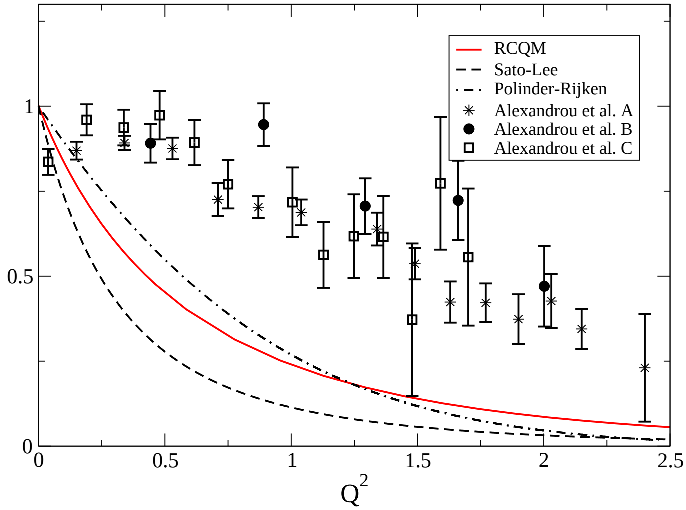

Fig. 3. Same as Fig. 2 but for the strong form factor $G_{\pi N \Delta}$.

As the vertex form factors represent an important input into a number of applications, we have also provided parametrizations in analytical forms as a function of the three-momentum transfer $\mathbf{q}^2$. It has turned out that an intermediate form between the usual monopole and dipole forms is most appropriate

$$G\left(\mathbf{q}^2\right) = \frac{1}{1 + \left(\frac{q}{\Lambda_1}\right)^2 + \left(\frac{q}{\Lambda_2}\right)^4} . \tag{9}$$

Our results in Figs. 2 and 3 are best reproduced with the parameter values given in Table 1.

Table 1. Coupling constants and cut-off parameters of the RCQM vertex form factors as parametrized according to the representation (9).

	$\frac{f_N^2}{4\pi}$	0.0691		$\frac{f_\Delta^2}{4\pi}$	0.188
N	Λ_1	0.451	Δ	Λ_1	0.594
	Λ_2	0.931		Λ_2	0.998

By this work we have obtained a parameter-free microscopic description of the strong πNN and πNΔ vertex form factors within a fully relativistic constituent quark model. Our study reveals that the structure of the πNΔ vertex is quite different from the πNN one, with cut-off parameters of up to 25% larger, contrary to what is often used in phenomenological models, where the πNN and πNΔ cut-offs are assumed of similar size [5–7] or even decreasing in the transition from πNN to πNΔ. Regarding the comparison with lattice-QCD results it will be most interesting, if the spread among them will be reduced by future calculations and how the final answer will turn out.

References

1. T. Melde, L. Canton and W. Plessas, Phys. Rev. Lett. **102**, 132002 (2009).
2. T. Melde, L. Canton, W. Plessas, and R. F. Wagenbrunn, Eur. Phys. J. A **25**, 97 (2005).
3. L. Y. Glozman, Z. Papp, W. Plessas, K. Varga, and R. F. Wagenbrunn, Phys. Rev. C **57**, 3406 (1998).
4. L. Y. Glozman, W. Plessas, K. Varga, and R. F. Wagenbrunn, Phys. Rev. D **58**, 094030 (1998).
5. T. Sato and T. S. H. Lee, Phys. Rev. C **54**, 2660 (1996).
6. H. Polinder and T. A. Rijken, Phys. Rev. C **72**, 065210 (2005).
7. H. Polinder and T. A. Rijken, Phys. Rev. C **72**, 065211 (2005).
8. C. Alexandrou, G. Koutsou, T. Leontiou, J. W. Negele, and A. Tsapalis, Phys. Rev. D **76**, 094511 (2007).
9. K. F. Liu, S. J. Dong, T. Draper, and W. Wilcox, Phys. Rev. Lett. **74**, 2172 (1995).
10. K. F. Liu *et al.*, Phys. Rev. D **59**, 112001 (1999).
11. G. Erkol, M. Oka and T. T. Takahashi, Phys. Rev. D **79**, 074509 (2009); arXiv:0805.3068.