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9 Can Spin-charge-family Theory Explain Baryon Number Non-conservation?

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Abstract. The *spin-charge-family* theory [1–12,14], in which spinors carry besides the Dirac spin also the second kind of the Clifford object, no charges, is a kind of the Kaluza-Klein theories [13]. The Dirac spinors of one Weyl representation in $d = (13+1)$ manifest [1,4,3,10,14,15] in $d = (3+1)$ at low energies all the properties of quarks and leptons assumed by the *standard model*. The second kind of spins explains the origin of families. Spinors interact with the vielbeins and the two kinds of the spin connection fields, the gauge fields of the two kinds of the Clifford objects, which manifest in $d = (3+1)$ besides the gravity and the known gauge vector fields also several scalar gauge fields. Scalars with the space index $s \in (7,8)$ carry the weak charge and the hyper charge ($\mp\frac{1}{2}, \pm\frac{1}{2}$, respectively), explaining the origin of the Higgs and the Yukawa couplings. It is demonstrated in this paper that the scalar fields with the space index $t \in (9, 10, \ldots, 14)$ carry the triplet colour charges, causing transitions of antileptons and antiquarks into quarks and back, enabling the appearance and the decay of baryons. These scalar fields are offering in the presence of the right handed neutrino condensate, which breaks the $\mathcal{CP}$ symmetry, the answer to the question about the matter-antimatter asymmetry.

Povzetek. V teoriji *spinov-nabojev-družin* [1–12,14] nosijo spinorji dve vrsti kvantnih števil, ki jih določata dve vrsti operatorjev γ^a: Diracovi operatorji γ^a in avtoričini $\tilde{\gamma}^a$, obe sta povezani z množenjem Cliffordovih objektov, ena vrsta z leve, druga z desne. Obe vrsti spina sta neodvisni in tvorita druga drugi ekvivalentne upodobitve. Analiza Lorentzove grupe $SO(13,1)$ s podgrupami te grupe pokaže, da vsebuje ena Weylova upodobitev Diracovih spinorjev v $d = (13+1)$ vse kvarke in leptone (ter antikvarke in antileptone) s kvantnimi števili kot jih predpiše *standardni model* pred elektrošibko zlomitvijo, le da so desnoročni nevtrini enakopravni partnerji elektronom [1,4,3,10,14,15]. Druga vrsta spina pojasni izvor družin. Spinorji interagirajo s tetradami in s polji dveh vrst spinskih povezav, ki so umeritvena polja obeh vrst operatorjev gamma. Po zlomitvi simetrij, tedaj pri opazljivih nizkih energijah, določajo ta polja, skupaj z vektorskimi svežnji, gravitacijo in vsa znana umeritvena vektorska polja. Določajo pa tudi skalarna polja. Skalarna polja s prostorskim indexom $s = (7,8)$ so šibki dubleti ($\tau^{13} = \mp\frac{1}{2}, Y = \pm\frac{1}{2}$), kar pojasni izvor Higgsovega skalarnega polja in Yukawinih sklopitev. Skalarna polja s prostorskim indeksom $t \in (9, 10, \ldots, 14)$ pa so barvni tripleti, ki povzročajo prehode antileptonov in antikvarkov v kvarke in obratno, kar omogoči nastanek in razpad barionov. Vsa skalarna polja nosijo glede na kvantna števila, ki jih določajo Diracovi γ^a in družinski $\tilde{\gamma}^a$, tudi družinska in Diracova kvantna števila v adjungirani upodobitvi grup. Lepota te teorije je, da en sam kondenzat iz dveh desnoročnih nevtrinov z družinskimi kvantnimi števili, ki niso družinska kvantna števila spodnjih štirih družin, zlomi diskretno simetrijo $\mathcal{CP}$ in poskrbi za maso vseh skalarnih polj, ter še neopaženega vektorskega polja. Ta skalarna polja ponujajo v prisotnosti kondenzata desnoročnih nevtrinov, ki zlomi simetrijo $\mathcal{CP}$, odgovor na vprašanje kako je v našem vesolju nastala opazljiva asimetrija med snovjo in antisnovjo. Skalarna polja s prostorskim indexom $s = (7, 8)$ zlomijo z neničelno vakuumsko pričakovano vrednostjo še šibki in hipernaboj, in spremenijo tudi lastno maso, ter tako pojasnijo vse privzetke standardnega modela. Ker teorija napoveduje dve ločeni gruči po štiri družine kvarkov in leptonov, pojasni stabilna od zgornjih širih družin izvor temne snovi. Teorija pa napoveduje tudi, da bodo na LHC izmerili četrto k trem že opaženim družinam, izmerili pa bodo tudi več skalarnih polj.

9.1 Introduction

The *spin-charge-family* [1–12,14] theory is offering, as a kind of the Kaluza-Klein like theories, the explanation for the charges of quarks and leptons (right handed neutrinos are in this theory the regular members of a family) and antiquarks and antileptons [15,16], and for the existence of the corresponding gauge vector fields. The theory explains, by using besides the Dirac kind of the Clifford algebra objects also the second kind of the Clifford algebra objects (there are only two kinds [5–7,3,17,18,20,19], associated with the left and the right multiplication of any Clifford object), the origin of families of quarks and leptons and correspondingly the origin of the scalar gauge fields causing the electroweak break. These scalar fields are responsible, after gaining nonzero vacuum expectation values, for the masses and mixing matrices of quarks and leptons [9–11] and for the masses of the weak vector gauge fields. They manifest, carrying the weak charge and the hyper charge equal to ($\pm\frac{1}{2}$, $\mp\frac{1}{2}$, respectively) [14], as the Higgs field and the Yukawa couplings of the *standard model*.

The *spin-charge-family* theory predicts two decoupled groups of four families [3,4,9–11]: The fourth of the lower group will be measured at the LHC [10], while the lowest of the upper four families constitutes the dark matter [12].

This theory also predicts the existence of the scalar fields which carry the triplet colour charges. All the scalars fields carry the fractional quantum numbers with respect to the scalar index $s \geq 5$, either the ones of $SU(2)$ or the ones of $SU(3)$, while they are with respect to other groups in the adjoint representations. Neither these scalar fields nor the scalars causing the electroweak break are the supersymmetric scalar partners of the quarks and leptons, since they do not carry all the charges of a family member.

These scalar fields with the triplet colour charges cause transitions of antileptons into quarks and antiquarks into quarks and back, offering, in the presence of the condensate of the two right handed neutrinos with the family quantum numbers belonging to the upper four families which breaks the CP symmetry, the explanation for the matter-antimatter asymmetry. This is the topic of the present paper.

Let me point out that the *spin-charge-family* theory overlaps in many points with other unifying theories [26–31], since all the unifying groups can be seen as the subgroups of the large enough orthogonal groups, with family groups included. But there are also many differences. While the theories built on chosen groups must for their choice propose the Lagrange densities designed for these groups and representations (which means that there must be a theory behind this effective Lagrange densities), the *spin-charge-family* theory starts with a very simple action, from where all the properties of spinors and the gauge vector and scalar fields follow, provided that the breaks of symmetries occur.

Consequently this theory differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up on different levels of the break of symmetries, in the unification scheme, in the family degrees of freedom and correspondingly also in the evolution of our universe.

It will be demonstrated in this paper that one condensate of two right handed neutrinos makes all the scalar gauge fields and all the vector gauge fields massive on the scale of the appearance of the condensate, except the vector gauge fields which appear in the *standard model* action before the electroweak break as massless fields. The scalar gauge fields, which cause the electroweak break while gaining nonzero vacuum expectation values and changing their masses, then explain masses of quarks and leptons and of the weak bosons.

It is an extremely encouraging fact, that one scalar condensate and the nonzero vacuum expectation values of some scalar fields, those with the weak and the hyper charge equal to by the *standard model* required charges for the Higgs's scalar, can bring the simple starting action in $d = (13+1)$ to manifest in $d = (3+1)$ in the low energy regime the observed phenomena of fermions and bosons, explaining the assumptions of the *standard model* and can possibly answer also the open questions, like the ones of the appearance of family members, of families, of the dark matter and of the matter-antimatter asymmetry.

The paper leaves, however, many a question connected with the break of symmetries open. Although the scales of breaks of symmetries can roughly be estimated, for the trustworthy predictions a careful study of the properties of fermions and bosons in the expanding universe is needed. It stays to be checked under which conditions in the expanding universe, the starting fields (fermions with the two kinds of spins and the corresponding vielbeins and the two kind of the spin connection fields) after the spontaneous breaks manifest in the low energy regime the observed phenomena. This is a very demanding study, a first simple step of which was done in the refs. [12,22]. The present paper is a step towards understanding the matter-antimatter asymmetry within the *spin-charge-family* theory.

In the subsection 9.1.1 I present the *action* and the *assumptions* of the *spin-charge-family* theory, with the comments added.

In sections (9.2, 9.4, 9.5, 9.3) the properties of the scalar and vector gauge fields and of the condensate are discussed. In appendices the discrete symmetries of the *spin-charge-family* theory and the technique used for representing spinors, with the one Weyl representation of $SO(13,1)$ and the families in $SO(7,1)$ included, is briefly presented. The final discussions are presented in sect. 9.7.

9.1.1 The action of the *spin-charge-family* theory and the assumptions

In this subsection all the assumptions of the *spin-charge-family* theory are presented and commented. This subsection follows to some extend a similar subsection of

the ref. [14].
i. The space-time is $d(=(13+1))$ dimensional. Spinors carry besides the internal degrees of freedom, determined by the Dirac γ^a's operators, also the second kind of the Clifford algebra operators [5–7,4], called $\tilde{\gamma}^a$'s.
ii. In the simple action [3,1] fermions ψ carry in $d=(13+1)$ only two kinds of spins, no charges, and *interact correspondingly with only the two kinds of the spin connection gauge fields, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$, and the vielbeins, $f^{\alpha}{}_{a}$.*

$$\begin{aligned}
S &= \int d^d x\, E\, \mathcal{L}_f + \\
&\quad \int d^d x\, E\, (\alpha\, R + \tilde{\alpha}\, \tilde{R})\,, \\
\mathcal{L}_f &= \frac{1}{2}\,(\bar{\psi}\,\gamma^a p_{0a}\psi) + h.c., \\
p_{0a} &= f^{\alpha}{}_a p_{0\alpha} + \frac{1}{2E}\{p_\alpha, E f^{\alpha}{}_a\}_-, \\
p_{0\alpha} &= p_\alpha - \frac{1}{2} S^{ab}\omega_{ab\alpha} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{ab\alpha}, \\
R &= \frac{1}{2}\{f^{\alpha[a}f^{\beta b]}\;(\omega_{ab\alpha,\beta} - \omega_{ca\alpha}\,\omega^{c}{}_{b\beta})\} + h.c.\,, \\
\tilde{R} &= \frac{1}{2}\, f^{\alpha[a}f^{\beta b]}\;(\tilde{\omega}_{ab\alpha,\beta} - \tilde{\omega}_{ca\alpha}\tilde{\omega}^{c}{}_{b\beta}) + h.c.\,.
\end{aligned} \tag{9.1}$$

Here [1] $f^{\alpha[a}f^{\beta b]} = f^{\alpha a}f^{\beta b} - f^{\alpha b}f^{\beta a}$. S^{ab} and $\tilde{S}^{ab}$ are generators (Eqs.(9.5, 9.37, 9.37) of the groups $SO(13,1)$ and $\widetilde{SO}(13,1)$, respectively, expressible by γ^a and $\tilde{\gamma}^a$.
iii. The manifold $M^{(13+1)}$ breaks first into $M^{(7+1)}$ times $M^{(6)}$ (which manifests as $SU(3)\times U(1)$), affecting both internal degrees of freedom, $SO(13+1)$ and $\widetilde{SO}(13+1)$. After this break there are $2^{((7+1)/2-1)}$ massless families, all the rest families get heavy masses [2].
Both internal degrees of freedom, the ordinary $SO(13+1)$ one (where γ^a determine spins and charges of spinors) and the $\widetilde{SO}(13+1)$ (where $\tilde{\gamma}^a$ determine family quantum numbers), break simultaneously with the manifolds.
iv. There are additional breaks of symmetry: The manifold $M^{(7+1)}$ breaks further

[1] $f^{\alpha}{}_a$ are inverted vielbeins to $e^a{}_\alpha$ with the properties $e^a{}_\alpha f^\alpha{}_b = \delta^a{}_b,\; e^a{}_\alpha f^\beta{}_a = \delta^\beta_\alpha$, $E = \det(e^a{}_\alpha)$. Latin indices $a, b, .., m, n, .., s, t, ..$ denote a tangent space (a flat index), while Greek indices $\alpha, \beta, .., \mu, \nu, ..\sigma, \tau, ..$ denote an Einstein index (a curved index). Letters from the beginning of both the alphabets indicate a general index ($a, b, c, ..$ and $\alpha, \beta, \gamma, ..$), from the middle of both the alphabets the observed dimensions $0, 1, 2, 3$ ($m, n, ..$ and $\mu, \nu, ..$), indices from the bottom of the alphabets indicate the compactified dimensions ($s, t, ..$ and $\sigma, \tau, ..$). We assume the signature $\eta^{ab} = diag\{1, -1, -1, \cdots, -1\}$.

[2] A toy model [22,23,15] was studied in $d=(5+1)$ with the same action as in Eq.'(9.1). For a particular choice of vielbeins and for a class of spin connection fields the manifold M^{5+1} breaks into $M^{(3+1)}$ times an almost S^2, while $2^{((3+1)/2-1)}$ families stay massless and mass protected. Equivalent assumption, although not yet proved that it really works, is made also in the case that $M^{(13+1)}$ breaks first into $M^{(7+1)} \times M^{(6)}$. The study is in progress.

into $M^{(3+1)} \times M^{(4)}$.
v. There is a scalar condensate of two right handed neutrinos with the family quantum numbers of the upper four families, bringing masses of the scale above the unification scale, to all the vector and scalar gauge fields, which interact with the condensate.
vi. There are nonzero vacuum expectation values of the scalar fields with the scalar indices $(7,8)$, which cause the electroweak break and bring masses to the fermions and weak gauge bosons, conserving the electromagnetic and colour charge.

Comments on the assumptions:

i.: There are, as already written above, two (only two) kinds of the Clifford algebra objects. The Dirac one (Eq.(9.35)) (γ^a) will be used to describe spins of spinors (fermions) in $d = (13+1)$, manifesting in $d = (3+1)$ the spin and all the fermion charges, the second one (Eq.(9.35)) ($\tilde{\gamma}^a$) will describe families of spinors. The representations of γ^a's and $\tilde{\gamma}^a$'s are orthogonal to one another [3]. There are correspondingly two groups determining internal degrees of freedom of spinors in $d = (13+1)$: The Lorentz group $SO(13,1)$ and the group $\widetilde{SO}(13,1)$.
One Weyl representation of $SO(13,1)$ contains, if analysed [1,3,4,15] with respect to the *standard model* groups, all the family members, assumed by the *standard model*, with the right handed neutrinos included (the family members are presented in table 9.3). It contains the left handed weak ($SU(2)_I$) charged and $SU(2)_{II}$ chargeless colour triplet quarks and colourless leptons (neutrinos and electrons), the right handed weakless and $SU(2)_{II}$ charged quarks and leptons, as well as the right handed weak charged and $SU(2)_{II}$ chargeless colour antitriplet antiquarks and (anti)colourless antileptons, and the left handed weakless and $SU(2)_{II}$ charged antiquarks and antileptons. The reader can easily check the properties of the representations of spinors (table 9.3), presented in the "technique" (appendix 9.9) way, if using Eqs. (9.5, 9.8, 9.9, 9.11, 9.14).
Each family member carries the family quantum numbers, originating in $\tilde{\gamma}^a$'s degrees of freedom. Correspondingly $\tilde{S}^{ab}$ changes the family quantum numbers, leaving the family member quantum number unchanged.
ii.: This starting action enables to represent the *standard model* as an effective low energy manifestation of the *spin-charge-family* theory, which explains all the *standard model* assumptions, with the families included. There are gauge vector fields, massless before the electroweak break: gravity, colour $SU(3)$ octet vector gauge fields, weak $SU(2)$ (it will be named $SU(2)_I$) triplet vector gauge field and "hyper" $U(1)$ (it will be named $U(1)_I$) singlet vector gauge fields. All are superposition of $f^{\alpha}{}_{c}\, \omega_{ab\alpha}$. There are (eight rather than the observed three) families of quarks and leptons, massless before the electroweak break.
These eight families are indeed two decoupled groups of four families, in the fundamental representations with respect to twice $\widetilde{SU}(2) \times \widetilde{SU}(2)$ groups, the

[3] One can learn in Eq. (9.44)of appendix (9.9) that S^{ab} transforms one state of the representation into another state of the same representation, while $\tilde{S}^{ab}$ transforms the state into the state belonging to another representation.

subgroups of $\widetilde{SO}(3,1)\times \widetilde{SO}(4) \in \widetilde{SO}(7,1)$. The scalar gauge fields, determining the mass matrices of quarks and leptons, carry with respect to the scalar index $s \in (7,8)$ the weak and the hyper charge of the scalar Higgs, while they carry if they are the superposition of $f^{\sigma}{}_{s}\, \tilde{\omega}_{ab\sigma}$ two kinds of the family quantum numbers in the adjoint representations, representing two (orthogonal) groups, each of the group contains two triplets (with respect to $\widetilde{SU}(2)_{\widetilde{SO}(3,1)} \times \widetilde{SU}(2)_{\widetilde{SO}(4)}$).
The scalar fields with the quantum numbers (Q, Q', Y', which are the superposition of $f^{\sigma}{}_{s}\, \omega_{ab\sigma}$) are the three singlets, again carrying the weak and the hyper charge of the scalar Higgs. One group of two triplets determine, together with the three singlets, after gaining nonzero expectation values, the Higgs's scalar and the Yukawa couplings of the *standard model*. The starting action contains also the additional $SU(2)_{II}$ (from $SO(4)$) vector gauge field and the scalar fields with the space index $s \in (5,6)$ and $t \in (9,10,11,12)$, as well as the auxiliary vector gauge fields expressible (Eqs. (9.56, 9.55) in the appendix 9.10) with vielbeins. They all remain either auxiliary (if there are no spinor sources manifesting their quantum numbers) or become massive after the appearance of the condensate.
iii., iv.: The assumed break from $M^{(13+1)}$ first into $M^{(7+1)}$ times $M^{(6)}$ (manifesting the symmetry $SU(3)\times U(1)_{II}$) explains why the weak and the hyper charge are connected with the handedness of spinors. In the spinor representation of $SO(7,1)$ there are left handed weak charged quarks and leptons with the hyper charges ($\frac{1}{6}, -\frac{1}{2}$, respectively) and the right handed weak chargeless quarks with the hyper charge either $\frac{2}{3}$ or $-\frac{1}{3}$, while the right handed weak chargeless leptons carry the hyper charge either 0 or (−1). A further break from $M^{(7+1)}$ into $M^{(3+1)} \times M^{4)}$, manifesting the symmetry $SO(3,1)\times SU(2)_{I} \times SU(2)_{II} \times U(1)_{II} \times SU(3)$, explains the observed properties of the family members - the coloured quarks, left handed weak charged and $SU(2)_{II}$ chargeless and right handed weak chargeless and $SU(2)_{II}$ charged and colourless leptons, again left handed weak charged and $SU(2)_{II}$ chargeless and right handed weak chargeless and $SU(2)_{II}$ charged, quarks with the "spinor" charge $\frac{1}{6}$ and leptons with the "spinor" charge $-\frac{1}{2}$ - and of the observed vector gauge fields and the scalar fields (through Higgs's scalar and Yukawa couplings).
Since the left handed members distinguish from the right handed partners in the weak and the hyper charges, the family members of all the families stay massless and mass protected up to the electroweak break [4]. Antiparticles are accessible from particles by the $\mathbb{C}_{\mathcal{N}}$ and $\mathcal{P}_{\mathcal{N}}$, as explained in refs. [15,16] and briefly also in the appendix (9.8). This discrete symmetry operator does not contain $\tilde{\gamma}^{a}$'s degrees of freedom. To each family member there corresponds the antimember, with the same family quantum number.
v.: It is a condensate of the two right handed neutrinos with the quantum numbers of the upper four families (table 9.2), appearing in the energy region

[4] As long as the left handed family members and their right handed partners carry different conserved charges, they can not behave as massive particles, they are mass protected. It is the appearance of nonzero vacuum expectation values of the scalar fields, carrying the weak and the hyper charge, which cause non conservation of these two charges, which makes the superposition of the left and the right handed family members possible, and breaks the mass protection.

above the unification scale, which makes all the scalar gauge fields (those with the space index $(5,6,7,8)$, as well as those with the space index $(9,\ldots,14)$) and the vector gauge fields, manifesting nonzero quantum numbers τ^4, τ^{23}, Q ,Y, $\tilde{\tau}^4$, $\tilde{\tau}^{23}$, $\tilde{Q}$,$\tilde{Y}$,$\tilde{N}^3_R$ (Eqs. (9.8, 9.9, 9.11, 9.12, 9.13, 9.14)) massive.
vi.: At the electroweak break the scalar fields with the space index $s = (7,8)$, triplets with respect to the family index (originating in $\tilde{\omega}_{abs}$, Eq. (9.16)) and the three singlets carrying the charges (Q, Q', Y') (originating in $\omega_{ts's}$, Eq. (9.15)), all with the weak and the hyper charge equal to ($\mp\frac{1}{2}, \pm\frac{1}{2}$, respectively), get nonzero vacuum expectation values, changing also their masses and breaking the weak and the hyper charge symmetry. These scalars determine mass matrices of twice the four families, as well as the masses of the weak bosons.
All the rest scalar fields keep masses of the condensate scale and are correspondingly unobservable in the low energy regime [5]. The fourth family to the observed three ones will (sooner or later) be observed at the LHC. Its properties are under the consideration [10], while the stable of the upper four families is the candidate for the dark matter constituents.

The above assumptions enable that the starting action (Eq. (9.1)) manifests effectively in $d = (3+1)$ in the low energy regime fermion and boson fields as assumed by the *standard model*.

To see this [3,1,4–8,2,9,10,12,14], let us formally rewrite the Lagrange density for a Weyl spinor of (Eq.(9.1)), as follows

$$
\begin{aligned}
\mathcal{L}_f &= \bar{\psi}\gamma^m (p_m - \sum_{Ai} g^A \tau^{Ai} A^{Ai}_m)\psi + \\
&\{ \sum_{s=7,8} \bar{\psi}\gamma^s p_{0s}\, \psi\} + \\
&\{ \sum_{t=5,6,9,\ldots,14} \bar{\psi}\gamma^t p_{0t}\, \psi\}, \\
p_{0s} &= p_s - \frac{1}{2} S^{s's"}\omega_{s's"s} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abs}\,, \\
p_{0t} &= p_t - \frac{1}{2} S^{t't"}\omega_{t't"t} - \frac{1}{2}\tilde{S}^{ab}\tilde{\omega}_{abt}\,,
\end{aligned}
\qquad (9.2)
$$

where $m \in (0,1,2,3)$, $s \in 7,8$, $(s', s") \in (5,6,7,8)$, (a,b) (appearing in $\tilde{S}^{ab}$) run within $\in (0,1,2,3)$ and $\in (5,6,7,8)$, $t \in (5,6,9,\ldots,13,14)$, $(t', t") \in (5,6,7,8)$ and $\in (9,10,\ldots,14)$. ψ represents all family members of all the families. The generators of the charge groups τ^{Ai} (expressed in Eqs. (9.3), (9.9), (9.11) in terms of S^{ab}) fulfil the commutation relations

$$
\begin{aligned}
\tau^{Ai} &= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab}\,, \\
\{\tau^{Ai}, \tau^{Bj}\}_- &= i\delta^{AB} f^{Aijk}\tau^{Ak}\,.
\end{aligned}
\qquad (9.3)
$$

[5] Correspondingly $d = (13+1)$ manifests in $d = (3+1)$ spins and charges as if there would be $d = (9+1)$, since the plane $(5,6)$ and the plane in which the vector τ^4 lies, are unobservable at low energies.

The spin generators are defined in Eq. (9.8). These group generators determine all the internal degrees of freedom of one family members as seen from the point of view of $d = (3+1)$: The colour charge ($SU(3)$ with the generators $\vec{\tau}^3$) and the "spinor charge" ($U(1)_{II}$) with the generator τ^4 originating in $SO(6)$, the weak charge $SU(2)_I$ with the generators $\vec{\tau}^1$ and the second $SU(2)_{II}$ charge with the generators $\vec{\tau}^2$ originating in $SO(4)$ ($SU(2)_{II}$ breaks in the presence of the condensate into $U(1)_I$, defining together with τ^4 the hyper charge Y ($= \tau^{23} + \tau^4$) and the spin determined by $SO(3,1)$.
The condensate of two right handed neutrinos with the family quantum numbers of the upper four families bring masses (of the unifying scale $\geq 10^{16}$ GeV or above) to all the scalar and those vector gauge fields which are not observed at so far measurable energies.
The scalar fields causing, when getting nonzero vacuum expectation values, the electroweak phase transitions changing at the transition also their own masses, bring masses to the eight families and to the weak bosons. We shall comment all these fields in what follows.

The first line of Eq. (9.2) describes [1,3] before the electroweak break the dynamics of eight families of massless fermions in interaction with the massless colour $\vec{A}^3_m$, weak $\vec{A}^1_m$ and hyper A^Y_m ($= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m$) gauge fields, all are the superposition of ω_{abm} [6].

The second line of the same equation (Eq. (9.2)) determines the mass term, which after the electroweak break brings masses to all the family members of the eight families and to the weak bosons. The scalar fields responsible - after getting nonzero vacuum expectation values - for masses of the family members and of the weak bosons are namely included in the second line of Eq. (9.2) as $(-\frac{1}{2} S^{s's"} \omega_{s's"s} - \frac{1}{2} \tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s = \in (7,8)$, $(s', s") \in (5,6,7,8)$, $(\tilde{a}, \tilde{b}) \in (\tilde{0}, \tilde{1}, \ldots, \tilde{8}))$ [7]. The properties of these scalar fields are discussed in sect. (9.4), where the proof is presented that they all carry the weak charge and the hyper charge as the *standard model* Higgs's scalar, while they are either triplets with respect to the family quantum numbers or singlets with respect to the charges Q, Q' and Y'. While the two triplets ($\vec{\tilde{A}}^1_s$, $\vec{\tilde{A}}^{\tilde{N}_L}_s$) interact with the lower four families, interact ($\vec{\tilde{A}}^2_s$, $\vec{\tilde{A}}^{\tilde{N}_R}_s$) with the upper four families. These twice two triplets are superposition of $\frac{1}{2}\, \tilde{S}^{\tilde{a}\tilde{b}}\, \tilde{\omega}_{\tilde{a}\tilde{b}s}$, $s = \in (7,8)$, Eq. (9.16). The three singlets (A^Q_s, $A^{Q'}_s$ and $A^{Y'}_m$) are superposition of $\omega_{s's"s}$, Eq.(9.15). They interact with the family members of all the families, "seeing" family members charges.

The third line of Eq. (9.2) represents fermions in interaction with all the rest scalar fields. Scalar fields become massive after interacting with the condensate. Those which do not gain nonzero vacuum expectation values, keep the heavy masses of the order of the scale of the condensate up to low energies. The massive

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[6] These superposition can easily be found by using Eqs. (9.11, 9.9). They are explicitly written in the ref. [3]. The interaction with the condensate makes the fields $A^{Y'}_m$, Eq. (9.14), A^{21}_m and A^{22}_m very massive (at the scale of the condensate).

[7] To point out that S^{ab} and $\tilde{S}^{ab}$ belong to two different kinds of the Clifford algebra objects are the indices (a, b) are in $\tilde{S}^{ab}$ in this paragraph written as $(\tilde{a}, \tilde{b})$. Normally only (a, b) will be used for S^{ab} and $\tilde{S}^{ab}$.

scalars with the space index $t \in (5,6)$ transform (table 9.3) u_R-quarks into d_L-quarks and ν_R-leptons into e_L-leptons and back, as well as $\bar{u}_R$-antiquarks into $\bar{d}_L$-antiquarks and back and $\bar{\nu}_R$-antileptons into $\bar{e}_L$-antileptons and back, breaking in the presence of the condensate the Q global symmetry. Those scalar fields with the space index $t = (9, 10, \cdots, 14)$ transform antileptons into quarks and antiquarks into quarks and back. They are offering in the presence of the scalar condensate breaking the $\mathcal{CP}$ symmetry the explanation for the observed matter-antimatter asymmetry, as we shall show in sect. 9.2.

Let us write down the part of the fermion action which in the presence of the condensate offers the explanation for the observed matter/antimatter asymmetry.

$$\mathcal{L}_{f'} = \psi^{\dagger} \gamma^0 \gamma^t \{ \sum_{t=(9,10,\ldots 14)} [p_t - (\frac{1}{2} S^{s' s"} \omega_{s' s" t} + \frac{1}{2} S^{t' t''} \omega_{t' t" t} + \frac{1}{2} \tilde{S}^{ab} \tilde{\omega}_{abt})]\} \psi , \tag{9.4}$$

where $(s', s") \in (5,6,7,8)$, $(t, t', t") \in (9, 10, \ldots, 14)$ and $(a, b) \in (0, 1, 2, 3)$ and $\in (5,6,7,8)$, in agreement with the assumed breaks in sect. (9.1). Again operators $\tilde{S}^{ab}$ determine family quantum numbers and S^{ab} determine family members quantum numbers. Correspondingly the superposition of the scalar fields $\tilde{\omega}_{abt}$ and the superposition of the scalar fields ω_{abt} carry the quantum numbers determined by either the superposition of $\tilde{S}^{ab}$ or by the superposition S^{ab} in the adjoint representations, while they carry the colour charge, determined by the space index $t \in (9, 10, \ldots, 14)$, in the triplet representation of the $SU(3)$ group, as we shall see. Similarly the scalars with the space index $s \in (7, 8)$ carry the weak and the hyper charge in the doublets representations.

The condensate of two right handed neutrinos with the family quantum numbers of the upper four families carries (table 9.2) $\tau^4 = 1$, $\tau^{23} = -1$, $\tau^{13} = 0$, $Y = 0$, $Q = 0$, and the family quantum numbers of the upper four families and gives masses to scalar and vector gauge fields with the nonzero corresponding quantum numbers. The only vector gauge fields which stay massless up to the electroweak break are the hyper charge field (A^Y_m), the weak charge field ($\vec{A}^1_m$) and the colour charge field ($\vec{A}^3_m$).

The *standard model* **subgroups of the $SO(13+1)$ and $\widetilde{SO}(13+1)$ groups and the corresponding gauge fields** This section follows to large extend the refs. cite-JMP,NscalarsweakY2014. To calculate quantum numbers of one Weyl representation presented in table 9.3 in terms of the generators of the *standard model* charge groups τ^{Ai} $(= \sum_{a,b} c^{Ai}{}_{ab}\, S^{ab})$ one must look for the coefficients $c^{Ai}{}_{ab}$ (Eq. (9.3)). Similarly also the spin and the family degrees of freedom can be expressed.

The same coefficients $c^{Ai}{}_{ab}$ determine operators which apply on spinors and on vectors. The difference among the three kinds of operators - vector and two kinds of spinor - lies in the difference among S^{ab}, $\tilde{S}^{ab}$ and $\mathcal{S}^{ab}$.

While S^{ab} for spins of spinors is equal to

$$S^{ab} = \frac{i}{4} (\gamma^a \gamma^b - \gamma^b \gamma^a) , \quad \{\gamma^a , \gamma^b\}_+ = 2\eta^{ab} , \tag{9.5}$$

and $\tilde{S}^{ab}$ for families of spinors is equal to

$$\tilde{S}^{ab} = \frac{i}{4}(\tilde{\gamma}^a \tilde{\gamma}^b - \tilde{\gamma}^b \tilde{\gamma}^a), \quad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}, \\ \{\gamma^a, \tilde{\gamma}^b\}_+ = 0, \tag{9.6}$$

one must take, when $\mathcal{S}^{ab}$ apply on the spin connections ω_{bde} ($= f^{\alpha}{}_e\ \omega_{bd\alpha}$) and $\tilde{\omega}_{\tilde{b}\tilde{d}e}$ ($= f^{\alpha}{}_e\ \tilde{\omega}_{\tilde{b}\tilde{d}\alpha}$), on either the space index e or the indices $(b, d, \tilde{b}, \tilde{d})$, the operator

$$(\mathcal{S}^{ab})^c{}_e A^{d\dots e\dots g} = i(\eta^{ac}\delta^b_e - \eta^{bc}\delta^a_e)\, A^{d\dots e\dots g}. \tag{9.7}$$

This means that the space index (e) of ω_{bde} transforms according to the requirement of Eq. (9.7), and so do b, d and $\tilde{b}, \tilde{d}$. Here I used again the notation $\tilde{b}, \tilde{d}$ to point out that S^{ab} and $\tilde{S}^{ab}$ ($= \tilde{S}^{\tilde{a}\tilde{b}}$) are the generators of two independent groups[14].

One finds [1,3–8,2] for the generators of the spin and the charge groups, which are the subgroups of $SO(13,1)$, the expressions:

$$\vec{N}_{\pm}(= \vec{N}_{(L,R)}) := \frac{1}{2}(S^{23} \pm iS^{01}, S^{31} \pm iS^{02}, S^{12} \pm iS^{03}), \tag{9.8}$$

where the generators $\vec{N}_{\pm}$ determine representations of the two $SU(2)$ invariant subgroups of $SO(3,1)$, the generators $\vec{\tau}^1$ and $\vec{\tau}^2$,

$$\vec{\tau}^1 := \frac{1}{2}(S^{58} - S^{67}, S^{57} + S^{68}, S^{56} - S^{78}), \tag{9.9}$$

$$\vec{\tau}^2 := \frac{1}{2}(S^{58} + S^{67}, S^{57} - S^{68}, S^{56} + S^{78}), \tag{9.10}$$

determine representations of the $SU(2)_I \times SU(2)_{II}$ invariant subgroups of the group $SO(4)$, which is further the subgroup of $SO(7,1)$ ($SO(4), SO(3,1)$ are subgroups of $SO(7,1)$), and the generators $\vec{\tau}^3$, τ^4 and $\tilde{\tau}^4$

$$\begin{aligned}\vec{\tau}^3 &:= \frac{1}{2}\{S^{9\,12} - S^{10\,11}, S^{9\,11} + S^{10\,12}, S^{9\,10} - S^{11\,12}, \\ &\quad S^{9\,14} - S^{10\,13}, S^{9\,13} + S^{10\,14}, S^{11\,14} - S^{12\,13}, \\ &\quad S^{11\,13} + S^{12\,14}, \frac{1}{\sqrt{3}}(S^{9\,10} + S^{11\,12} - 2S^{13\,14})\}, \\ \tau^4 &:= -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14}), \\ \tilde{\tau}^4 &:= -\frac{1}{3}(\tilde{S}^{9\,10} + \tilde{S}^{11\,12} + \tilde{S}^{13\,14}),\end{aligned} \tag{9.11}$$

determine representations of $SU(3) \times U(1)$, originating in $SO(6)$, and of $\tilde{\tau}^4$ originating in $\widetilde{SO}(6)$.

One correspondingly finds the generators of the subgroups of $\widetilde{SO}(7,1)$,

$$\vec{\tilde{N}}_{L,R} := \frac{1}{2}(\tilde{S}^{23} \pm i\tilde{S}^{01}, \tilde{S}^{31} \pm i\tilde{S}^{02}, \tilde{S}^{12} \pm i\tilde{S}^{03}), \tag{9.12}$$

which determine representations of the two $\widetilde{SU}(2)$ invariant subgroups of $\widetilde{SO}(3,1)$, while

$$\vec{\tilde{\tau}}^{1} := \frac{1}{2}(\tilde{S}^{58} - \tilde{S}^{67}, \tilde{S}^{57} + \tilde{S}^{68}, \tilde{S}^{56} - \tilde{S}^{78}),$$
$$\vec{\tilde{\tau}}^{2} := \frac{1}{2}(\tilde{S}^{58} + \tilde{S}^{67}, \tilde{S}^{57} - \tilde{S}^{68}, \tilde{S}^{56} + \tilde{S}^{78}), \tag{9.13}$$

determine representations of $\widetilde{SU}(2)_{I} \times \widetilde{SU}(2)_{II}$ of $\widetilde{SO}(4)$. Both, $\widetilde{SO}(3,1)$ and $\widetilde{SO}(4)$, are the subgroups of $\widetilde{SO}(7,1)$.

One further finds [3]

$$Y = \tau^4 + \tau^{23}, \; Y' = -\tau^4 \tan^2\vartheta_2 + \tau^{23}, \; Q = \tau^{13} + Y, \; Q' = -Y \tan^2\vartheta_1 + \tau^{13},$$
$$\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23}, \; \tilde{Y}' = -\tilde{\tau}^4 \tan^2\tilde{\vartheta}_2 + \tilde{\tau}^{23}, \; \tilde{Q} = \tilde{Y} + \tilde{\tau}^{13}, \; \tilde{Q}' = -\tilde{Y} \tan^2\tilde{\vartheta}_1 + \tilde{\tau}^{13}. \tag{9.14}$$

The scalar fields, responsible [1–3] - after getting in the electroweak break nonzero vacuum expectation values - for the masses of the family members and of the weak bosons, and presented in the second line of Eq. (9.2), can be expressed in terms of ω_{abc} fields and $\tilde{\omega}_{abc}$ fields as presented in Eq. (9.15), 9.16).

One can find the below expressions by taking into account Eqs. (9.9, 9.11, 9.12, 9.13) and Eq. (9.14).

$$\begin{aligned}
-\frac{1}{2} S^{s's"} \omega_{s's"s} &= -(g^{23}\tau^{23} A^{23}_s + g^{13}\tau^{13} A^{13}_s + g^{4}\tau^{4} A^{4}_s),\\
g^{13}\tau^{13} A^{13}_s + g^{23}\tau^{23} A^{23}_s + g^{4}\tau^{4} A^{4}_s &= g^{Q}\, Q A^{Q}_s + g^{Q'}\, Q' A^{Q'}_s + g^{Y'}\, Y' A^{Y'}_s,\\
A^{4}_s &= -(\omega_{9\,10\,s} + \omega_{11\,12\,s} + \omega_{13\,14\,s}),\\
A^{13}_s &= (\omega_{56s} - \omega_{78s}), \quad A^{23}_s = (\omega_{56s} + \omega_{78s}),\\
A^{Q}_s &= \sin\vartheta_1\, A^{13}_s + \cos\vartheta_1\, A^{Y}_s,\\
A^{Q'}_s &= \cos\vartheta_1\, A^{13}_s - \sin\vartheta_1\, A^{Y}_s,\\
A^{Y}_s &= \sin\vartheta_2\, A^{23}_s + \cos\vartheta_2\, A^{4}_s,\\
A^{Y'}_s &= \cos\vartheta_2\, A^{23}_s - \sin\vartheta_2\, A^{4}_s,\\
&(s \in (7,8)).
\end{aligned} \tag{9.15}$$

In Eq. (9.15) the coupling constants were explicitly written to see the analogy with the gauge fields in the *standard model*.

$$\begin{aligned}
-\frac{1}{2}\tilde{S}^{\tilde{a}\tilde{b}}\,\tilde{\omega}_{\tilde{a}\tilde{b}s} &= -(\vec{\tilde{\tau}}^{\tilde{1}}\,\vec{\tilde{A}}^{\tilde{1}}_s + \vec{\tilde{N}}_{\tilde{L}}\,\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s + \vec{\tilde{\tau}}^{\tilde{2}}\,\vec{\tilde{A}}^{\tilde{2}}_s + \vec{\tilde{N}}_{\tilde{R}}\,\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s),\\
\vec{\tilde{A}}^{\tilde{1}}_s &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} - \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} + \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} - \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{L}}}_s &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} + i\,\tilde{\omega}_{\tilde{0}\tilde{3}s}),\\
\vec{\tilde{A}}^{\tilde{2}}_s &= (\tilde{\omega}_{\tilde{5}\tilde{8}s} + \tilde{\omega}_{\tilde{6}\tilde{7}s}, \tilde{\omega}_{\tilde{5}\tilde{7}s} - \tilde{\omega}_{\tilde{6}\tilde{8}s}, \tilde{\omega}_{\tilde{5}\tilde{6}s} + \tilde{\omega}_{\tilde{7}\tilde{8}s}),\\
\vec{\tilde{A}}^{\tilde{N}_{\tilde{R}}}_s &= (\tilde{\omega}_{\tilde{2}\tilde{3}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{1}s}, \tilde{\omega}_{\tilde{3}\tilde{1}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{2}s}, \tilde{\omega}_{\tilde{1}\tilde{2}s} - i\,\tilde{\omega}_{\tilde{0}\tilde{3}s}),\\
&(s \in (7,8)).
\end{aligned} \tag{9.16}$$

Scalar fields from Eq. (9.16) couple to the family quantum numbers, while those from Eq. (9.15) distinguish among family members.

Expressions for the vector gauge fields in terms of the spin connection fields and the vielbeins, which correspond to the colour charge ($\vec{A}^3_m$), the $SU(2)_{II}$ charge ($\vec{A}^2_m$), the weak charge ($SU(2)_I$) ($\vec{A}^1_m$) and the $U(1)_{II}$ charge originating in $SO(6)$ ($\vec{A}^4_m$), can be found by taking into account Eqs. (9.9, 9.11). Equivalently one finds the vector gauge fields in the "tilde" sector. One really can use just the expressions from Eqs. (9.15, 9.16), if replacing the scalar index s with the vector index m.

9.2 Properties of scalar and vector gauge fields, contributing to transitions of antileptons into quarks

In this - the main - part of the present paper the properties, quantum numbers and discrete symmetries of those scalar and vector gauge fields appearing in the action (Eqs.(9.1, 9.2), 9.4) of the *spin-charge-family* theory [1–9,12] are studied, which cause transitions of antileptons into quarks and back, and antiquarks into quarks and back.

These scalar gauge fields carry the triplet or antitriplet colour charge (see table 9.1) and the fractional hyper and electromagnetic charge.

The Lagrange densities from Eqs. (9.1, 9.2, 9.4) manifest $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance (appendix (9.8)). All the vector and the spinor gauge fields are before the appearance of the condensate massless and reactions creating particles from antiparticles and back goes in both directions equivalently. Correspondingly there is no matter-antimatter asymmetry.

The *spin-charge-family* theory breaks the matter-antimatter symmetry by the appearance of the condensate (sect. 9.3) and also by nonzero vacuum expectation values of the scalar fields causing the electroweak phase transition (sect. 9.4). I shall show that there is the condensate of two right handed neutrinos which breaks this symmetry, giving masses to all the scalar gauge fields and to all those vector gauge fields which would be in contradiction with the observations.

Let us start by analysing the Lagrange density presented in Eq. (9.4) before the appearance of the condensate. The term $\gamma^t \frac{1}{2} S^{s's"} \omega_{s's"t}$ in Eq. (9.4) can be rewritten, if taking into account Eq. (9.42), as follows

$$
\begin{aligned}
\gamma^t \frac{1}{2} S^{s's"} \omega_{s's"t} &= \sum_{+,-} \sum_{(t\,t')} \overset{tt'}{(\oplus)} \frac{1}{2} S^{s's"} \omega_{s"s" \overset{tt'}{(\oplus)}}\,, \\
\omega_{s"s" \overset{tt'}{(\oplus)}} &:= \omega_{s"s" \overset{tt'}{(\pm)}} = (\omega_{s's"t} \mp i\, \omega_{s's"t'})\,, \\
\overset{tt'}{(\oplus)} &:= \overset{tt'}{(\pm)} = \frac{1}{2} (\gamma^t \pm \gamma^{t'})\,, \\
(t\,t') &\in ((9\,10), (11\,12), (13\,14))\,.
\end{aligned}
\tag{9.17}
$$

I introduced the notations $\overset{tt'}{(\oplus)}$ and $\omega_{s"s" \overset{tt'}{(\oplus)}}$ to distinguish among different superposition of states in equations below.

Using Eqs. (9.9, 9.11) the expression $\overset{tt'}{(\bigoplus)}\; \frac{1}{2} S^{s's"}\, \omega_{s"s"\overset{tt'}{(\oplus)}}$ can be further rewritten as follows

$$
\begin{aligned}
&\overset{tt'}{(\bigoplus)}\quad \frac{1}{2} S^{s's"}\,\omega_{s"s"\overset{tt'}{(\oplus)}} = \\
&\overset{tt'}{(\bigoplus)}\,\{\tau^{2+}\,A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-}\,A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23}\,A^{23}_{\overset{tt'}{(\oplus)}} + \tau^{1+}\,A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-}\,A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13}\,A^{13}_{\overset{tt'}{(\oplus)}}\},\\
&A^{2\boxplus}_{\overset{tt'}{(\oplus)}} = (\omega_{58\overset{tt'}{(\oplus)}} + \omega_{67\overset{tt'}{(\oplus)}}) \boxminus i(\omega_{57\overset{tt'}{(\oplus)}} - \omega_{68\overset{tt'}{(\oplus)}}),\\
&A^{23}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} + \omega_{78\overset{tt'}{(\oplus)}}),\\
&A^{1\boxplus}_{\overset{tt'}{(\oplus)}} = \omega_{58\overset{tt'}{(\oplus)}} - \omega_{67\overset{tt'}{(\oplus)}}) \boxminus i(\omega_{57\overset{tt'}{(\oplus)}} + \omega_{68\overset{tt'}{(\oplus)}}),\\
&A^{13}_{\overset{tt'}{(\oplus)}} = (\omega_{56\overset{tt'}{(\oplus)}} - \omega_{78\overset{tt'}{(\oplus)}}).
\end{aligned}
\tag{9.18}
$$

Equivalently one expresses the term $\gamma^t\, \frac{1}{2}\,\tilde{S}^{ab}\,\tilde{\omega}_{abt}$ in Eq. (9.4), by using Eqs. (9.12, 9.13), as

$$
\begin{aligned}
&\gamma^t \frac{1}{2}\,\tilde{S}^{ab}\,\tilde{\omega}_{abt} = \overset{tt'}{(\bigoplus)}\;\frac{1}{2}\,\tilde{S}^{ab}\,\tilde{\omega}_{ab\overset{tt'}{(\oplus)}} = \\
&\overset{tt'}{(\bigoplus)}\,\{\tilde{\tau}^{2+}\,\tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-}\,\tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23}\,\tilde{A}^{23}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1+}\,\tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-}\,\tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13}\,\tilde{A}^{13}_{\overset{tt'}{(\oplus)}} + \\
&\tilde{N}^{+}_{R}\,\tilde{A}^{N_R+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R}\,\tilde{A}^{N_R-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R}\,\tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{+}_{L}\,\tilde{A}^{N_L+}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L}\,\tilde{A}^{N_L-}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L}\,\tilde{A}^{N_L3}_{\overset{tt'}{(\oplus)}}\},\\
&\tilde{A}^{N_R\boxplus}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{01\overset{tt'}{(\oplus)}}) \boxminus i(\tilde{\omega}_{31\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{02\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} - i\,\tilde{\omega}_{03\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_L\boxplus}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{23\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{01\overset{tt'}{(\oplus)}}) \boxminus i(\tilde{\omega}_{31\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{02\overset{tt'}{(\oplus)}}),\\
&\tilde{A}^{N_R3}_{\overset{tt'}{(\oplus)}} = (\tilde{\omega}_{12\overset{tt'}{(\oplus)}} + i\,\tilde{\omega}_{03\overset{tt'}{(\oplus)}}),
\end{aligned}
\tag{9.19}
$$

with $\tilde{A}^{2\boxplus}_{\overset{tt'}{(\oplus)}}, \tilde{A}^{23}_{\overset{tt'}{(\oplus)}}, \tilde{A}^{1\boxplus}_{\overset{tt'}{(\oplus)}}$ and $\tilde{A}^{13}_{\overset{tt'}{(\oplus)}}$ following from expressions for $A^{2\boxplus}_{\overset{tt'}{(\oplus)}}, A^{23}_{\overset{tt'}{(\oplus)}}, A^{1\boxplus}_{\overset{tt'}{(\oplus)}}$ and $A^{13}_{\overset{tt'}{(\oplus)}}$, respectively, in (Eq.(9.18)), if replacing $\omega_{s"s"\overset{tt'}{(\oplus)}}$ by $\tilde{\omega}_{s"s"\overset{tt'}{(\oplus)}}$.

There is the additional term in Eq. (9.4): $\gamma^t\, \frac{1}{2}\, S^{t't"}\,\omega_{t't"t}$. This term can be written with respect to the generators $S^{t't"}$ as one colour octet scalar field and one $U(1)_{II}$ scalar field (Eq. 9.11)

$$
\begin{aligned}
&\gamma^t\,\frac{1}{2}\,S^{t"t'"}\,\omega_{t"t'"t} = \sum_{+,-}\sum_{(t\,t')}\overset{tt'}{(\bigoplus)}\,\{\vec{\tau}^{3}\cdot\vec{A}^{3}_{\overset{tt'}{(\oplus)}} + \tau^{4}\cdot A^{4}_{\overset{tt'}{(\oplus)}}\},\\
&(t\,t') \in ((9\,10), 11\,12), 13\,14))\,.
\end{aligned}
\tag{9.20}
$$

Taking all above equations (9.17, 9.18, 9.19, 9.20) into account Eq. (9.4) can be rewritten, if we leave out $p_{\overset{tt'}{(\oplus)}}$ since in the low energy limit the momentum does not play any role, as follows

$$\begin{aligned}
\mathcal{L}_{f''} = \psi^{\dagger}\gamma^{0}(-)\{\sum_{+,-}\sum_{(t\,t')} \overset{tt'}{(\oplus)} \cdot \\
[\tau^{2+} A^{2+}_{\overset{tt'}{(\oplus)}} + \tau^{2-} A^{2-}_{\overset{tt'}{(\oplus)}} + \tau^{23} A^{23}_{\overset{tt'}{(\oplus)}} \\
+ \tau^{1+} A^{1+}_{\overset{tt'}{(\oplus)}} + \tau^{1-} A^{1-}_{\overset{tt'}{(\oplus)}} + \tau^{13} A^{13}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{\tau}^{2+} \tilde{A}^{2+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{2-} \tilde{A}^{2-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{23} \tilde{A}^{23}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{\tau}^{1+} \tilde{A}^{1+}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{1-} \tilde{A}^{1-}_{\overset{tt'}{(\oplus)}} + \tilde{\tau}^{13} \tilde{A}^{13}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{N}^{+}_{R} \tilde{A}^{N_R +}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{R} \tilde{A}^{N_R -}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{R} \tilde{A}^{N_R 3}_{\overset{tt'}{(\oplus)}} \\
+ \tilde{N}^{+}_{L} \tilde{A}^{N_L +}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{-}_{L} \tilde{A}^{N_L -}_{\overset{tt'}{(\oplus)}} + \tilde{N}^{3}_{L} \tilde{A}^{N_L 3}_{\overset{tt'}{(\oplus)}} \\
+ \tau^{3i} A^{3i}_{\overset{tt'}{(\oplus)}} + \tau^{4} A^{4}_{\overset{tt'}{(\oplus)}}\,]\}\psi\,, \qquad (9.21)
\end{aligned}$$

where (t, t') run in pairs over $[(9, 10), \dots (13, 14)]$ and the summation must go over $+$ and $-$ of $\overset{tt'}{(\oplus)}$.

Let us calculate now quantum numbers of the scalar and vector gauge fields appearing in Eq. (9.21) by taking into account that the spin of gauge fields is determined according to Eq. (9.7) ($((S^{ab})^{c}{}_{d}A^{d\dots e\dots g} = i(\eta^{ac}\delta^{b}_{d} - \eta^{bc}\delta^{a}_{d})\,A^{d\dots e\dots g}$, for each index ($\in (d\dots g)$) of a bosonic field $A^{d\cdots g}$ separately). We must take into account also the relation among S^{ab} and the charges (the relations are, of course, the same for bosons and fermions) (Eqs. (9.8, 9.9, 9.11)).

On table 9.1 properties of the scalar gauge fields appearing in Eq. (9.21) are presented.

The scalar fields with the scalar index $s = (9, 10, \cdots, 14)$, presented in table 9.1, carry one of the triplet colour charges and the "spinor" charge equal to twice the quark "spinor" charge, or the antitriplet colour charges and the anti "spinor" charge. They carry in addition the quantum numbers of the adjoint representations originating in S^{ab} or in $\tilde{S}^{ab}$. Although carrying the colour charge in one of the triplet or antitriplet states, these fields can not be interpreted as superpartners of the quarks as required by, let say, the $N = 1$ supersymmetry. The hyper charges and the electromagnetic charges are namely not those required by the supersymmetric partners to the family members.

Let us have a look what do the scalar fields, appearing in Eq. (9.21) and in table 9.1, do when being applied on the left handed members of the Weyl representation presented on table 9.3, containing quarks and leptons and antiquarks and antileptons [4,21,15]. Let us choose the 57^{th} line of table 9.3, which represents in the spinor technique the left handed positron, $\bar{e}^{+}_{L}$. If we make, let say, the choice of the term $(\gamma^{0} \overset{910}{(+)}\ \tau^{2\boxminus})\, A^{2\boxminus}_{\overset{9\,10}{(\oplus)}}$ (the scalar field $A^{2\boxminus}_{\overset{9\,10}{(\oplus)}}$ is presented in the 7^{th} line in table 9.1 and in the second line of Eq. (9.21)), the family quantum numbers will

field	prop.	τ^4	τ^{13}	τ^{23}	(τ^{33}, τ^{38})	Y	Q	$\tilde{\tau}^4$	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$A^{1\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	$\boxplus\ 1$	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3} + \boxplus\ 1$	0	0	0	0	0
$A^{13}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$A^{1\boxplus}_{11\,12}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	$\boxplus\ 1$	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3} + \boxplus\ 1$	0	0	0	0	0
$A^{13}_{11\,12}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$A^{1\boxplus}_{13\,14}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	$\boxplus\ 1$	0	$(0, \oplus\ \frac{1}{\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3} + \boxplus\ 1$	0	0	0	0	0
$A^{13}_{13\,14}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(0, \oplus\ \frac{1}{\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$A^{2\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	$\boxplus\ 1$	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3} + \boxplus\ 1$	$\oplus\ \frac{1}{3} + \boxplus\ 1$	0	0	0	0	0
$A^{23}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{1\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	$\boxplus\ 1$	0	0	0
$\tilde{A}^{13}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{2\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	$\boxplus\ 1$	0	0
$\tilde{A}^{23}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{N_L\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	$\boxplus\ 1$	0
$\tilde{A}^{N_L 3}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$\tilde{A}^{N_R\boxplus}_{9\,10}(\oplus)$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	$\boxplus\ 1$
$\tilde{A}^{N_R 3}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$A^{3i}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\boxplus\ 1 + \oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$A^{4}_{9\,10}(\oplus)$ $\cdots$	scalar	$\oplus\ \frac{1}{3}$	0	0	$(\oplus\ \frac{1}{2}, \oplus\ \frac{1}{2\sqrt{3}})$	$\oplus\ \frac{1}{3}$	$\oplus\ \frac{1}{3}$	0	0	0	0	0
$\vec{A}^3_m$	vector	0	0	0	octet	0	0	0	0	0	0	0
A^4_m	vector	0	0	0	0	0	0	0	0	0	0	0

Table 9.1. Quantum numbers of the scalar gauge fields carrying the space index $t = (9, 10, \cdots, 14)$, appearing in Eq. (9.21), are presented. To the colour charge of all these scalar fields the space degrees of freedom contribute one of the triplets values. These scalars are with respect to the two $SU(2)$ charges, ($\vec{\tau}^1$ and $\vec{\tau}^2$), and the two $\widetilde{SU}(2)$ charges, ($\vec{\tilde{\tau}}^1$ and $\vec{\tilde{\tau}}^2$), triplets (that is in the adjoint representations of the corresponding groups), and they all carry twice the "spinor" number (τ^4) of the quarks. The quantum numbers of the two vector gauge fields, the colour and the $U(1)_{II}$ ones, are added.

not be affected and they can be any. The state carries the "spinor" (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = \frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hyper charge $(Y(= \tau^4 + \tau^{23}))$ is 1 and the electromagnetic charge $Q(= Y + \tau^{13})$ is 1.

So, what does the term $\gamma^0 \overset{910}{(+)}\; \tau^{2\boxminus}\; A^{2\boxminus}_{\underset{(\oplus)}{9\,10}}$ make on this spinor? Making use of Eqs. (9.44, 9.46, 9.54) of appendix 9.9 one easily finds that operator $\gamma^0 \overset{910}{(+)}\; \tau^{2-}$ transforms the left handed positron into $\overset{03}{(+i)}\overset{12}{(+)} | \overset{56}{[-]}\overset{78}{[-]} || \overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$, which is d^{c1}_R, presented on line 3 of table 9.3. Namely, γ^0 transforms $\overset{03}{[-i]}$ into $\overset{03}{(+i)}$, $\overset{910}{(+)}$ transforms $\overset{9\,10}{[-]}$ into $\overset{9\,10}{(+)}$, while τ^{2-} $(= -\overset{56}{(-)}\overset{78}{(-)})$ transforms $\overset{56}{(+)}\overset{78}{(+)}$ into $\overset{56}{[-]}\overset{78}{[-]}$. The state d^{c1}_R carries the "spinor" (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$ and the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hyper charge is $(Y = \tau^4 + \tau^{23} =) -\frac{1}{3}$ and the electromagnetic charge $(Q = Y + \tau^{13} =) -\frac{1}{3}$. The scalar field $A^{2\boxminus}_{\underset{(\oplus)}{9\,10}}$ carries just the needed quantum numbers as we can see in the 7^{th} line of table 9.1.

If the antiquark $\bar{u}^{\bar{c}2}_L$, from the line 43 (it is not presented, but one can very easily construct it) in table 9.3, with the "spinor" charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = 0$, the second $SU(2)_{II}$ charge $\tau^{23} = -\frac{1}{2}$, the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hyper charge $Y(= \tau^4 + \tau^{23} =) -\frac{2}{3}$ and the electromagnetic charge $Q(= Y + \tau^{13} =) -\frac{2}{3}$ submits the $A^{2\boxminus}_{\underset{(\oplus)}{9\,10}}$ scalar field, it transforms into u^{c3}_R from the line 17 of table 9.3, carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = 0$, $\tau^{23} = \frac{1}{2}$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{2}{3}$ and $Q = \frac{2}{3}$. These two quarks, d^{c1}_R and u^{c3}_R can bind together with u^{c2}_R from the 9^{th} line of the same table (at low enough energy, after the electroweak transition, and if they belong in a superposition with the left handed partners to the first family) into the colour chargeless baryon - a proton. This transition is presented in figure 9.1.

The opposite transition at low energies would make the proton decay.

Let us look at one more example. The 63^{th} line of table 9.3 represents in the spinor technique the right handed positron, $\bar{e}^+_R$. Since we shall again not have a look on a transition, in which scalar fields with the nonzero family quantum numbers are involved, the family quantum number of this positron is not important. The state carries the "spinor" (lepton) number $\tau^4 = \frac{1}{2}$, the weak charge $\tau^{13} = \frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$ and the colour charge $(\tau^{33}, \tau^{38}) = (0, 0)$. Correspondingly, its hyper charge $(Y = \tau^4 + \tau^{23})$ is $\frac{1}{2}$ and the electromagnetic charge $Q = Y + \tau^{13}$ is 1.

What does, let say, the term $\gamma^0 \overset{910}{(+)}\; \tau^{1\boxminus}\; A^{1\boxminus}_{\underset{(\oplus)}{9\,10}}$ (the scalar field $A^{1\boxminus}_{\underset{(\oplus)}{9\,10}}$ is presented in the first line of table 9.1) make on $\bar{e}^+_R$? Making use of Eqs. (9.44, 9.46, 9.54) of appendix 9.9 one easily finds that the right handed positron transforms under the application of $\gamma^0\, \tau^{1-} \overset{910}{(+)}$ into $\overset{03}{[-i]}\overset{12}{(+)} | \overset{56}{[-]}\overset{78}{(+)} || \overset{9\,10}{(+)}\;\overset{11\,12}{(-)}\;\overset{13\,14}{(-)}$, which is d^{c1}_L presented on line 5 of table 9.3. Namely, γ^0 transforms $\overset{03}{(+i)}$ into $\overset{03}{[-i]}$, $\overset{910}{(+)}$

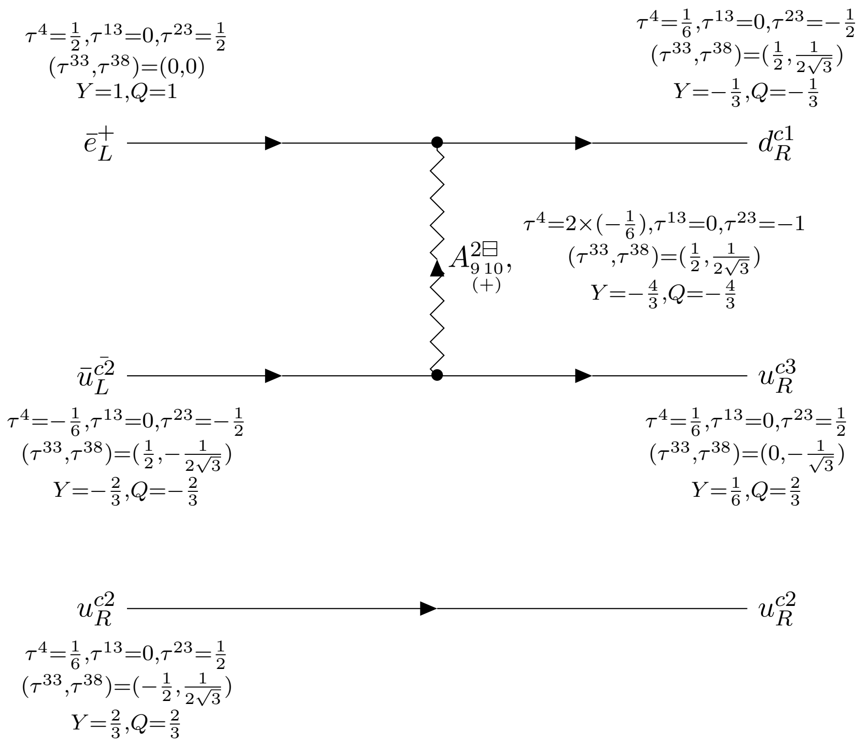

Fig. 9.1. The birth of a proton out of an positron $\bar{e}_L^{+}$, antiquark $\bar{u}_L^{\bar{c}2}$ and quark (spectator) u_R^{c2}. The family quantum number can be any.

transforms $\stackrel{9\,10}{[-]}$ into $\stackrel{9\,10}{(+)}$, while $\tau^{1\boxminus}$ (=$\stackrel{56}{(-)}\,\stackrel{78}{(+)}$) transforms $\stackrel{56}{(+)}\,\stackrel{56}{[-]}$ into $\stackrel{56}{[-]}\,\stackrel{56}{(+)}$. The state d_L^{c1} carries the "spinor" (quark) number $\tau^4 = \frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$ and the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, \frac{1}{2\sqrt{3}})$. Correspondingly its hyper charge is $(Y = \tau^4 + \tau^{23} =)\ \frac{1}{6}$ and the electromagnetic charge $(Q = Y + \tau^{13} =) -\frac{1}{3}$. The scalar field $A^{1\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ carries all the needed quantum numbers, as one can see in figure 9.1.

If the antiquark $\bar{u}_R^{\bar{c}2}$, from the line 47 in table 9.3 (the reader can easily find the expression $\stackrel{03}{(+i)}\,\stackrel{12}{(+)}\mid\stackrel{56}{[-]}\,\stackrel{78}{(+)}\parallel\stackrel{9\,10}{(+)}\ \ \stackrel{11\,12}{(-)}\ \ \stackrel{13\,14}{[+]}$), with the "spinor" charge $\tau^4 = -\frac{1}{6}$, the weak charge $\tau^{13} = -\frac{1}{2}$, the second $SU(2)_{II}$ charge $\tau^{23} = 0$, the colour charge $(\tau^{33}, \tau^{38}) = (\frac{1}{2}, -\frac{1}{2\sqrt{3}})$, the hypercharge $(Y = \tau^4 + \tau^{23} =) -\frac{1}{6}$ and the electromagnetic charge $(Q = Y + \tau^{13} =) -\frac{2}{3}$, submits the $A^{1\boxminus}_{\stackrel{9\,10}{(\oplus)}}$ scalar field, it transforms into u_L^{c3} from the line 23 of table 9.3 ($\stackrel{03}{[-i]}\,\stackrel{12}{(+)}\mid\stackrel{56}{(+)}\,\stackrel{78}{[-]}\parallel\stackrel{9\,10}{[-]}\ \ \stackrel{11\,12}{(-)}\ \ \stackrel{13\,14}{[+]}$), carrying the quantum numbers $\tau^4 = \frac{1}{6}$, $\tau^{13} = \frac{1}{2}$, $\tau^{23} = 0$, $(\tau^{33}, \tau^{38}) = (0, -\frac{1}{\sqrt{3}})$, $Y = \frac{1}{6}$ and $Q = \frac{2}{3}$. These two quarks, d_L^{c1} and u_L^{c3}, can bind (at low enough energy, when making after the electroweak transition the superposition with the right handed partners) together with u_L^{c2} from the 15^{th} line of the same table, into the colour chargeless baryon - a proton. This transition is presented in figure 9.2.

The opposite transition would make the proton decay.

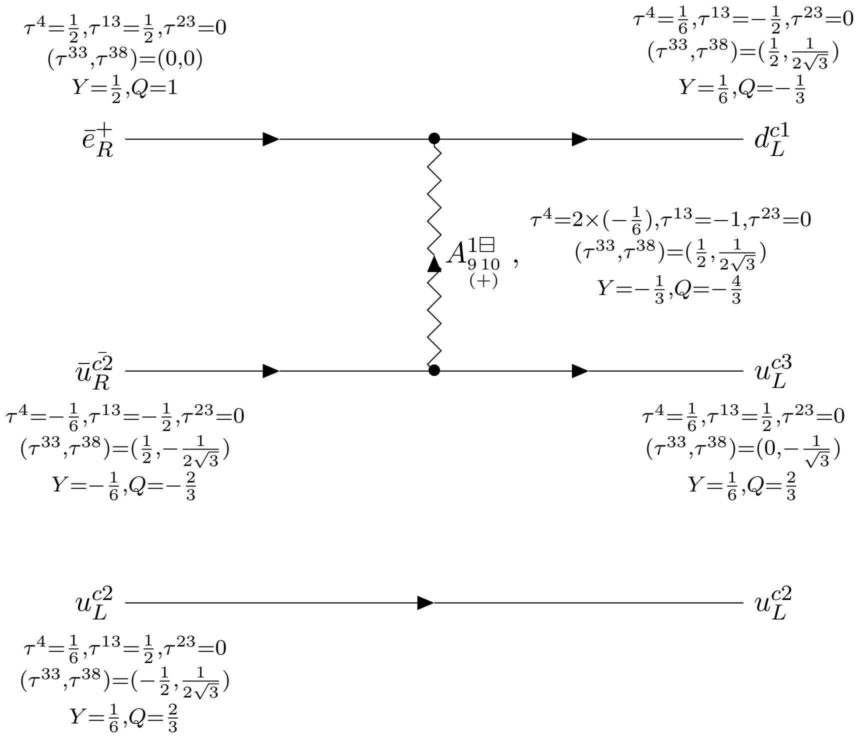

Fig. 9.2. The birth of a proton out of a positron $\bar{e}^+_R$, antiquark $\bar{u}^{\bar{c2}}_R$ and quark (spectator) u^{c2}_L. The family quantum number can be any.

Similar transitions go also with other scalars from Eq. (9.21) and table 9.1. The $\vec{\tilde{A}}^{1}_{\stackrel{t' t''}{(+)}}$, $\vec{\tilde{A}}^{2}_{\stackrel{t' t''}{(+)}}$, $\vec{\tilde{A}}^{N_L}_{\stackrel{t' t''}{(+)}}$ and $\vec{\tilde{A}}^{N_L}_{\stackrel{t' t''}{(+)}}$ fields cause transitions among the family members, changing a particular member into the antimember of another colour and of another family. The term $\gamma^0 \stackrel{910}{(+)}\; \tilde{N}^-_R\; A^{\tilde{N}_R -}_{\stackrel{9\,10}{(\oplus)}}$ transforms $\bar{e}^+_R$ into u^{c1}_L, changing the family quantum numbers.

The action from Eqs. (9.1, 9.2, 9.4) manifests $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ invariance. All the vector and the spinor gauge fields are massless.

Since no one of the scalar fields from table 9.1 have been observed and also no vector gauge fields like $\vec{A}^2_m$, A^4_m and other scalar and vector fields, we shall discuss this topic in sect. 9.5, it must exist a mechanism, which makes the non observed scalar and vector gauge fields massive enough.

Scalar fields from table 9.1 carry the colour and the electromagnetic charge. Therefore their nonzero vacuum expectation values would not be in agreement with the observed phenomena. One, however, notices that all the scalar gauge fields from table 9.1 and several other scalar and vector gauge fields (see sect. 9.5) couple to the condensate with the nonzero quantum number τ^4 and τ^{23} and nonzero family quantum numbers.

It is not difficult to recognize that the desired condensate must have spin zero, $Y = \tau^4 + \tau^{23} = 0$, $Q = Y + \tau^{13} = 0$ and $\vec{\tau}^1 = 0$ in order that in the low energy limit the *spin-charge-family* theory would manifest effectively as the *standard model*.

I make a choice of the two right handed neutrinos of the VIII^{th} family coupled into a scalar, with $\tau^4 = -1$, $\tau^{23} = 1$, correspondingly $Y = 0$, $Q = 0$ and $\vec{\tau}^1 = 0$, and with family quantum numbers (Eqs. (9.13, 9.12)) $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$, $\tilde{N}^3_R = 1$, and correspondingly with $\tilde{Y} = \tilde{\tau}^4 + \tilde{\tau}^{23} = 0$, $\tilde{Q} = \tilde{Y} + \tilde{\tau}^{13} = 0$, and $\vec{\tilde{\tau}}^1 = 0$. The condensate carries the family quantum numbers of the upper four families.

The condensate made out of spinors couples to spinors differently than to antispinors - "anticondensate" would namely carry $\tau^4 = 1$, and $\tau^{23} = -1$ - breaking correspondingly the $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ symmetry: The reactions creating particles from antiparticles are not any longer symmetric to those creating antiparticle from particles.

Such a condensate leaves the hyper field A^Y_m $(= \sin\vartheta_2\, A^{23}_m + \cos\vartheta_2\, A^4_m)$ (for the choice that $\sin\vartheta_2 = \cos\vartheta_2$ and $g^4 = g^2$, there is no justification for such a choice, $A^Y_m = \frac{1}{\sqrt{2}}\,(A^{23}_m + A^4_m)$) massless, while it gives masses to $A^{2\pm}_m$ and $A^{Y'}_m$ $(= \frac{1}{\sqrt{2}}\,(A^4_m - A^{23}_m)$ for $\sin\vartheta_2 = \cos\vartheta_2)$ and it gives masses also to all the scalar gauge fields from table 9.1, since they all couple to the condensate through τ^4.

The weak vector gauge fields, $\vec{A}^1_m$, the hyper charge vector gauge fields, A^Y_m, and the colour vector gauge fields, $\vec{A}^1_m$, stay massless.

The scalar fields with the scalar space index $s = (7, 8)$ - those which couple to all eight families, those which couple only to the upper and those which couple only to the lower four families - carrying the weak and the hyper charges of the Higgs's scalar - wait for getting nonzero vacuum expectation values to change their masses while causing the electroweak break.

The condensate does what is needed so that in the low energy regime the *spin-charge-family* manifests as an effective theory which agrees with the *standard model* to the extend that it is in agreement with the observed phenomena, explaining the *standard model* assumptions and predicting new fermion and boson fields.

It also may hopefully explain also the observed matter-antimatter asymmetry if the conditions in the expanding universe would be appropriate (9.6). The work needed to check these conditions in the expanding universe within the *spin-charge-family* theory is very demanding. Although we do have some experience with following the history of the expanding universe [12], this study needs much more efforts, not only in the calculations, but also in understanding the mechanism of appearing the condensate, relations among the velocity of the expansion, the temperature and the dimension of space-time in the period of the appearance of the condensate. This study has not yet been really started.

9.3 Properties of the condensate

In table 9.2 the properties of the condensate of the two right handed neutrinos $(|\nu^{\text{VIII}}_R >_1 |\nu^{\text{VIII}}_R >_2)$, one with spin up and another with spin down (table 9.3, line

25 and 26), carrying the family quantum numbers of the VIII^{th} family (table 9.4), are presented. The condensate carries the quantum numbers of $SU(2)_{II}$, $\tau^{23} = 1$ (Eq. (9.9)), of $U(1)_{II}$ originating in $SO(6)$, $\tau^4 = -1$ (Eq.9.11), correspondingly $Y = 0$, $Q = 0$, and the family quantum numbers (table 9.4) $\tilde{\tau}^4 = -1$ (Eq. (9.11)), $\tilde{\tau}^{23} = 1$ (Eq. (9.13)), and $\tilde{N}^3_R = 1$ (Eq. (9.12)). Each of the two neutrinos could belong to a different family of the upper four families. In this case the family quantum numbers of the condensate change.

The condensate is presented in the first line of table 9.2 as a member of a triplet of the group $SU(2)_{II}$ with the generators τ^{2i}. Correspondingly the condensate couples to all the vector gauge fields which carry nonzero τ^{2i}, τ^4, $\tilde{\tau}^{2i}$, $\tilde{N}^i_R$ and $\tilde{\tau}^4$. The fields $A^Y_m, \vec{A}^3_m$ and $\vec{A}^1_m$ stay massless. The condensate couples also to all the scalar gauge fields with the scalar indices $s \in (5, 6, 7, 8, 9, \ldots, 14)$, since they all carry nonzero either τ^4 or τ^{23}.

state	S^{03}	S^{12}	τ^{13}	τ^{23}	τ^4	Y	Q	$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{\tau}^4$	$\tilde{Y}$	$\tilde{Q}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$
$(\lvert\nu^{\mathbf{VIII}}_{\mathbf{1R}} >_{\mathbf{1}} \lvert\nu^{\mathbf{VIII}}_{\mathbf{2R}} >_{\mathbf{2}})$	0	0	0	1	−1	0	0	0	1	−1	0	0	0	1
$(\lvert\nu^{VIII}_{1R} >_1 \lvert e^{VIII}_{2R} >_2)$	0	0	0	0	−1	−1	−1	0	1	−1	0	0	0	1
$(\lvert e^{VIII}_{1R} >_1 \lvert e^{VIII}_{2R} >_2)$	0	0	0	−1	−1	−2	−2	0	1	−1	0	0	0	1

Table 9.2. The condensate of the two right handed neutrinos ν_R, with the VIII^{th} family quantum number, coupled to spin zero and belonging to a triplet with respect to the generators τ^{2i}, is presented, together with its two partners. The condensate carries $\vec{\tau}^1 = 0$, $\tau^{23} = 1$, $\tau^4 = -1$ and $Q = 0 = Y$. The triplet carries $\tilde{\tau}^4 = -1$, $\tilde{\tau}^{23} = 1$ and $\tilde{N}^3_R = 1$, $\tilde{N}^3_L = 0$, $\tilde{Y} = 0$, $\tilde{Q} = 0$. The family quantum numbers are presented in table 9.4.

Coupling of the scalar gauge fields to the condensate is proportional to

$$
\begin{aligned}
&(< \nu^{VIII}_{1R}|_1 \; < \nu^{VIII}_{2R}|)\,(\gamma^0 \; \overset{tt'}{(\oplus)} \; \tau^{Ai}\, A^{Ai}_{\underset{(\oplus)}{tt'}})^\dagger (\gamma^0 \; \overset{tt'}{(\oplus)} \; \tau^{Ai}\, A^{Ai}_{\underset{(\oplus)}{tt'}})(|\nu^{VIII}_{1R} >_1 \; |\nu^{VIII}_{2R} >) \\
&\propto \; (A^{Ai}_{\underset{(\oplus)}{tt'}})^\dagger \, (A^{Ai}_{\underset{(\oplus)}{tt'}})\,, \\
&(tt') \in [(56), (78), (9\,10), \ldots, (13\,14)]\,.
\end{aligned}
\tag{9.22}
$$

The condensate does break the $\mathbb{C}_{\mathcal{N}} \cdot \mathcal{P}_{\mathcal{N}}$ symmetry. (The "anticondensate" would namely carry $\tau^{23} = -1$ and $\tau^4 = 1$).

The condensate gives masses to all the scalars from table 9.1, either because they couple to the condensate due to τ^4 or due to τ^4 and τ^{23} quantum numbers. It gives masses also to all the scalar fields with $s \in (5, 6, 7, 8)$, since they couple to the condensate due to the nonzero τ^{23}. The scalar fields with the quantum numbers of the upper four families couple in addition through their family quantum numbers.

The condensate couples also to all the vector gauge fields except to the gauge colour octet field $\vec{A}^3_m$, the hyper charge vector fields A^Y_m and the weak charge vector triplet fields $\vec{A}^1_m$, since they carry zero τ^{23}, τ^4 and Y quantum numbers.

The spin connection fields, of either "tilde" ($\tilde{S}^{ab}$) or "nontilde" (S^{ab}) origin, which do not couple to the spinor condensate, are auxiliary fields, expressible with vielbeins fields (abstract (9.10)).

Below the scalar and vector gauge fields are presented, which get masses through the interaction with the condensate.

$$
\begin{aligned}
& A^{2\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; A^{23}_{\substack{tt'\\(\oplus)}}\,,\quad A^{1\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; A^{13}_{\substack{tt'\\(\oplus)}}\,,\quad \vec{A}^{3}_{\substack{tt'\\(\oplus)}}\,,\\
& \tilde{A}^{2\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; \tilde{A}^{23}_{\substack{tt'\\(\oplus)}}\,,\quad \tilde{A}^{1\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; \tilde{A}^{13}_{\substack{tt'\\(\oplus)}}\,,\\
& \tilde{A}^{N_L\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; \tilde{A}^{N_L 3}_{\substack{tt'\\(\oplus)}}\,,\quad \tilde{A}^{N_R\boxplus}_{\substack{tt'\\(\oplus)}}\,,\; \tilde{A}^{N_R 3}_{\substack{tt'\\(\oplus)}}\,,\\
& (tt') \in [(9\,10),(11\,12),(13\,14)]\,,\\
& A^{2\boxplus}_{\substack{ss'\\(\oplus)}}\,,\quad A^{Y'}_{\substack{ss'\\(\oplus)}} = \frac{1}{\sqrt{2}}\,(A^{23}_{\substack{ss'\\(\oplus)}} - A^{4}_{\substack{ss'\\(\oplus)}})\,,\\
& (ss') \in [(56),(78)]\,,\\
& A^{2\boxplus}_{m}\,,\; A^{Y'}_{m} = \frac{1}{\sqrt{2}}\,(A^{23}_{m} - A^{4}_{m})\,,\\
& \vec{\tilde{A}}^{2}_{m}\,, \tilde{A}^{4}_{m}\,, \vec{\tilde{A}}^{N_R}_{m}\,,\\
& m \in (0,1,2,3)\,.
\end{aligned}
\tag{9.23}
$$

In expression for $A^{Y'}_{m,s}$ $\vartheta_2 = \frac{\pi}{4}$ is chosen, just for simplicity, with no justification so far.

It stays as an open question what does make the right handed neutrinos to form such a condensate in the history of the universe.

Since A^{Ai}_s, $s \in (5,6)$ couple to the condensate and get masses, while (by assumption) they do not get nonzero vacuum expectation values during the electroweak break (what changes the masses of the scalar fields A^{Ai}_s, $s \in (7,8)$) the restriction in the sum in Eq. (9.2) is justified.

The scalar fields, causing the birth of baryons, have the triplet colour charges. They resemble the supersymmetric partners of the quarks, but since they do not carry all the quantum numbers of the quarks, they are not.

9.4 Properties of scalar fields which determine mass matrices of fermions

This section is a short overview of the ref. [14].

There are two kinds of the scalar gauge fields, which gain at the electroweak break nonzero vacuum expectation values and determine correspondingly the masses of the families of quarks and leptons and to the masses of gauge weak bosons: The kind originating in $\tilde{\omega}_{\tilde{a}\tilde{b}s}$ and the kind originating in $\omega_{tt's}$, ω_{56s} and ω_{78s}, both kinds have the space index $s = (7,8)$ and both carry the weak and the hyper charge as the Higgs's scalar. These scalar fields are presented in the Lagrange density for fermions (Eq. (9.2)) in the second line. The "tilde" kind influences the family quantum numbers of fermions, the "Dirac" kind influences the family members quantum numbers.

The two triplets ($\vec{\tilde{A}}^{1}_{s}$, $\vec{\tilde{A}}^{N_L}_{s}$) influence the lower four families (the lowest three already observed), while ($\vec{\tilde{A}}^{2}_{s}$, $\vec{\tilde{A}}^{N_R}_{s}$) influence the upper four families, the stable of which constitute the dark matter. Recognizing that $\vec{\tilde{\tau}}^1\, \vec{\tilde{A}}^{1}_{s} + \vec{\tilde{N}}_L\, \vec{\tilde{A}}^{N_L}_{s} + \vec{\tilde{\tau}}^2\, \vec{\tilde{A}}^{2}_{s} + \vec{\tilde{N}}_R\, \vec{\tilde{A}}^{N_R}_{s} = \frac{1}{2}\, \tilde{S}^{ab}\, \tilde{\omega}_{abs}$, $s = (7, 8)$, one easily finds, taking into account Eqs. (9.12, 9.13), the expressions

$$
\begin{aligned}
\vec{\tilde{A}}^{1}_{s} &= (\tilde{\omega}_{58s} - \tilde{\omega}_{67s}, \tilde{\omega}_{57s} + \tilde{\omega}_{68s}, \tilde{\omega}_{56s} - \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^{N_L}_{s} &= (\tilde{\omega}_{23s} + i\,\tilde{\omega}_{01s}, \tilde{\omega}_{31s} + i\,\tilde{\omega}_{02s}, \tilde{\omega}_{12s} + i\,\tilde{\omega}_{03s})\,,\\
\vec{\tilde{A}}^{2}_{s} &= (\tilde{\omega}_{58s} + \tilde{\omega}_{67s}, \tilde{\omega}_{57s} - \tilde{\omega}_{68s}, \tilde{\omega}_{56s} + \tilde{\omega}_{78s})\,,\\
\vec{\tilde{A}}^{N_R}_{s} &= (\tilde{\omega}_{23s} - i\,\tilde{\omega}_{01s}, \tilde{\omega}_{31s} - i\,\tilde{\omega}_{02s}, \tilde{\omega}_{12s} - i\,\tilde{\omega}_{03s})\,,\\
s &= (7, 8)\,,
\end{aligned}
\tag{9.24}
$$

presented already in Eq. (9.16). Similarly one finds, taking into account Eqs. (9.8, 9.9, 9.11, 9.14), the expressions for A^{Q}_{s}, A^{Y}_{s} and $A^{Y'}_{s}$, presented in Eqs. (9.15).

The scalar fields A^{Q}_{s}, A^{Y}_{s} and $A^{Y'}_{s}$ distinguish among the family members, coupling to the family members quantum numbers through $Q\ (= \tau^{13} + Y)$, $Y\ (= \tau^{23} + \tau^{4})$ and $Y' = \tau^{23} - \tan\vartheta_2\, \tau^{4}$, $\tau^{4} = -\frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$. The scalars originating in $\tilde{\omega}_{abs}$ and distinguishing among families, couple the family quantum numbers through ($\vec{\tilde{\tau}}^1$ and $\vec{\tilde{N}}_L$), or through ($\vec{\tilde{\tau}}^2$ and $\vec{\tilde{N}}_R$), all in the adjoint representations of the corresponding groups.

Let us now prove that all the scalar fields with the space (scalar with respect to $d = (3+1)$) index $s = (7, 8)$ carry the weak and the hyper charge (τ^{13}, Y) equal to either $(-\frac{1}{2}, \frac{1}{2})$ or to $(\frac{1}{2}, -\frac{1}{2})$. Let us first simplify the notation, using a common name A^{Ai}_{s} for all the scalar fields with the scalar index $s = (7, 8)$

$$
A^{Ai}_{s} = (A^{Q}_{s}, A^{Q'}_{s}, A^{Y'}_{s}, \tilde{A}^{4}_{s}, \vec{\tilde{A}}^{2}_{s}, \vec{\tilde{A}}^{1}_{s}, \vec{\tilde{A}}^{N_R}_{s}, \vec{\tilde{A}}^{N_L}_{s})\,,
\tag{9.25}
$$

and let us rewrite the term $\sum_{s=7,8} \bar{\psi}\, \gamma^{s}\, p_{0s}\, \psi$ in Eq. (9.2) as follows

$$
\begin{aligned}
&\sum_{s=7,8} \bar{\psi}\, \gamma^{s}\, p_{0s}\, \psi\,,\\
&= \bar{\psi}\{ \overset{78}{(+)}\, p_{0+} + \overset{78}{(-)}\, p_{0-} \}\psi\,,\\
&p_{0\pm} = (p_{07} \mp i p_{08})\,,\\
&(p_{07} \mp i p_{08}) = (p_{7} \mp i p_{8}) - \tau^{Ai}\,(A^{Ai}_{7} \mp i A^{Ai}_{8})\\
&\overset{78}{(\pm)} = \frac{1}{2}\,(\gamma^{7} \pm i\gamma^{8})\,.
\end{aligned}
\tag{9.26}
$$

Let us now apply the operators Y, Q, Eq. (9.14), and $\tau^{13} = \frac{1}{2}\,(S^{56} - S^{78})$, Eq. (9.9), on the fields $A^{Ai}_{\overset{78}{(\pm)}} = (A^{Ai}_{7} \mp i\, A^{Ai}_{8})$. One finds

$$
\begin{aligned}
\tau^{13}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \pm\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Y\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= \mp\frac{1}{2}\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8})\,,\\
Q\,(A^{Ai}_{7} \mp i\,A^{Ai}_{8}) &= 0\,,
\end{aligned}
\tag{9.27}
$$

This is, with respect to the weak, the hyper and the electromagnetic charge, just what the standard model assumes for the Higgs' scalars. The proof is complete.

One can check also, using Eq. (9.44), that $\gamma^0 \stackrel{78}{(-)}$ transforms the u_R^{c1} from the first line of table 9.3 into u_L^{c1} from the seventh line of the same table, or ν_R from the 25^{th} line into the ν_L from the 31^{th} line of the same table.

The scalars $A^{Ai}_{\stackrel{78}{(-)}}$ obviously bring the weak charge $\frac{1}{2}$ and the hyper charge $-\frac{1}{2}$ to the right handed family members (u_R, ν_R), and the scalars $A^{Ai}_{\stackrel{78}{(+)}}$ bring the weak charge $-\frac{1}{2}$ and the hyper charge $\frac{1}{2}$ to (d_R, e_R).

Let us now prove that the scalar fields of Eq. (9.25) are either triplets with respect to the family quantum numbers $(\vec{\tilde{N}}_R, \vec{\tilde{N}}_L, \vec{\tilde{\tau}}^2, \vec{\tilde{\tau}}^1$; Eqs. (9.12, 9.13)) or singlets as the gauge fields of $Q = \tau^{13} + Y$, $Q' = \tau^{13} - Y \tan^2\vartheta_1$ and $Y' = \tau^{23} - \tan^2\vartheta_2\,\tau^4$. One can prove this by applying $\vec{\tilde{\tau}}^2, \vec{\tilde{\tau}}^1, \vec{\tilde{N}}_R, \vec{\tilde{N}}_L$ and Q, Q', Y' on their eigenstates. Let us calculate, as an example, $\tilde{N}^3_L$ and Q on $\tilde{A}^{N_L 3}_{\stackrel{78}{(\pm)}}$ and on $A^{Q}_{\stackrel{78}{(\pm)}}$, taking into account Eqs. (9.12, 9.11,9.9, 9.7)

$$
\begin{aligned}
\tilde{N}^3_L\, \tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}} &= \boxplus\, \tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}}\,, \quad \tilde{N}^3_L\, \tilde{A}^{N_L 3}_{\stackrel{78}{(\pm)}} = 0\,,\\
Q\, A^{Q}_{\stackrel{78}{(\pm)}} &= 0\,,\\
\tilde{A}^{N_L \boxplus}_{\stackrel{78}{(\pm)}} &= \{(\tilde{\omega}_{23 \stackrel{78}{(\pm)}} + i\,\tilde{\omega}_{01 \stackrel{78}{(\pm)}}) \boxplus i\,(\tilde{\omega}_{31 \stackrel{78}{(\pm)}} + i\tilde{\omega}_{02 \stackrel{78}{(\pm)}})\}\,,\\
\tilde{A}^{N_L 3}_{\stackrel{78}{(\pm)}} &= (\tilde{\omega}_{12 \stackrel{78}{(\pm)}} + i\tilde{\omega}_{03 \stackrel{78}{(\pm)}})\\
A^{Q}_{\stackrel{78}{(\pm)}} &= \sin\vartheta_1\, A^{13}_{\stackrel{78}{(\pm)}} + \sin\vartheta_1 (-)(\omega_{9\,10 \stackrel{78}{(\pm)}} + \omega_{11\,12 \stackrel{78}{(\pm)}} + \omega_{13\,14 \stackrel{78}{(\pm)}})\,, \quad (9.28)
\end{aligned}
$$

with $Q = S^{56} + \tau^4 = S^{56} - \frac{1}{3}(S^{9\,10} + S^{11\,12} + S^{13\,14})$, and with τ^4 defined in Eq. (9.11)).

Nonzero vacuum expectation values of the scalar fields (Eq. (9.25)), which carry the scalar index $s = (7, 8)$, and correspondingly the weak and the hyper charges, break the mass protection mechanism of quarks and leptons of the lower and the upper four families. In the loop corrections contribute to all the matrix elements of mass matrices of any family members besides $\tilde{A}^{Ai}_s$ and the scalar fields which are the gauge fields of Q, Q', Y' also the vector gauge fields.

The gauge fields of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ contribute only to masses of the upper four families, while the gauge fields of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ contribute only to masses of the lower four families. The triplet scalar fields with the scalar index $s = (7, 8)$ and the family charges $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ transform any family member belonging to the group of the upper four families into the same family member belonging to another family of the same group of four families, changing the right handed member into the left handed partner, while those triplets with the family charges $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ transform any family member of particular handedness and belonging to the lower four families into its partner of opposite handedness, belonging to another family of the lower four families.

The scalars $A^{Q}_{\substack{78\\(\pm)}}$ (Eq. (9.28)), $A^{Q'}_{\substack{78\\(\pm)}}$ $(= \cos\vartheta_1 A^{13}_{\substack{78\\(\pm)}} - \sin\vartheta_1 A^{4}_{\substack{78\\(\pm)}})$ and $A^{Y'}_{\substack{78\\(\pm)}}$ (Eq. (9.23)) contribute to all eight families, distinguishing among the family members and not among the families.

The mass matrix of any family member, belonging to any of the two groups of the four families, manifests - due to the $\widetilde{SU}(2)_{(R,L)} \times \widetilde{SU}(2)_{(II,I)}$ (either (R, II) or (L, I)) structure of the scalar fields, which are the gauge fields of the $\vec{\tilde{N}}_{R,L}$ and $\vec{\tilde{\tau}}^{2,1}$ - the symmetry presented in Eq. (9.29)

$$\mathcal{M}^{\alpha} = \begin{pmatrix} -a_1 - a & e & d & b \\ e & -a_2 - a & b & d \\ d & b & a_2 - a & e \\ b & d & e & a_1 - a \end{pmatrix}^{\alpha} . \tag{9.29}$$

In the ref. gn2014 the mass matrices for quarks, which are in the agreement with the experimental data, are presented and predictions made.

9.5 Condensate and nonzero vacuum expectation values of scalar fields make spinors and most of scalar and vector gauge fields massive

Let us shortly overview properties of the scalar and the vector gauge fields after **a.** two right handed neutrinos (coupled to spin zero and with the family quantum numbers (table 9.4) of the upper four families) make a condensate (table 9.2) at the scale $\geq 10^{16}$ GeV and after **b.** the electroweak break, when the scalar fields with the space index $s = (7, 8)$ get nonzero vacuum expectation values.

All the scalar gauge fields $A^{Ai}_{t}, t \in (5, 6, 7, 8, 9, \ldots, 14)$ (Eqs. (9.2, 9.21, 9.23), table 9.1) interact with the condensate through the quantum numbers τ^4 and τ^{23}, those with the family quantum numbers of the upper four families interact also through the family quantum numbers $\vec{\tilde{\tau}}^{2}$ or $\vec{\tilde{N}}_{R}$, getting masses of the order of the condensate scale (Eq.(9.23)).

At the electroweak break the scalar fields A^{Ai}_{s}, $s \in (7, 8)$, from Eqs. (9.25, 9.25) get nonzero vacuum expectation values, changing correspondingly their own masses and determining masses of quarks and leptons, as well as of the weak vector gauge fields.

The vector gauge fields $A^{2\boxplus}_{m}$, $A^{Y'}_{m}$, $\vec{\tilde{A}}^{2\boxplus}_{m}$, $\tilde{A}^{Y'}_{m}$ and $\vec{\tilde{A}}^{N_R}_{m}$ (Eq. (9.23)) get masses due to the interaction with the condensate through τ^{23} and τ^4, the first two, and/or also due to the family quantum numbers of the upper four families, the last three, respectively.

The vector gauge fields $\vec{A}^{3}_{m}$, $\vec{A}^{1}_{m}$, and A^{Y}_{m} stay massless up to the electroweak break when the scalar gauge fields, which are weak doublets with the hypercharge making electromagnetic charge Q equal to zero, give masses to the weak bosons $(A^{1\boxplus}_{m} = \frac{1}{\sqrt{2}} (A^{11}_{m} \mp i A^{11}_{m})$ and $A^{Q'}_{m} = \cos\vartheta_1 A^{13}_{m} - \sin\vartheta_1 A^{4}_{m})$, while the electromagnetic vector field $(A^{Q}_{m} = \sin\vartheta_1 A^{13}_{m} + \cos\vartheta_1 A^{4}_{m})$ and the colour vector gauge field stay massless.

At the electroweak break, when the nonzero vacuum expectation values of the scalar fields break the weak and the hypercharge global symmetry, also all the eight families of quarks and leptons get masses. Until the electroweak break the families were mass protected, since the right handed partners distinguished from the left handed ones in the weak and hyper charges, what disabled them to make the superposition manifesting masses.

9.6 Sakharov conditions as seen in view of the *spin-charge-family* theory

The condensate of the right handed neutrinos, as well as the nonzero vacuum expectation values of the scalar fields $A^{Ai}_{\substack{78\\(\pm)}}$ - if leading to the complex matrix elements of the mixing matrices - cause the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ violation terms, which generate the matter-antimatter asymmetry.

It is the question whether both generators of the matter-antimatter asymmetry - the condensate and the complex phases of the mixing matrices of quarks and leptons (this last alone can not with one complex phase and also very probably not with the three complex phases of the lower four families) - can explain at all the observed matter-antimatter asymmetry of the "ordinary" matter, that is the matter of mostly the first family of quarks and leptons.

The lowest of the upper four families determine the dark matter. For the dark matter any relation among matter and antimatter is so far experimentally allowed.

Both origins of the matter-antimatter asymmetry - the condensate and the nonzero vacuum expectation values of the scalar fields carrying the weak and the hyper charge - (are assumed to) appear spontaneously.

Sakharov [24] states that for the matter-antimatter asymmetry three conditions must be fulfilled:

a. ($\mathcal{C}_{\mathcal{N}}$ and) $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ must not be conserved.

b. Baryon number non conserving processes must take place.

c. Thermal non equilibrium must be present not to equilibrate the number of baryons and antibaryons.

Sakharov uses for **c.** the requirement that CPT must be conserved and that $\{CPT, H\}_- = 0$. In a thermal equilibrium the average number of baryons $< n_B >= Tr(e^{-\beta H} n_B) = Tr(e^{-\beta H} CPT n_B (CPT)^{-1}) = < \bar{n}_B >$. Therefore $< n_B > - < \bar{n}_B >= 0$ at the thermal equilibrium and there is no excess of baryons with respect to antibaryons. In the expanding universe, however, the temperature is changing with time. It is needed that the discrete symmetry $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ is broken to break the symmetry between matter and antimatter, if the universe starts with no matter-antimatter asymmetry.

The *spin-charge-family* theory starting action (Eq.(9.1)) is invariant under the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry. The scalar fields (Eq.(9.21)) of this theory cause transitions, in which a quark is born out of a positron (figures (9.1, 9.2)) and a quark is born out of antiquark, and back. These reactions go in both directions with the same probability, until the spontaneous break of the $\mathbb{C}_{\mathcal{N}}\,\mathcal{P}_{\mathcal{N}}$ symmetry is caused by the appearance of the condensate of the two right handed neutrinos (table 9.2).

But after the appearance of the condensate (and in addition of the appearance of the non zero vacuum expectation values of the scalar fields with the space index $s \in (7,8)$), family members "see" the vacuum differently than the antimembers. And this *might* explain the matter-antimatter asymmetry. It is also predicting the proton decay.

It is, of course, the question whether both phenomena can at all explain the observed matter-antimatter asymmetry. I agree completely with the referee of this paper that before answering the question whether or not the *spin-charge-family* theory explains this observed phenomena, one must do a lot of additional work to find out: i. Which is the order of phase transition, which leads to the appearance of the condensate. ii. How strong is the thermal nonequilibrium, which leads to the matter-antimatter asymmetry during the phase transition. iii. How rapid is the appearance of the matter-antimatter asymmetry in comparison with the expansion of the universe. iv. Does the later history of the expanding universe enable that the produced asymmetry survives up to today.

Although we do have some experience with solving the Boltzmann equations for fermions and antifermions [12] to follow the history of the dark matter within the *spin-charge-family* theory, the study of the history of the universe from the very high temperature to the baryon production within the same theory in order to see the matter-antimatter asymmetry in the present time is much more demanding task. These is under consideration, but still at a very starting point since a lot of things must be understood before starting with the calculations.

What I can conclude is that the *spin-charge-family* theory does offer the opportunity also for the explanation for the observed matter-antimatter asymmetry.

9.7 Conclusions

The *spin-charge-family* [1,3–8,2,9,12,14,15] theory is a kind of the Kaluza-Klein theories in $d = (13+1)$ but with the families introduced by the second kind of gamma operators - the $\tilde{\gamma}^a$ operators in addition to the Dirac γ^a in $d = (13+1)$. The theory assumes a simple starting action (Eq. (9.1)) in $d = (13+1)$. This simple action manifests in the low energy regime, after the breaks of symmetries (subsection 9.1.1), all the degrees of freedom assumed in the *standard model*, offering the explanation for all the properties of quarks and leptons (right handed neutrinos are in this theory the regular members of each family) and antiquarks and antileptons. The theory explains the existence of the observed gauge vector fields. It explains the origin of the scalar fields (the Higgs and the Yukawa couplings) responsible for the quark and lepton masses and the masses of the weak bosons and carrying the weak and the hyper charge of the *standard model* Higgs ([14]).

The theory is offering the explanation also for the matter-antimatter asymmetry and for the appearance of the dark matter.

The *spin-charge-family* theory predicts two decoupled groups of four families [3,4,9,12]: The fourth of the lower group of four families will be measured at the LHC [10] and the lowest of the upper four families constitutes the dark matter [12] and was already seen. It also predicts that there might be several scalar fields observable at the LHC. The upper four families manifest, due to their high masses, a new "nuclear force" among their baryons.

All these degrees of freedom are already contained in the simple starting action. The scalar fields with the weak and the hyper charges equal to $(\mp\frac{1}{2}, \pm\frac{1}{2})$, respectively (section 9.4), have the space index $s = (7, 8)$, while they carry in addition to the weak and the hyper charges also the family quantum numbers, originating in $\tilde{S}^{ab}$ (they form two groups of twice $SU(2)$ triplets), or the family members quantum numbers, originating in S^{ab} (they form three singlets with the quantum numbers (Q, Q', Y')). These scalar fields cause the transitions of the right handed family members into the left handed partners and back. Those with the family quantum numbers cause at the same time transitions among families within each of the two family groups of four families. They all gain in the electroweak break nonzero vacuum expectation values, giving masses to both groups of four families of quarks and leptons and to weak bosons (changing also their own masses).

There are in this theory also the scalar fields with the space index $s = (5, 6)$; They carry with respect to this degree of freedom they the weak charge equal to the hyper charge $(\mp\frac{1}{2}, \mp\frac{1}{2}$, respectively). They carry also additional quantum numbers Eq.(9.23) like all the scalar fields: The family quantum numbers, originating in $\tilde{S}^{ab}$ and the family members quantum numbers originating in S^{ab}.

And there are also the scalar fields with the scalar index $s = (9, 10, \cdots, 14)$. These scalars carry the triplet colour charge with respect to the space index and the additional quantum numbers (table 9.1), originating in family quantum numbers $\tilde{S}^{ab}$ and in family members quantum numbers S^{ab}.

There are no additional scalar gauge fields.

There are the vector gauge fields with respect to $d = (3 + 1)$: A^{Ai}_{m}, with Ai staying for the groups $SU(3)$ and $U(1)$ (both originating in $SO(6)$ of $SO(13, 1)$), for the groups $SU(2)_{II}$ and $SU(2)_{I}$ (both originating in $SO(4)$ of $SO(7, 1)$) and for the groups $SU(2) \times SU(2)$ ($\in SO(3, 1)$), in both sectors, the S^{ab} and $\tilde{S}^{ab}$ ones.

The condensate of the two right handed neutrinos with the family charges of the upper four families (table 9.2) gives masses to all the scalar and vector gauge fields, except to the colour octet vector, the hyper singlet vector and the weak triplet vector gauge fields, to which the condensate does not couple. It gives masses also to all the vector gauge fields to which the condensate couples. Those vector gauge fields of either S^{ab} or $\tilde{S}^{ab}$ origin, which do not couple to the condensate, are expressible with the corresponding vielbeins (appendices 9.55, 9.56). The condensate breaks the $\mathbb{C}_{\mathcal{N}}$ $\mathcal{P}_{\mathcal{N}}$ symmetry (sections (9.3, 9.8)).

There are no additional vector gauge fields in this theory.

Nonzero vacuum expectation values of the scalar gauge fields with the space index $s = (7, 8)$ and the quantum numbers as explained in the fourth paragraph of this section change in the electroweak break their masses, while all the other scalars or vectors either stay massless (the colour octet, the electromagnetic field), or keep the masses of the scale of the condensate. The only before the electroweak massless vector fields, which become at the electroweak break massive, are the heavy bosons.

It is extremely encouraging that the simple starting action of the spin-charge-family *offers at low energies the explanations for so many observed phenomena,* although the starting assumptions (section 9.1.1) wait to be derived from the initial and boundary conditions of the expanding universe.

This paper is a step towards understanding the matter-antimatter asymmetry within the *spin-charge-family* theory, predicting also the proton decay. The theory obviously offers the possibility that the scalar gauge fields with the space index $s = (9, 10, \cdots, 14)$ explain, after the appearance of the condensate, the matter-antimatter asymmetry. To prove, however, that this indeed happen, requires the additional study: Following the universe through the phase transitions which breaks the $\mathbb{C}_{\mathcal{N}}\ \mathcal{P}_{\mathcal{N}}$ symmetry at the level of the condensate and further through the electroweak phase transition up to today, to check how much of the matter-antimatter asymmetry is left. The experience when following the history of the expanding universe to see whether the *spin-charge-family* theory can explain the dark matter content [12] is of some help. However, answering the question to which extend this theory can explain the observed matter-antimatter asymmetry requires a lot of additional understanding and a lot of work.

Let me conclude with the recognition, pointed out already in the introduction, that the *spin-charge-family* theory overlaps in many points with other unifying theories [26–31], since all the unifying groups can be recognized as the subgroups of the large enough orthogonal groups, with family groups included. But there are also many differences: The *spin-charge-family* theory starts with a very simple action, from where all the properties of spinors and the gauge vector and scalar fields follow, provided that the breaks of symmetries occur in the desired way. Consequently it differs from other unifying theories in the degrees of freedom of spinors and scalar and vector gauge fields which show up on different levels of the break of symmetries, in the unification scheme, in the family degrees of freedom and correspondingly also in the evolution of our universe.

9.8 APPENDIX: Discrete symmetry operators [15]

I present here the discrete symmetry operators in the second quantized picture, for the description of which the Dirac sea is used. I follow the reference [15]. The discrete symmetry operators of this reference are designed for the Kaluza-Klein like theories, in which the total angular momentum in higher than $(3+1)$ dimensions manifest as charges in $d = (3+1)$. The dimension of space-time is even, as it is in the case of the *spin-charge-family* theory.

$$
\begin{aligned}
\mathcal{C}_{\mathcal{N}} &= \prod_{\Im\gamma^m, m=0}^{3} \gamma^m\ \Gamma^{(3+1)}\, K\, I_{x^6, x^8, \ldots, x^d}\,, \\
\mathcal{T}_{\mathcal{N}} &= \prod_{\Re\gamma^m, m=1}^{3} \gamma^m\ \Gamma^{(3+1)}\, K\, I_{x^0}\, I_{x^5, x^7, \ldots, x^{d-1}}\,, \\
\mathcal{P}_{\mathcal{N}} &= \gamma^0\, \Gamma^{(3+1)}\, \Gamma^{(d)}\, I_{\vec{x}_3}\,.
\end{aligned} \tag{9.30}
$$

The operator of handedness in even d dimensional spaces is defined as

$$\Gamma^{(d)} := (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\,\gamma^a)\,, \tag{9.31}$$

with products of γ^a in ascending order. We choose γ^0, γ^1 real, γ^2 imaginary, γ^3 real, γ^5 imaginary, γ^6 real, alternating imaginary and real up to γ^d real. Operators I operate as follows:

$$\begin{aligned}
I_{x^0} x^0 &= -x^0;\\
I_x x^a &= -x^a;\\
I_{x^0} x^a &= (-x^0, \vec{x});\\
I_{\vec{x}} \vec{x} &= -\vec{x};\\
I_{\vec{x}_3} x^a &= (x^0, -x^1, -x^2, -x^3, x^5, x^6, \dots, x^d);\\
I_{x^5, x^7, \dots, x^{d-1}} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &=\\
(x^0, x^1, x^2, x^3, -x^5, x^6, -x^7, \dots, -x^{d-1}, x^d);&\\
I_{x^6, x^8, \dots, x^d} (x^0, x^1, x^2, x^3, x^5, x^6, x^7, x^8, \dots, x^{d-1}, x^d) &=\\
(x^0, x^1, x^2, x^3, x^5, -x^6, x^7, -x^8, \dots, x^{d-1}, -x^d), d = 2n.&
\end{aligned}$$

$\mathcal{C}_\mathcal{N}$ transforms the state, put on the top of the Dirac sea, into the corresponding negative energy state in the Dirac sea.

The operator, it is named [1,16,15] $\mathbb{C}_\mathcal{N}$, is needed, which transforms the starting single particle state on the top of the Dirac sea into the negative energy state and then empties this negative energy state. This hole in the Dirac sea is the antiparticle state put on the top of the Dirac sea. Both, a particle and its antiparticle state (both put on the top of the Dirac sea), must solve the Weyl equations of motion.

This $\mathbb{C}_\mathcal{N}$ is defined as a product of the operator [1,16] "$\mathit{emptying}$", (making transformations into a completely different Fock space)

$$\text{"emptying"} = \prod_{\Re\gamma^a} \gamma^a\, K = (-)^{\frac{d}{2}+1} \prod_{\Im\gamma^a} \gamma^a\, \Gamma^{(d)} K\,, \tag{9.32}$$

and $\mathcal{C}_\mathcal{N}$

$$\begin{aligned}
\mathbb{C}_\mathcal{N} &= \prod_{\Re\gamma^a, a=0}^{d} \gamma^a \;\; K \prod_{\Im\gamma^m, m=0}^{3} \gamma^m \;\; \Gamma^{(3+1)}\, K\, I_{x^6, x^8, \dots, x^d}\\
&= \prod_{\Re\gamma^s, s=5}^{d} \gamma^s \;\; I_{x^6, x^8, \dots, x^d}\,.
\end{aligned} \tag{9.33}$$

We shall need inded only the product of operators $\mathbb{C}_\mathcal{N}\mathcal{P}_\mathcal{N}$, $\mathcal{T}_\mathcal{N}$ and $\mathbb{C}_\mathcal{N}\mathcal{P}_\mathcal{N}\,\mathcal{T}_\mathcal{N}$, since either $\mathbb{C}_\mathcal{N}$ or $\mathcal{P}_\mathcal{N}$ have in even dimensional spaces with $d = 2(2n+1)$ an odd number of γ^a operators, transforming accordingly states from the representation of one handedness in $d = 2(2n+1)$ into the Weyl of another handedness.

$$\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}} = \gamma^0 \prod_{\Im\gamma^s, s=5}^{d} \gamma^s \; I_{\vec{x}_3}\, I_{x^6,x^8,\dots,x^d}\,,$$
$$\mathbb{C}_{\mathcal{N}}\mathcal{P}_{\mathcal{N}}\mathcal{T}_{\mathcal{N}} = \prod_{\Im\gamma^a, a=0}^{d} \gamma^a \; K\, I_x\,. \tag{9.34}$$

9.9 APPENDIX: Short presentation of technique [6,18,20]

I make in this appendix a short review of the technique [18,20], initiated and developed [5–8] when proposing the *spin-charge-family* theory [5,6,8,4,1,2,12,9] assuming that all the internal degrees of freedom of spinors, with family quantum number included, are describable in the space of d-anticommuting (Grassmann) coordinates [6], if the dimension of ordinary space is d. There are two kinds of operators in the Grassmann space, fulfilling the Clifford algebra, which anticommute with one another. The technique was further developed in the present shape together with H.B. Nielsen [18,20] by identifying one kind of the Clifford objects with γ^s's and another kind with $\tilde{\gamma}^a$'s.

The objects γ^a and $\tilde{\gamma}^a$ have properties

$$\{\gamma^a, \gamma^b\}_+ = 2\eta^{ab}\,, \qquad \{\tilde{\gamma}^a, \tilde{\gamma}^b\}_+ = 2\eta^{ab}\,, \quad , \quad \{\gamma^a, \tilde{\gamma}^b\}_+ = 0\,,$$
$$\tilde{\gamma}^a B := i(-)^{n_B} B\gamma^a \;|\psi_0>,$$
$$B = a_0 + a_a\gamma^a + a_{ab}\gamma^a\gamma^b + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\dots\gamma^{a_d})|\psi_0> \tag{9.35}$$

for any d, even or odd. I is the unit element in the Clifford algebra. The two kinds of the Clifford algebra objects are connected with the left and the right multiplication of any Clifford algebra objects B. In Eq. (9.35) B is expressed as a polynomial of γ^a, $(-)^{n_B} = +1, -1$, when the object B has a Clifford even (+1) or odd (+1) character, respectively. $|\psi_0>$ is a vacuum state on which the operators γ^a apply.

If B is a Clifford algebra object, let say a polynomial of γ^a, then one finds

$$(\tilde{\gamma}^a B : = i(-)^{n_B}\, B\gamma^a\,)\;|\psi_0>,$$
$$B = a_0 + a_{a_0}\gamma^{a_0} + a_{a_1 a_2}\gamma^{a_1}\gamma^{a_2} + \cdots + a_{a_1\cdots a_d}\gamma^{a_1}\cdots\gamma^{a_d}\,, \tag{9.36}$$

where $|\psi_0>$ is a vacuum state, defined in Eq. (9.50) and $(-)^{n_B}$ is equal to 1 for the term in the polynomial which has an even number of γ^b's, and to -1 for the term with an odd number of γ^b's.

In this last stage we constructed a spinor basis as products of nilpotents and projections formed as odd and even objects of γ^a's, respectively, and chosen to be eigenstates of a Cartan subalgebra of the Lorentz groups defined by γ^a's and $\tilde{\gamma}^a$'s.

The technique can be used to construct a spinor basis for any dimension d and any signature in an easy and transparent way. Equipped with the graphic presentation of basic states, the technique offers an elegant way to see all the quantum numbers of states with respect to the two Lorentz groups, as well as transformation properties of the states under any Clifford algebra object.

The Clifford algebra objects S^{ab} and $\tilde{S}^{ab}$ close the algebra of the Lorentz group

$$\begin{aligned} S^{ab} &:= (i/4)(\gamma^a\gamma^b - \gamma^b\gamma^a)\,, \\ \tilde{S}^{ab} &:= (i/4)(\tilde{\gamma}^a\tilde{\gamma}^b - \tilde{\gamma}^b\tilde{\gamma}^a)\,, \\ \{S^{ab}, \tilde{S}^{cd}\}_- &= 0\,, \\ \{S^{ab}, S^{cd}\}_- &= i(\eta^{ad}S^{bc} + \eta^{bc}S^{ad} - \eta^{ac}S^{bd} - \eta^{bd}S^{ac})\,, \\ \{\tilde{S}^{ab}, \tilde{S}^{cd}\}_- &= i(\eta^{ad}\tilde{S}^{bc} + \eta^{bc}\tilde{S}^{ad} - \eta^{ac}\tilde{S}^{bd} - \eta^{bd}\tilde{S}^{ac})\,, \end{aligned} \tag{9.37}$$

We assume the "Hermiticity" property for γ^a's and $\tilde{\gamma}^a$'s

$$\gamma^{a\dagger} = \eta^{aa}\gamma^a\,, \qquad \tilde{\gamma}^{a\dagger} = \eta^{aa}\tilde{\gamma}^a\,, \tag{9.38}$$

in order that γ^a and $\tilde{\gamma}^a$ are compatible with (9.35) and formally unitary, i.e. $\gamma^{a\,\dagger}\,\gamma^a = I$ and $\tilde{\gamma}^{a\,\dagger}\tilde{\gamma}^a = I$.

One finds from Eq.(9.38) that $(S^{ab})^\dagger = \eta^{aa}\eta^{bb}S^{ab}$.

Recognizing from Eq.(9.37) that two Clifford algebra objects S^{ab}, S^{cd} with all indices different commute, and equivalently for $\tilde{S}^{ab}, \tilde{S}^{cd}$, we select the Cartan subalgebra of the algebra of the two groups, which form equivalent representations with respect to one another

$$\begin{aligned} S^{03}, S^{12}, S^{56}, \cdots, S^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4, \\ S^{03}, S^{12}, \cdots, S^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,, \\ \tilde{S}^{03}, \tilde{S}^{12}, \tilde{S}^{56}, \cdots, \tilde{S}^{d-1\;d}, &\quad \text{if} \quad d = 2n \geq 4\,, \\ \tilde{S}^{03}, \tilde{S}^{12}, \cdots, \tilde{S}^{d-2\;d-1}, &\quad \text{if} \quad d = (2n+1) > 4\,. \end{aligned} \tag{9.39}$$

The choice for the Cartan subalgebra in $d < 4$ is straightforward. It is useful to define one of the Casimirs of the Lorentz group - the handedness Γ ($\{\Gamma, S^{ab}\}_- = 0$) in any d

$$\begin{aligned} \Gamma^{(d)} &:= (i)^{d/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n, \\ \Gamma^{(d)} &:= (i)^{(d-1)/2} \prod_a (\sqrt{\eta^{aa}}\gamma^a), \quad \text{if} \quad d = 2n+1\,. \end{aligned} \tag{9.40}$$

One proceeds equivalently for $\tilde{\Gamma}^{(d)}$, subtituting γ^a's by $\tilde{\gamma}^a$'s. We understand the product of γ^a's in the ascending order with respect to the index a: $\gamma^0\gamma^1\cdots\gamma^d$. It follows from Eq.(9.38) for any choice of the signature η^{aa} that $\Gamma^\dagger = \Gamma,\ \Gamma^2 = I$. We also find that for d even the handedness anticommutes with the Clifford algebra objects γ^a ($\{\gamma^a, \Gamma\}_+ = 0$) , while for d odd it commutes with γ^a ($\{\gamma^a, \Gamma\}_- = 0$).

To make the technique simple we introduce the graphic presentation as follows

$$\begin{aligned} \stackrel{ab}{(k)} &:= \frac{1}{2}(\gamma^a + \frac{\eta^{aa}}{ik}\gamma^b)\,, \qquad \stackrel{ab}{[k]} := \frac{1}{2}(1 + \frac{i}{k}\gamma^a\gamma^b)\,, \\ \stackrel{+}{\circ} &:= \frac{1}{2}(1+\Gamma)\,, \qquad \stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma), \end{aligned} \tag{9.41}$$

where $k^2 = \eta^{aa}\eta^{bb}$. It follows then

$$\gamma^a = \stackrel{ab}{(k)} + \stackrel{ab}{(-k)}, \qquad \gamma^b = ik\eta^{aa}\,(\stackrel{ab}{(k)} - \stackrel{ab}{(-k)}),$$
$$S^{ab} = \frac{k}{2}(\stackrel{ab}{[k]} - \stackrel{ab}{[-k]}) \tag{9.42}$$

One can easily check by taking into account the Clifford algebra relation (Eq.9.35) and the definition of S^{ab} and $\tilde{S}^{ab}$ (Eq.9.37) that if one multiplies from the left hand side by S^{ab} or $\tilde{S}^{ab}$ the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, it follows that

$$S^{ab}\ \stackrel{ab}{(k)} = \frac{1}{2}\,k\ \stackrel{ab}{(k)}, \qquad S^{ab}\ \stackrel{ab}{[k]} = \frac{1}{2}\,k\ \stackrel{ab}{[k]},$$
$$\tilde{S}^{ab}\ \stackrel{ab}{(k)} = \frac{1}{2}\,k\ \stackrel{ab}{(k)}, \qquad \tilde{S}^{ab}\ \stackrel{ab}{[k]} = -\frac{1}{2}\,k\ \stackrel{ab}{[k]}, \tag{9.43}$$

which means that we get the same objects back multiplied by the constant $\frac{1}{2}k$ in the case of S^{ab}, while $\tilde{S}^{ab}$ multiply $\stackrel{ab}{(k)}$ by k and $\stackrel{ab}{[k]}$ by $(-k)$ rather than (k). This also means that when $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$ act from the left hand side on a vacuum state $|\psi_0\rangle$ the obtained states are the eigenvectors of S^{ab}. We further recognize that γ^a transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[-k]}$, never to $\stackrel{ab}{[k]}$, while $\tilde{\gamma}^a$ transform $\stackrel{ab}{(k)}$ into $\stackrel{ab}{[k]}$, never to $\stackrel{ab}{[-k]}$

$$\gamma^a\ \stackrel{ab}{(k)} = \eta^{aa}\ \stackrel{ab}{[-k]},\ \gamma^b\ \stackrel{ab}{(k)} = -ik\ \stackrel{ab}{[-k]},\ \gamma^a\ \stackrel{ab}{[k]} = \stackrel{ab}{(-k)},\ \gamma^b\ \stackrel{ab}{[k]} = -ik\eta^{aa}\ \stackrel{ab}{(-k)},$$
$$\tilde{\gamma^a}\ \stackrel{ab}{(k)} = -i\eta^{aa}\ \stackrel{ab}{[k]},\ \tilde{\gamma^b}\ \stackrel{ab}{(k)} = -k\ \stackrel{ab}{[k]},\ \tilde{\gamma^a}\ \stackrel{ab}{[k]} = \ i\ \stackrel{ab}{(k)},\ \tilde{\gamma^b}\ \stackrel{ab}{[k]} = -k\eta^{aa}\ \stackrel{ab}{(k)} \tag{9.44}$$

From Eq.(9.44) it follows

$$S^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)} = -\frac{i}{2}\eta^{aa}\eta^{cc}\ \stackrel{ab}{[-k]}\stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{aa}\eta^{cc}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]},$$
$$S^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]} = \frac{i}{2}\ \stackrel{ab}{(-k)}\stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{[k]} = -\frac{i}{2}\ \stackrel{ab}{(k)}\stackrel{cd}{(k)},$$
$$S^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\ \stackrel{ab}{[-k]}\stackrel{cd}{(-k)}, \qquad \tilde{S}^{ac}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]} = -\frac{i}{2}\eta^{aa}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)},$$
$$S^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\ \stackrel{ab}{(-k)}\stackrel{cd}{[-k]}, \qquad \tilde{S}^{ac}\ \stackrel{ab}{[k]}\stackrel{cd}{(k)} = \frac{i}{2}\eta^{cc}\ \stackrel{ab}{(k)}\stackrel{cd}{[k]}\ . \tag{9.45}$$

From Eqs. (9.45) we conclude that $\tilde{S}^{ab}$ generate the equivalent representations with respect to S^{ab} and opposite.

Let us deduce some useful relations

$$\begin{array}{llll}
\stackrel{ab}{(k)}\stackrel{ab}{(k)} = 0, & \stackrel{ab}{(k)}\stackrel{ab}{(-k)} = \eta^{aa}\ \stackrel{ab}{[k]}, & \stackrel{ab}{(-k)}\stackrel{ab}{(k)} = \eta^{aa}\ \stackrel{ab}{[-k]}, & \stackrel{ab}{(-k)}\stackrel{ab}{(-k)} = 0, \\
\stackrel{ab}{[k]}\stackrel{ab}{[k]} = \stackrel{ab}{[k]}, & \stackrel{ab}{[k]}\stackrel{ab}{[-k]} = 0, & \stackrel{ab}{[-k]}\stackrel{ab}{[k]} = 0, & \stackrel{ab}{[-k]}\stackrel{ab}{[-k]} = \stackrel{ab}{[-k]}, \\
\stackrel{ab}{(k)}\stackrel{ab}{[k]} = 0, & \stackrel{ab}{[k]}\stackrel{ab}{(k)} = \stackrel{ab}{(k)}, & \stackrel{ab}{(-k)}\stackrel{ab}{[k]} = \stackrel{ab}{(-k)}, & \stackrel{ab}{(-k)}\stackrel{ab}{[-k]} = 0, \\
\stackrel{ab}{(k)}\stackrel{ab}{[-k]} = \stackrel{ab}{(k)}, & \stackrel{ab}{[k]}\stackrel{ab}{(-k)} = 0, & \stackrel{ab}{[-k]}\stackrel{ab}{(k)} = 0, & \stackrel{ab}{[-k]}\stackrel{ab}{(-k)} = \stackrel{ab}{(-k)}\ .
\end{array} \tag{9.46}$$

We recognize in the first equation of the first line and the first and the second equation of the second line the demonstration of the nilpotent and the projector character of the Clifford algebra objects $\stackrel{ab}{(k)}$ and $\stackrel{ab}{[k]}$, respectively. Defining

$$\stackrel{ab}{(\tilde{\pm} i)} = \frac{1}{2}\,(\tilde{\gamma}^a \mp \tilde{\gamma}^b)\,, \quad \stackrel{ab}{(\tilde{\pm} 1)} = \frac{1}{2}\,(\tilde{\gamma}^a \pm i\tilde{\gamma}^b)\,, \tag{9.47}$$

one recognizes that

$$\stackrel{ab}{(\tilde{k})}\stackrel{ab}{(k)} = 0\,, \quad \stackrel{ab}{(\tilde{-k})}\stackrel{ab}{(k)} = -i\eta^{aa}\stackrel{ab}{[k]}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[k]} = i\stackrel{ab}{(k)}\,, \quad \stackrel{ab}{(\tilde{k})}\stackrel{ab}{[-k]} = 0\,. \tag{9.48}$$

Recognizing that

$$\stackrel{ab}{(k)}^{\dagger} = \eta^{aa}\stackrel{ab}{(-k)}\,, \quad \stackrel{ab}{[k]}^{\dagger} = \stackrel{ab}{[k]}\,, \tag{9.49}$$

we define a vacuum state $|\psi_0>$ so that one finds

$$\begin{aligned} &< \stackrel{ab}{(k)}^{\dagger} \stackrel{ab}{(k)} >= 1\,, \\ &< \stackrel{ab}{[k]}^{\dagger} \stackrel{ab}{[k]} >= 1\,. \end{aligned} \tag{9.50}$$

Taking into account the above equations it is easy to find a Weyl spinor irreducible representation for d-dimensional space, with d even or odd.

For d even we simply make a starting state as a product of d/2, let us say, only nilpotents $\stackrel{ab}{(k)}$, one for each S^{ab} of the Cartan subalgebra elements (Eq.(9.39)), applying it on an (unimportant) vacuum state. For d odd the basic states are products of $(d-1)/2$ nilpotents and a factor $(1 \pm \Gamma)$. Then the generators S^{ab}, which do not belong to the Cartan subalgebra, being applied on the starting state from the left, generate all the members of one Weyl spinor.

$$\begin{array}{c}
\stackrel{0d}{(k_{0d})}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{[-k_{12}]}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\,\psi_0 \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\,\psi_0 \\
\vdots \\
\stackrel{0d}{[-k_{0d}]}\stackrel{12}{(k_{12})}\stackrel{35}{(k_{35})}\cdots\stackrel{d-1\,d-2}{[-k_{d-1\,d-2}]}\,\psi_0 \\
\stackrel{od}{(k_{0d})}\stackrel{12}{[-k_{12}]}\stackrel{35}{[-k_{35}]}\cdots\stackrel{d-1\,d-2}{(k_{d-1\,d-2})}\,\psi_0 \\
\vdots
\end{array} \tag{9.51}$$

All the states have the handedness Γ, since $\{\Gamma, S^{ab}\} = 0$. States, belonging to one multiplet with respect to the group $SO(q, d-q)$, that is to one irreducible representation of spinors (one Weyl spinor), can have any phase. We made a choice of the simplest one, taking all phases equal to one.

The above graphic representation demonstrate that for d even all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. There are $S^{am}, S^{an}, S^{bm}, S^{bn}$, which do this. The procedure gives $2^{(d/2-1)}$ states. A Clifford algebra object γ^a being applied from the left hand side, transforms a Weyl spinor of one handedness into a Weyl spinor of the opposite handedness. Both Weyl spinors form a Dirac spinor.

For d odd a Weyl spinor has besides a product of $(d-1)/2$ nilpotents or projectors also either the factor $\stackrel{+}{\circ} := \frac{1}{2}(1+\Gamma)$ or the factor $\stackrel{-}{\bullet} := \frac{1}{2}(1-\Gamma)$. As in the case of d even, all the states of one irreducible Weyl representation of a definite handedness follow from a starting state, which is, for example, a product of $(1+\Gamma)$ and $(d-1)/2$ nilpotents $\stackrel{ab}{(k_{ab})}$, by transforming all possible pairs of $\stackrel{ab}{(k_{ab})}\stackrel{mn}{(k_{mn})}$ into $\stackrel{ab}{[-k_{ab}]}\stackrel{mn}{[-k_{mn}]}$. But γ^a's, being applied from the left hand side, do not change the handedness of the Weyl spinor, since $\{\Gamma, \gamma^a\}_- = 0$ for d odd. A Dirac and a Weyl spinor are for d odd identical and a "family" has accordingly $2^{(d-1)/2}$ members of basic states of a definite handedness.

We shall speak about left handedness when $\Gamma = -1$ and about right handedness when $\Gamma = 1$ for either d even or odd.

While S^{ab} which do not belong to the Cartan subalgebra (Eq. (9.39)) generate all the states of one representation, generate $\tilde{S}^{ab}$ which do not belong to the Cartan subalgebra(Eq. (9.39)) the states of $2^{d/2-1}$ equivalent representations.

Making a choice of the Cartan subalgebra set (Eq.(9.39)) of the algebra S^{ab} and $\tilde{S}^{ab}$ a left handed ($\Gamma^{(13,1)} = -1$) eigen state of all the members of the Cartan subalgebra, representing a weak chargeless u_R-quark with spin up, hyper charge (2/3) and colour $(1/2\,,1/(2\sqrt{3}))$, for example, can be written as

$$\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\;\stackrel{11\,12}{(-)}\;\stackrel{13\,14}{(-)}\; |\psi\rangle = \frac{1}{2^7}(\gamma^0-\gamma^3)(\gamma^1+i\gamma^2)|(\gamma^5+i\gamma^6)(\gamma^7+i\gamma^8)|| (\gamma^9+i\gamma^{10})(\gamma^{11}-i\gamma^{12})(\gamma^{13}-i\gamma^{14})|\psi\rangle\,. \quad (9.52)$$

This state is an eigen state of all S^{ab} and $\tilde{S}^{ab}$ which are members of the Cartan subalgebra (Eq. (9.39)).

The operators $\tilde{S}^{ab}$, which do not belong to the Cartan subalgebra (Eq. (9.39)), generate families from the starting u_R quark, transforming u_R quark from Eq. (9.52) to the u_R of another family, keeping all the properties with respect to S^{ab} unchanged. In particular $\tilde{S}^{01}$ applied on a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$ from Eq. (9.52) generates a state which is again a right handed u_R-quark, weak chargeless, with spin up, hyper charge (2/3) and the colour charge $(1/2\,,1/(2\sqrt{3}))$

$$\tilde{S}^{01}\;\stackrel{03}{(+i)}\stackrel{12}{(+)} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)} = -\frac{i}{2}\;\stackrel{03}{[+i]}\stackrel{12}{[+]} | \stackrel{56}{(+)}\stackrel{78}{(+)} || \stackrel{9\,10}{(+)}\stackrel{11\,12}{(-)}\stackrel{13\,14}{(-)}\;. \quad (9.53)$$

Below some useful relations [4] are presented

$$
\begin{aligned}
&N^{\pm}_{+} = N^{1}_{+} \pm i\,N^{2}_{+} = -\overset{03}{(\mp i)}\overset{12}{(\pm)}\,, \quad N^{\pm}_{-} = N^{1}_{-} \pm i\,N^{2}_{-} = \overset{03}{(\pm i)}\overset{12}{(\pm)}\,,\\
&\tilde{N}^{\pm}_{+} = -\overset{03}{\tilde{(\mp i)}}\overset{12}{\tilde{(\pm)}}\,, \quad \tilde{N}^{\pm}_{-} = \overset{03}{\tilde{(\pm i)}}\overset{12}{\tilde{(\pm)}}\,,\\
&\tau^{1\pm} = (\mp)\,\overset{56}{(\pm)}\overset{78}{(\mp)}\,, \quad \tau^{2\mp} = (\mp)\,\overset{56}{(\mp)}\overset{78}{(\mp)}\,,\\
&\tilde{\tau}^{1\pm} = (\mp)\,\overset{56}{\tilde{(\pm)}}\overset{78}{\tilde{(\mp)}}\,, \quad \tilde{\tau}^{2\mp} = (\mp)\,\overset{56}{\tilde{(\mp)}}\overset{78}{\tilde{(\mp)}}\,.
\end{aligned} \tag{9.54}
$$

I present at the end one Weyl representation of $SO(13+1)$ and the family quantum numbers of the two groups of four families.

One Weyl representation of $SO(13+1)$ contains left handed weak charged and the second $SU(2)$ chargeless coloured quarks and colourless leptons and right handed weak chargeless and the second $SU(2)$ charged quarks and leptons (electrons and neutrinos). It carries also the family quantum numbers, not mentioned in this table. The table is taken from the reference [15].

i		$\vert{}^{a}\psi_i>$ Octet, $\Gamma^{(1,7)}=1$, $\Gamma^{(6)}=-1$, of quarks and leptons	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^{4}	Y	Q
1	u_R^{c1}	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
2	u_R^{c1}	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
3	d_R^{c1}	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{[-]}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
4	d_R^{c1}	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{[-]}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$-\frac{1}{3}$	$-\frac{1}{3}$
5	d_L^{c1}	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{[-]}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
6	d_L^{c1}	$\overset{03}{(+i)}\overset{12}{[-]}\,\vert\,\overset{56}{[-]}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{3}$
7	u_L^{c1}	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
8	u_L^{c1}	$\overset{03}{(+i)}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{(-)}\overset{13\,14}{(-)}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$
9	u_R^{c2}	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{(-)}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
10	u_R^{c2}	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{(-)}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
17	u_R^{c3}	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
18	u_R^{c3}	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{(-)}\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{2}{3}$
...												
25	ν_R	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
26	ν_R	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$-\frac{1}{2}$	0	0
27	e_R	$\overset{03}{(+i)}\overset{12}{(+)}\,\vert\,\overset{56}{[-]}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
28	e_R	$\overset{03}{[-i]}\overset{12}{[-]}\,\vert\,\overset{56}{[-]}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$-\frac{1}{2}$	-1	-1
29	e_L	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{[-]}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
30	e_L	$\overset{03}{(+i)}\overset{12}{[-]}\,\vert\,\overset{56}{[-]}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	-1
31	ν_L	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
32	ν_L	$\overset{03}{(+i)}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{(+)}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$-\frac{1}{2}$	$-\frac{1}{2}$	0
33	$\bar{d}_L^{\bar{c}1}$	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
34	$\bar{d}_L^{\bar{c}1}$	$\overset{03}{(+i)}\overset{12}{[-]}\,\vert\,\overset{56}{(+)}\overset{78}{(+)}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
35	$\bar{u}_L^{\bar{c}1}$	$\overset{03}{[-i]}\overset{12}{(+)}\,\vert\,\overset{56}{[-]}\overset{78}{[-]}\,\Vert\,\overset{9\,10}{[-]}\overset{11\,12}{[+]}\overset{13\,14}{[+]}$ -	1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$

Continued on next page

i		$\vert^a\psi_i>$	$\Gamma^{(3,1)}$	S^{12}	$\Gamma^{(4)}$	τ^{13}	τ^{23}	τ^{33}	τ^{38}	τ^4	Y	Q
		Octet, $\Gamma^{(1,7)}=1,\ \Gamma^{(6)}=-1,$ of quarks and leptons										
36	$\bar{u}^{\bar{c}1}_{L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{2}{3}$	$-\frac{2}{3}$
37	$\bar{d}^{\bar{c}1}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
38	$\bar{d}^{\bar{c}1}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{3}$
39	$\bar{u}^{\bar{c}1}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
40	$\bar{u}^{\bar{c}1}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{[+]}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{2}{3}$
41	$\bar{d}^{\bar{c}2}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{[+]}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
. . .												
49	$\bar{d}^{\bar{c}3}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{(+)}\ \stackrel{11\,12}{[+]}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{3}$
. . .												
57	$\bar{e}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
58	$\bar{e}_{L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	1	1
59	$\bar{\nu}_{L}$	$\stackrel{03}{[-i]}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
60	$\bar{\nu}_{L}$	$\stackrel{03}{(+i)}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	0	0
61	$\bar{\nu}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
62	$\bar{\nu}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{[-]}\stackrel{78}{(+)}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0
63	$\bar{e}_{R}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1
64	$\bar{e}_{R}$	$\stackrel{03}{[-i]}\stackrel{12}{[-]}\vert\stackrel{56}{(+)}\stackrel{78}{[-]}\vert\vert\stackrel{9\,10}{[-]}\ \stackrel{11\,12}{(-)}\ \stackrel{13\,14}{(-)}$	1	$-\frac{1}{2}$	-1	$\frac{1}{2}$	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1

Table 9.3. The left handed ($\Gamma^{(13,1)}=-1$) multiplet of spinors - the members of the $SO(13,1)$ group, manifesting the subgroup $SO(7,1)$ - of the colour charged quarks and anti-quarks and the colourless leptons and anti-leptons, is presented in the massless basis using the technique presented in Appendix 9.9. It contains the left handed ($\Gamma^{(3,1)}=-1$) weak charged ($\tau^{13}=\pm\frac{1}{2}$) and $SU(2)_{II}$ chargeless ($\tau^{23}=0$) quarks and the right handed weak chargeless and $SU(2)_{II}$ charged ($\tau^{23}=\pm\frac{1}{2}$) quarks of three colours ($c^i=(\tau^{33},\tau^{38})$) with the "spinor" charge ($\tau^4=\frac{1}{6}$) and the colourless left handed weak charged leptons and the right handed weak chargeless leptons with the "spinor" charge ($\tau^4=-\frac{1}{2}$). S^{12} defines the ordinary spin $\pm\frac{1}{2}$. The vacuum state $|vac>_{fam}$, on which the nilpotents and projectors operate, is not shown. The reader can find this Weyl representation also in the refs. [21,3]. Left handed antiquarks and anti leptons are weak chargeless and carry opposite charges.

The eight families of the first member of the eight-plet of quarks from Table 9.3, for example, that is of the right handed u_{1R} quark, are presented in the left column of Table 9.4 [3]. In the right column of the same table the equivalent eight-plet of the right handed neutrinos ν_{1R} are presented. All the other members of any of the eight families of quarks or leptons follow from any member of a particular family by the application of the operators $N^{\pm}_{R,L}$ and $\tau^{(2,1)\pm}$ on this particular member.

The eight-plets separate into two group of four families: One group contains doublets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$, these families are singlets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$. Another group of families contains doublets with respect to $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$, these families are singlets with respect to $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$.

The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^2$ couple only to the four families which are doublets with respect to these two groups. The scalar fields which are the gauge scalars of $\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^1$ couple only to the four families which are doublets with respect to these last two groups.

					$\tilde{\tau}^{13}$	$\tilde{\tau}^{23}$	$\tilde{N}^3_L$	$\tilde{N}^3_R$	$\tilde{\tau}^4$
I	$u^{c1}_{R\,1}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,2}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,2}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,3}$	$\stackrel{03}{(+i)}\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
I	$u^{c1}_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,4}$	$\stackrel{03}{[+i]}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	0	$-\frac{1}{3}$
II	$u^{c1}_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,5}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \vert \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,6}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{[+]}\stackrel{78}{[+]} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,7}$	$\stackrel{03}{[+i]}\stackrel{12}{[+]} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{3}$
II	$u^{c1}_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{[-]}\stackrel{13\,14}{[-]}$	$\nu_{R\,8}$	$\stackrel{03}{(+i)}\stackrel{12}{(+)} \vert \stackrel{56}{(+)}\stackrel{78}{(+)} \Vert \stackrel{9\,10}{(+)}\stackrel{11\,12}{(+)}\stackrel{13\,14}{(+)}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	$-\frac{1}{3}$

Table 9.4. Eight families of the right handed u^{c1}_R (9.3) quark with spin $\frac{1}{2}$, the colour charge ($\tau^{33} = 1/2$, $\tau^{38} = 1/(2\sqrt{3})$, and of the colourless right handed neutrino ν_R of spin $\frac{1}{2}$ (9.2) are presented in the left and in the right column, respectively. They belong to two groups of four families, one (I) is a doublet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a singlet with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$), the other (II) is a singlet with respect to ($\vec{\tilde{N}}_L$ and $\vec{\tilde{\tau}}^{(1)}$) and a doublet with with respect to ($\vec{\tilde{N}}_R$ and $\vec{\tilde{\tau}}^{(2)}$). All the families follow from the starting one by the application of the operators ($\tilde{N}^{\pm}_{R,L}$, $\tilde{\tau}^{(2,1)\pm}$), Eq. (9.54). The generators ($N^{\pm}_{R,L}$, $\tau^{(2,1)\pm}$) (Eq. (9.54)) transform u_{1R} to all the members of one family of the same colour. The same generators transform equivalently the right handed neutrino ν_{1R} to all the colourless members of the same family.

9.10 APPENDIX: Expressions for the spin connection fields in terms of vielbeins and the spinor sources [14]

The expressions for the spin connection of both kind, $\omega_{ab\alpha}$ and $\tilde{\omega}_{ab\alpha}$ in terms of the vielbeins and the spinor sources of both kinds are presented, obtained by the variation of the action Eq.(9.1). The expression for the spin connection $\omega_{ab\alpha}$ is taken from the ref. [32].

$$
\begin{aligned}
\omega_{ab\alpha} = & -\frac{1}{2E}\Big\{ e_{e\alpha} e_{b\gamma}\, \partial_\beta (E f^{\gamma[e} f^{\beta}{}_{a]}) + e_{e\alpha} e_{a\gamma}\, \partial_\beta (E f^{\gamma}{}_{[b} f^{\beta e]}) \\
& \qquad - e_{e\alpha} e^{e}{}_{\gamma}\, \partial_\beta \left(E f^{\gamma}{}_{[a} f^{\beta}{}_{b]}\right) \Big\} \\
& - \frac{e_{e\alpha}}{4} \Big\{ \bar{\Psi} \Big(\gamma_e\, S_{ab} + \frac{3i}{2} (\delta^e_b \gamma_a - \delta^e_a \gamma_b) \Big) \Psi \Big\} \\
& - \frac{1}{d-2} \Big\{ e_{a\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{b]}\right) + \frac{1}{2} \bar{\Psi} \gamma^d S_{db}\, \Psi \Big] \\
& \qquad - e_{b\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{a]}\right) + \frac{1}{2} \bar{\Psi} \gamma^d S_{da}\, \Psi \Big\} \Big] \,. \qquad (9.55)
\end{aligned}
$$

One notices that if there are no spinor sources, carrying the spinor quantum numbers S^{ab}, then $\omega_{ab\alpha}$ is completely determined by the vielbeins.

Equivalently one obtains expressions for the spin connection fields carryin family quantum numbers

$$
\begin{aligned}
\tilde{\omega}_{ab\alpha} = & -\frac{1}{2E}\Big\{ e_{e\alpha} e_{b\gamma}\, \partial_\beta (E f^{\gamma[e} f^{\beta}{}_{a]}) + e_{e\alpha} e_{a\gamma}\, \partial_\beta (E f^{\gamma}{}_{[b} f^{\beta e]}) \\
& \qquad - e_{e\alpha} e^{e}{}_{\gamma}\, \partial_\beta \left(E f^{\gamma}{}_{[a} f^{\beta}{}_{b]}\right) \Big\} \\
& - \frac{e_{e\alpha}}{4} \Big\{ \bar{\Psi} \Big(\gamma_e\, \tilde{S}_{ab} + \frac{3i}{2} (\delta^e_b \gamma_a - \delta^e_a \gamma_b) \Big) \Psi \Big\} \\
& - \frac{1}{d-2} \Big\{ e_{a\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{b]}\right) + \frac{1}{2} \bar{\Psi} \gamma^d\, \tilde{S}_{db}\, \Psi \Big] \\
& \qquad - e_{b\alpha} \Big[\frac{1}{E} e^{d}{}_{\gamma} \partial_\beta \left(E f^{\gamma}{}_{[d} f^{\beta}{}_{a]}\right) + \frac{1}{2} \bar{\Psi} \gamma^d\, \tilde{S}_{da}\, \Psi \Big\} \Big] \,. \qquad (9.56)
\end{aligned}
$$

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