

PRAKTIČNA METODA ZA OPTIMALNO ZASNOVO PASOVNEGA TEMELJA Z OPTIMIZACIJO S KOLONIJO MRAVELJ

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Ključne besede

optimalno načrtovanje; temeljenje; stabilnost; nelinearno programiranje; optimizacija kolonije mravelj

Izvleček

Namen članka je predstaviti praktično metodo optimalne zasnove pasovnih temeljev obremenjenih z vertikalnimi in horizontalnimi obtežbami. Problem načrtovanja z iskanjem optimalne velikosti temelja in minimalne količine armature je izražen v nelinearni minimalizacijski obliki. Na vrhu pasovnega temelja delujejo vertikalne in horizontalne obtežbe. V problemu načrtovanja nastopajo štiri neodvisne spremenljivke, in sicer širina temelja, debelina temelja, globina temeljenja in količina armature. Zahtevane geotehnične omejitve vključujejo preveritve nosilnosti, prevrnitve, kot tudi globalni zdrs in lokalni zdrs na vogalih temeljev. Kratkotrajna stabilnost in dolgotrajna stabilnost sta sočasno formulirani. Konstrukcijske omejitve zajemajo kontrolo odpora prereza na strižne sile in upogibne momente.

Formulacija omejitvenega problema vodi do nelinearnega programiranja katerega cilj je zmanjšati skupno maso materiala temelja, vključno z betonom in armaturo. Optimalna rešitev je dobljena z algoritmom optimizacije s kolonijami mravelj, MIDACO. Predlagana metoda optimizacije je prikazana na primeru dejanskega načrtovanje temelja za podporo velikega stroja, ki se premika po tirih.

A PRACTICAL METHOD FOR THE OPTIMAL DESIGN OF CONTINUOUS FOOTING USING ANT-COLONY OPTIMIZATION

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Keywords

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Abstract

The objective of this paper is to present a practical method for the optimal design of a continuous footing subjected to vertical and horizontal loads. The design problem of finding the optimal size of footing as well as the minimum steel reinforcement is formulated in a nonlinear minimization form. The continuous footing is subjected to the vertical and horizontal loads acting on the top of the column. There are four design variables in the design problem, i.e., the width of the footing, the thickness of the footing, the soil-embedment depth, and the amount of steel reinforcement. The required geotechnical constraints include the bearing capacity, overturning, as well as global sliding and local sliding at the footing corners. Short-term stability and long-term stability are considered simultaneously in the same formulation. The structural constraints are enforced to control the shear force and bending moment within the section resistance. The formulation of the problem's constraints leads to the nonlinear programming, whose objective function is to minimize the total cost of the footing material, including the concrete and steel reinforcement. The optimal solution is solved using the ant-colony optimization algorithm MIDACO. The proposed optimization method is demonstrated through the actual design of the footing for supporting a large machine moving on rails.

1 INTRODUCTION

A shallow foundation is generally used to support a structure when the underlying soil has a relatively high shear strength. The conventional design of a shallow foundation subjected to a vertical loading is an iterative process that considers geotechnical analysis and structural reinforced-concrete design separately. The dimension of the footing must be assumed initially such that a geotechnical analysis of the bearing capacity is evaluated. Once the geotechnical considerations have been satisfied, the design of a structurally reinforced-concrete footing is then carried out. This process is repeated in order to determine the optimal dimensions of the footing as well as the amount of steel reinforcement. Thus, the conventional process for the optimal design of a footing is iterative and practical.

This paper studies the optimal design of a continuous footing subjected to vertical and horizontal loads. This foundation is used to support a large machine, such as a stacker or a reclaimer used in a bulk-material handling process, as shown in Fig. 1 [1, 2]. These machines move slowly on a rail on top of strip footing in order to pile up the bulk material as a stockpile (stacker, Fig. 1(a)) or to recover the material (reclaimer, Fig. 1(b)) from a stockpile. Bulk materials include coal, limestone, ores, etc.



(a) stacker machine [1]



(b) reclaimer machine [2]

Figure 1. Large machine used in bulk-material handling process.

The optimal design of a continuous footing supporting these machines is complex because it is involved with checking the safety factors of several geotechnical criteria. The horizontal load adds more complexities and more stability evaluation, in addition to the standard bearing capacity for the vertical load case. The geotechnical analyses must evaluate additional failure mechanisms, namely, overturning, global sliding, and local sliding at the edge or corner of the footing. In addition, the short-term stability and long-term stability of these failure mechanisms must be considered in the analysis.

The techniques of optimization have been applied to many problems in geotechnical engineering [3, 4, 5, 6, 7]. Various previous studies have mainly focused on the optimal design of retaining structures and some optimization methods were proposed and developed. Early research of the optimization of a retaining wall was pioneered by Rhomberg and Street [8]. Saribas and Erbatur [9] presented a detailed study of the optimum design for reinforced-concrete cantilever retaining walls with seven geometrical and reinforcement design vari-

ables, where the constrained nonlinear programming was solved by a specially prepared program. Ten modes of wall failure, including overturning, sliding, eccentricity, bearing capacity, shear and the bending moment of the toe slab, heel slab and stem of the wall were considered. A similar technique for the optimization of a retaining wall was studied by Basudkar and Lakshman [10]. Alshawi et al. [11] applied the optimization method to a tie-back retaining wall.

Ceranic et al. [12] studied the application of a simulated annealing algorithm to a problem with only geometrical design variables. Castillo et al. [13] and Babu and Basha [14] presented an approach for a reliability-based design optimization of a reinforced-concrete cantilever retaining wall, where the analysis was performed by treating the input parameters as random variables. Khajehzadeh et al. [15] presented the effectiveness of the particle-swarm optimization with a passive congregation algorithm to the economic design of a retaining wall, where the problem consisted of eight geometrical and reinforcement design variables and the constraints were the same as those of Babu and Basha [14]. The ant-colony optimization method was proposed by Ghazavi and Bonab [16] to determine the optimal design of a reinforced concrete retaining wall. Camp and Akin [17] employed a numerically simple optimization algorithm, the big-bang/big-crunch optimization for designing low-cost or low-weight cantilever reinforced-concrete retaining walls with base shear keys. A numerical model was proposed by Pourbaba et al. [18] to obtain the optimum cost of cantilever retaining walls having different cases of backfill, where the optimal solution of the economical sections was determined by the chaotic imperialist competitive algorithm, minimizing the cost of the sections. Papazafeiropoulos et al. [19] employed two-dimensional finite-element simulations together with the genetic algorithm to find the optimum design of cantilever reinforced-concrete retaining walls with considerations for earthquake loading, where the linear elastic soil, retaining wall stem and wall foundation were assumed to calculate the seismic earth pressures. The optimum design of gravity and reinforced retaining walls using the enhanced charged system search algorithm, the recently developed meta-heuristic algorithm, was introduced by Talatahari and Sheikholeslami [20] to obtain the least-cost sections with different cases of backfill. Very recently, Sadoglu [21] proposed an optimization technique for the optimal design of a symmetrical gravity retaining wall. It is clear that most of the previous investigations have heavily focused on the optimal design of cantilever or gravity retaining walls where different optimization algorithms were proposed to solve the formulated optimization problems.

Several researches on the optimal design of foundations were conducted in the past [22, 23, 24 and 25]. Some studies focused on optimal design with a finite-element analysis [22], while the remaining focused on its mathematical formulation and derivation [23, 24, and 25]. The optimization technique was also applied to a steel pile group foundation [26]. Very few researches have studied the practical application of optimization for a strip footing. Moreover, none of research has considered short-term and long-term geotechnical conditions in the same optimization. These are the main contributions of the proposed practical method in this study.

In this paper, a practical method for the optimal design of a continuous footing subjected to vertical and horizontal loads is proposed. The geotechnical and structural constraints are enforced in order to setup a feasible region of the decision variable, including the footing width, the footing thickness, the soil-embedment depth, and the main steel reinforcement. The short-term and long-term stability are considered simultaneously in the same numerical optimization. The proposed nonlinear programming problem is solved using a state-of-the-art, ant-colony optimization algorithm, MIDACO [27, 28, 29, 30]. The proposed practical method of numerical optimization is applied to determine the optimal design of a continuous footing that supports a very large machine moving slowly on rail such as stacker or reclaimer, where they are commonly used in the stockpile.

2. PROBLEM FORMULATION

Fig. 2 shows the problem geometry for determining the optimal dimension of a continuous footing subjected to the vertical load (P) and the horizontal load (H). In this analysis, the global variable (X) consists of four design variables as:

$$X = (x_1, x_2, x_3, x_4) \quad (1)$$

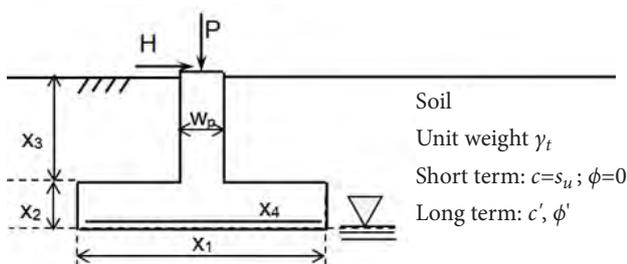


Figure 2. Problem geometry and design variables.

where

- x_1 = Footing width (m)
- x_2 = Footing thickness (m)
- x_3 = Soil-embedment depth (m)
- x_4 = Cross-sectional area of the main steel reinforcement (m^2/m)

Those four design variables represent the maximum unknowns of the continuous footing that can be optimized in the design. It should be noted that the width of the pedestal wall (W_p) is not considered as a design variable, but constitutes one of the input parameters because its size is controlled by the superstructure design.

2.1 Geotechnical constraints

Since this footing is subjected to both the vertical and horizontal loads, such a condition produces equivalently the inclined and eccentric loading, as shown in Fig. 3. The footing has the load eccentricity (e) measured from the centreline and the load inclination (α) with respect to the vertical line. The vertical component (Q_v) of the total load of the footing (Q) is the sum of the applied vertical load, the weight of the concrete footing, and the weight of the soil embedment. These expressions can be written in terms of the design variables as:

$$Q_v = P + \gamma_c x_1 x_2 + \gamma_t W_p x_3 + \gamma_t (x_1 - W_p) x_3 \quad (2)$$

$$e = H(x_2 + x_3) / Q_v \quad (3)$$

$$\alpha = \arctan(H / Q_v) \quad (4)$$

where

- P = The applied vertical load at the centre of the pedestal wall
- H = The applied horizontal force at the centre of the pedestal wall
- γ_c = Unit weight of concrete
- γ_t = Total unit weight of back-filled soil

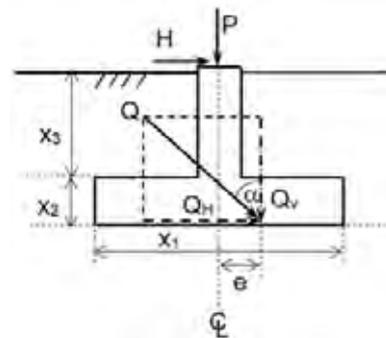


Figure 3. Equivalent problem of an inclined and eccentrically loaded strip footing.

It should be noted that the eccentricity distance e is calculated straightforwardly by taking the moment equilibrium at the centre of the footing base.

In this analysis it is assumed that the properties of the back-filled soil above the base of the footing are the same as those of the underlying soil. The net resulting inclined and eccentric load of the footing gives rise to the non-uniform, applied pressure underneath the footing base, as shown in Fig. 4. The maximum pressure (q_{max}) and the minimum pressure (q_{min}) at the footing corners can be calculated based on the vertical force equilibrium as:

$$q_{max} = \frac{Q_v}{x_1} \left(1 + \frac{6e}{x_1}\right) \quad (5)$$

$$q_{min} = \frac{Q_v}{x_1} \left(1 - \frac{6e}{x_1}\right) \quad (6)$$

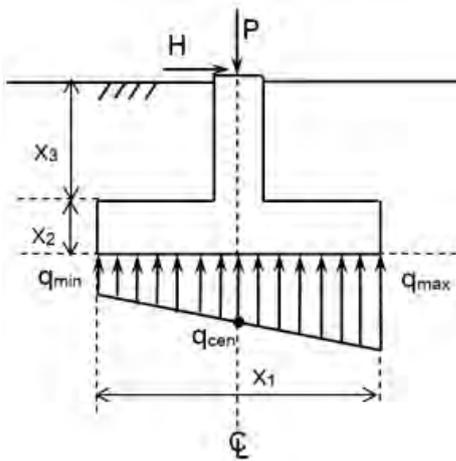


Figure 4. Pressure distribution under the footing.

Since the considered foundation is the strip footing, it is not necessary to include the correction factors for the footing shape. However, the depth factors (F_{cd} , F_{qd} , $F_{\gamma d}$), the inclination factors (F_{ci} , F_{qi} , $F_{\gamma i}$) and the effective width concept must be applied for calculating the ultimate bearing capacity (q_{ult}) of the footing. These correction factors together with the standard bearing-capacity factors (N_c , N_q , N_γ) can be found in most standard foundation textbooks [31, 32, 33]. The short- ($q_{ult,s}$) and long-term ($q_{ult,l}$) ultimate bearing capacity of this continuous footing can be approximated using Terzaghi's general bearing-capacity equation as:

$$q_{ult,s} = s_u N_c F_{cd} F_{ci} + q N_q F_{qd} F_{qi} + 0.5 B' \gamma N_\gamma F_{\gamma d} F_{\gamma i} \quad (7)$$

$$q_{ult,l} = c' N_c F_{cd} F_{ci} + q' N_q F_{qd} F_{qi} + 0.5 B' \gamma' N_\gamma F_{\gamma d} F_{\gamma i} \quad (8)$$

where

- s_u = Undrained shear strength of soil
- c' = Effective cohesion of soil
- ϕ' = Effective friction angle of soil
- γ' = Buoyant unit weight of soil = $\gamma_t - \gamma_w$
- γ_w = Unit weight of water
- q = Total surcharge = $\gamma_t(x_2 + x_3)$
- q' = Effective surcharge = $\gamma'(x_2 + x_3)$
- B' = Effective footing width = $x_1 - 2e$

For the short-term condition or the total stress analysis, the $\phi = 0$ concept is applied in equation (7). For the long-term condition or the effective stress analysis, the effective cohesion and the effective soil friction angle are substituted in equation (8). Even though the ground-water table is assumed to locate at the base of the footing, the effective unit weight is used to calculate the effective surcharge due to the effect of the perched water table.

The corresponding ultimate load per unit length for the short- ($Q_{ult,s}$) and the long-term ($Q_{ult,l}$) conditions of the continuous footing are calculated from the effective width of the footing as:

$$Q_{ult,s} = q_{ult,s} B' \quad (9)$$

$$Q_{ult,l} = q_{ult,l} B' \quad (10)$$

The factor of safety against a bearing-capacity failure defined in terms of stress for the short-term ($FS_{bs,s}$) and long-term ($FS_{bs,l}$) conditions are given as:

$$FS_{bs,s} = q_{ult,s} / q_{max} \quad (11)$$

$$FS_{bs,l} = q_{ult,l} / q_{max} \quad (12)$$

The geotechnical criterion requires that the safety factor against a bearing-capacity failure must be equal to or greater than the required value. In addition, the minimum applied stress must be compression or greater than zero in order not to cause tensile stress to the underlying soil. These two criteria can be written for the short-term and long-term constraints as:

$$G_1(X) = FS_{bs,s} - FS_{b,r} \geq 0 \quad (13)$$

$$G_2(X) = FS_{bs,l} - FS_{b,r} \geq 0 \quad (14)$$

$$G_3(X) = q_{min} \geq 0 \quad (15)$$

where $FS_{b,r}$ = Required safety factor for a bearing-capacity failure.

In addition to enforcing the safety factor against a bearing-capacity failure as a function of stress, it is advisable

to enforce this term as a function of force. The factors of safety against a bearing-capacity failure defined in terms of force for the short-term ($FS_{bf,s}$) and long-term conditions ($FS_{bf,l}$) are given as:

$$FS_{bf,s} = Q_{ult,s} / Q \quad (16)$$

$$FS_{bf,l} = Q_{ult,l} / Q \quad (17)$$

Thus, those safety factors for a bearing-capacity failure in terms of force must be equal to or greater than the required value as:

$$G_4(X) = FS_{bf,s} - FS_{b,r} \geq 0 \quad (18)$$

$$G_5(X) = FS_{bf,l} - FS_{b,r} \geq 0 \quad (19)$$

It should be noted that expressions (13), (14) and (18), (19) can produce the same result in the case that there is no horizontal load acting on the top of the footing. In other words, it corresponds to the case of the concentric load without eccentricity, which is in contrast to the strip footing considered here.

The next geotechnical consideration is the overturning stability of the footing since it is applied using the horizontal load. The driving moment (M_d) about the right corner of the footing for the overturning mechanism and the resisting moment (M_r) due to the self-weight of concrete, the soil embedment and the applied total vertical force are given as:

$$M_r = Q_v X_1 / 2 \quad (20)$$

$$M_d = H(x_2 + x_3) \quad (21)$$

The safety factor against the overturning mechanism (FS_{ov}) is defined in terms of the ratio of the resisting moment to the driving moment as:

$$FS_{ov} = M_r / M_d \quad (22)$$

The geotechnical criterion requires that the safety factor against the overturning mechanism (FS_{ov}) must be equal to or greater than the required value ($FS_{ov,r}$) as:

$$G_6(X) = FS_{ov} - FS_{ov,r} \geq 0 \quad (23)$$

The last geotechnical criterion is the sliding stability of the footing along its base. In this case, the passive earth force of the soil embedment is neglected for the reason of conservatism. The resisting sliding force for the short-term ($F_{r,s}$) and the long-term ($F_{r,l}$) conditions are given as:

$$F_{r,s} = s_{ui} B' \quad (24)$$

$$F_{r,l} = c'_i B' + Q_v \tan(\delta') \quad (25)$$

where

s_{ui} = Interface undrained shear strength between concrete and soil = $0.5s_u$

c'_i = Interface effective cohesion between concrete and soil = $0.5c'$

δ' = Interface friction between concrete and soil = $0.5\phi'$

The sliding failure mechanism is caused by the applied horizontal force. Thus, the factors of safety against the global sliding for the short-term ($FS_{gs,s}$) and long-term ($FS_{gs,l}$) conditions are defined as:

$$FS_{gs,s} = F_{r,s} / H \quad (26)$$

$$FS_{gs,l} = F_{r,l} / H \quad (27)$$

The geotechnical criterion requires that the safety factor against the global sliding mechanism must be equal to or higher than the required value ($FS_{gs,r}$) for both the short- and long-term conditions as:

$$G_7(X) = FS_{gs,s} \geq FS_{gs,r} \quad (28)$$

$$G_8(X) = FS_{gs,l} \geq FS_{gs,r} \quad (29)$$

In addition, to ensure an adequate safety factor against the global sliding along the footing base, it is advisable to enforce additional constraints of the local sliding at each footing corner. This requirement is important and necessary, particularly for the long-term condition since there is a significant difference in the applied normal stress among the two footing corners, i.e., q_{\max} (corner 1) and q_{\min} (corner 2). As a result, the interface shear resistance at each corner is different, which results in a possible progressive local sliding failure. On the other hand, the short-term condition gives rise to the interface shear resistance, which is independent of those applied normal stresses because of the $\phi = 0$ concept. Thus, the short-term stability of the global sliding gives the same constraint as that of the local sliding. Thus, there is no need to enforce the constraint of the short-term stability of the local sliding.

For a generality of the formulation, the local sliding is enforced for both the short-term and long-term conditions. The interface shear resistance at each corner of the footing for both the short-term condition (corner 1, $\tau_{i1,s}$; corner 2, $\tau_{i2,s}$) and the long-term condition (corner 1, $\tau_{i1,l}$; corner 2, $\tau_{i2,l}$) can be calculated as:

$$\tau_{i1,s} = \tau_{i2,s} = s_{ui} \quad (30)$$

$$\tau_{i1,l} = c'_i + q_{\max} \tan(\delta') \quad (31)$$

$$\tau_{i2,l} = c'_i + q_{\min} \tan(\delta') \quad (32)$$

Based on the effective width concept of the footing, the average applied shear stress (τ_{avg}) for each footing corner is given as:

$$\tau_{avg} = H / B' \quad (33)$$

The ratio of the interface shear resistance to the applied average shear stress at each footing corner defines the safety factor against the local sliding failure for the short-term condition (corner 1 $FS_{ls1,s}$; corner 2 $FS_{ls2,s}$) and the long-term condition (corner 1 $FS_{ls1,l}$; corner 2 $FS_{ls2,l}$). These expressions are given as:

$$FS_{ls1,s} = \tau_{i1,s} / \tau_{avg} \quad (34)$$

$$FS_{ls2,s} = \tau_{i2,s} / \tau_{avg} \quad (35)$$

$$FS_{ls1,l} = \tau_{i1,l} / \tau_{avg} \quad (36)$$

$$FS_{ls2,l} = \tau_{i2,l} / \tau_{avg} \quad (37)$$

The geotechnical criterion requires that the safety factor against the local sliding failure must be equal to or greater than that of the required value ($FS_{ls,r}$) as:

$$G_9(X) = FS_{ls1,s} \geq FS_{ls,r} \quad (38)$$

$$G_{10}(X) = FS_{ls2,s} \geq FS_{ls,r} \quad (39)$$

$$G_{11}(X) = FS_{ls1,l} \geq FS_{ls,r} \quad (40)$$

$$G_{12}(X) = FS_{ls2,l} \geq FS_{ls,r} \quad (41)$$

2.2 Structural constraints of reinforced concrete

Fig. 5 shows the shear-force and bending-moment diagrams along the base of the footing generated from a linear distribution of pressure under the footing. For convenience, by neglecting the reduction effect from the concrete weight and the weight of the back-filled soil, the applied shear force (V_{cen}) and the bending moment (M_{cen}) calculated from the overturning side at the centre of base footing are given as:

$$V_{cen} = (q_{max} + q_{cen})x_1 / 4 \quad (42)$$

$$M_{cen} = q_{cen} x_1^2 / 8 + (q_{max} + q_{cen}) / 12 \quad (43)$$

where $q_{cen} = 0.5(q_{max} + q_{min})$.

It should be noted that the shear force and the bending moment at the centre of the footing are slightly higher than those at the critical sections given by the design code [34]. According to the ACI code [34], there are two critical sections of the shear force: 1) the beam shear type; and 2) the punching shear type. The former and the latter happen at the distances $t = W_p/2 \pm d$ (V_b^+ and

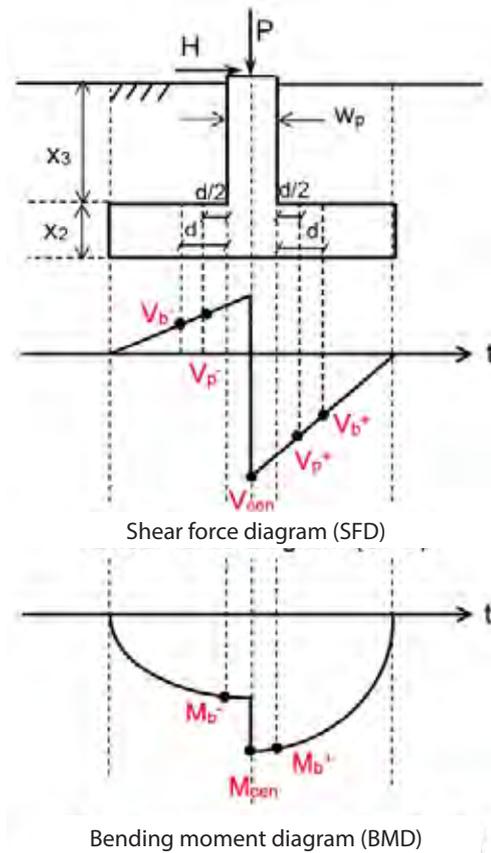


Figure 5. Shear-force and bending-moment diagrams along the base of the footing.

V_b^-) and $t = W_p/2 \pm d/2$ (V_p^+ and V_p^-) measured from the centre of the footing, where d is the effective depth of the base of the footing. The critical section of the bending moment happens at the pedestal edge or the distance $t = \pm W_p/2$ (M_b^+ and M_b^-) measured from the footing centre. Instead of using the exact calculation given by the design code, this analysis adopts the approximated values of the shear force (V_{cen}) and the bending moment (M_{cen}) at the centre of the footing shown in the expressions (42) and (43) as the substitutes for the values at the critical sections. If the calculations of the shear force and the moment follow the code, four constraints are needed to enforce the shear force and the moment on the left- and right-hand sides at the critical sections. However, since the modified calculations for the shear force and the moment at the centre give rise to the largest values, only two constraints are required. Thus, the use of maximum values for the shear force and the moment at the centre are adopted for reasons of convenience and conservatism.

In this paper, the classical design code of the working stress method (e.g., Ricketts et al. [35]) is used to evaluate the allowable shear resistance and the bending

moment of the reinforced concrete footing. Based on this method, the allowable shear force of the concrete footing can be calculated as:

$$V_c = 29\sqrt{f'_c/100}bd \quad (44)$$

where

f'_c = Unconfined compressive strength of concrete (kPa)

b = 1 unit length of footing (m)

d = Effective depth of slab footing = $x_3 - c_v$ (m)

c_v = Effective concrete covering (m)

The allowable moment resistances of the reinforced concrete footing calculated from the concrete (M_c) and the steel reinforcement (M_s) are defined as:

$$M_c = Rbd^2 \quad (45)$$

$$M_s = f_s j d x_4 \quad (46)$$

where

$R = 0.5f_c k j$

$f_s = 0.5f_y \leq 170000$ kPa

f_y = Tensile strength of steel (kPa)

$j = 1 - k/3$

$k = 1/(1 + f_s/(n f_c))$

$f_c = 0.45 f'_c$

$n = E_s/E_c$

E_s = Young's modulus of steel = 2.04×10^8 kPa

E_c = Young's modulus of concrete (kPa) =

$$1521000\sqrt{f'_c/100}$$

The first structural concrete criterion requires that the allowable shear force (V_c) of concrete must be equal to or greater than the approximated shear force at the critical section (V_{cen}). Moreover, the other criterion requires that the allowable moment resistance from the steel (M_s) must be equal to or greater than the approximated bending moment at the critical section (M_{cen}). These two criteria can be written as:

$$G_{13}(X) = V_c - V_{cen} \geq 0 \quad (47)$$

$$G_{14}(X) = M_s - M_{cen} \geq 0 \quad (48)$$

In order to design a single steel reinforcement at the bottom face of the footing base, it is necessary to enforce an additional constraint of the approximated bending moment at the critical section (M_{cen}) to be equal to or smaller than the allowable moment of the concrete (M_c) as:

$$G_{15}(X) = M_c - M_{cen} \geq 0 \quad (49)$$

Finally, in addition to the structural constraints of the reinforced concrete criteria, it is necessary to specify the maximum and minimum allowable limits of the

design variables since most optimization solvers require their searching ranges. The range of footing width, x_1 , is defined as $x_1 = 0.25-3.0$ m. The footing thickness, x_2 , and the soil embedment depth, x_3 , are in the range $0.5-2.0$ m. Finally, $0.2-5.0\%$ of the total area of the footing base is used for the main steel reinforcement, x_4 . These maximum and minimum limits are converted to the inequality constraints as follows:

$$G_{16}(X) = x_1 - 0.25 \geq 0 \quad (50)$$

$$G_{17}(X) = 3 - x_1 \geq 0 \quad (51)$$

$$G_{18}(X) = x_2 - 0.5 \geq 0 \quad (52)$$

$$G_{19}(X) = 2 - x_2 \geq 0 \quad (53)$$

$$G_{20}(X) = x_3 - 0.5 \geq 0 \quad (54)$$

$$G_{21}(X) = 2 - x_3 \geq 0 \quad (55)$$

$$G_{22}(X) = x_4 - 0.002bx_2 \geq 0 \quad (56)$$

$$G_{23}(X) = 0.05bx_2 - x_4 \geq 0 \quad (57)$$

2.3 Objective function and optimization form

The objective function ($F(X)$) of the proposed optimization problem for a continuous footing is to minimize the total cost of the material in the strip footing, including the price of the concrete for the entire footing and the price of the main steel reinforcement of the footing slab. This objective function can be written as:

$$\text{Minimize } F(X) = \text{Minimize } (u_c(x_1x_2 + W_p x_3) + u_s x_1 x_4 \gamma_s) \quad (58)$$

where

γ_s = Unit weight of steel

u_c = Unit price of concrete per unit volume (i.e., US\$/m³)

u_s = Unit price of steel reinforcement per unit weight (i.e., US\$/kN)

The resulting numerical optimization of the dimension and reinforcement for a continuous footing leads to the constrained nonlinear programming, which has the form:

$$\text{Minimize } F(X) \quad (59)$$

$$\text{Subject to: } G_i(X) \geq 0, \quad i = 1..23$$

It should be noted that both the objective function and the geotechnical and structural constraints are nonlinear in terms of four unknown design variables and there are 18 nonlinear constants in this optimization.

There are two classes of optimization algorithm that can be used to solve the formulated optimization problem shown in expression (59). The first class is to apply the gradient-based algorithm of the optimization (e.g., NLPsolve [36], FindMinimum [37], fmincon [38], and Knitro [39]), while the other is to employ the free-derivative type of optimization (e.g., MIDACO [27, 28, 29, 30], ga [40], Nminimize [41], and Particle-swarm optimization [42, 43]). The gradient-based approach to optimization has the advantages such that it can efficiently determine the optimal solution with a rapid speed with the help of the first and second derivatives of the objective function and the constraints. Several algorithms of the gradient-based optimization approach can be found in Venkataraman [44]. On the other hand, the gradient-based technique of the optimization suffers a major drawback of being the local optimization, where its searching can be trapped into a local optimal solution, not the global optimal solution. Thus, several trials of other values of the decision variables must be performed in order to determine the global optimal solution.

Instead of using the classical technique of local optimization, which requires a first- and second-derivatives approach and changes to the initial values of the variables, the optimal solution of the formulated optimization can also be solved using the technique of global optimization, such as evolutionary algorithms (Genetic algorithm [40, 45], Differential evolution [41]), swarm-based optimization algorithms (Particle-swarm optimization [42, 43], Ant-colony optimization [27, 28, 29, 30]). In this paper, the proposed optimization problem is coded in MATLAB and the optimal solution of the proposed formulation is solved using a state-of-the-art solver, MIDACO [27, 28, 29, 30].

MIDACO is an extended ant-colony optimization that is one of the swarm-based optimization algorithms. A distinct feature of this solver is that it employs an evolutionary metaheuristic search strategy to determine the global optimal solution from the search space in an intelligent and efficient way, as if ants seek the best path between their colony and a source of food. The search space is generated from the multi-kernel Gaussian probability density function. In addition, MIDACO is a self-adaptive algorithm to automatically determine the global optimal solution rather a local optimal solution. The major advantage of MIDACO is that there is no need to change the initial value of the decision variables by the users. Furthermore, the algorithm does not require the property of differentiability of the first or second derivatives for the nonlinear objective function or nonlinear equality or inequality constraints. Since MIDACO is a global optimization algorithm, it ensures that the computed solution from this software corresponds to the

global optimal solution of the continuous footing that is subjected to the vertical and horizontal loads. Details of this solver are not within the scope of this study, but can be found in [27, 28, 29, 30].

All the analyses of the optimal design for a continuous footing are carried out on a personal computer, Windows 7 operating system, Intel Core I7-4770 CPU, @ 3.40 GHz and 8 GB memory.

3 RESULTS AND DISCUSSIONS

Table 1 lists all the input parameters used for demonstrating the application of the proposed optimization method in practice. Those parameters represent the actual conditions of the strip footing design. This continuous footing is used to support the stacker machine moving on the rail on top of the strip footing. Based on the results of triaxial testing, the soil is classified as a hard clay, whose total stress parameters are $s_u = 150$ kPa and the effective soil parameters are $c' = 6$ kPa and $\phi' = 30^\circ$. The slow-moving stacker generates the static applied vertical and horizontal loads as $P = 400$ kN/m and $H = 40$ kN/m. The unit prices of the concrete and steel reinforcements are $u_c = 83.33$ US\$/m³ and $u_s = 733.33$ US\$/ton (metric), based on the average unit costs in Thailand.

Table 1. Input parameters for the optimal design of the continuous footing.

Input parameters	Value
Applied vertical load, P (kN/m)	400
Applied horizontal load, H (kN/m)	40
Width of pedestal, W_p (m)	0.7
Total unit weight of soil, γt (kN/m ³)	20.0
Undrained shear strength of soil, s_u (kPa)	150
Effective cohesion, c' (kPa)	6.0
Effective friction angle, ϕ'	30°
Unit weight of concrete, γ_c (kN/m ³)	24
Unconfined compressive strength of concrete, f'_c (kPa)	28000
Effective concrete covering, c_v (m)	0.09
Unit weight of steel, γ_s (kN/m ³)	78.5
Tensile strength of steel, f_y (kPa)	400000
Unit price of concrete, u_c (US\$/m ³)	83.33
Unit price of steel reinforcement, u_s (US\$/ton)	733.33
Required safety factor for bearing, $FS_{b,r}$	3.0
Required safety factor for overturning, $FS_{ov,r}$	2.5
Required safety factor for global sliding, $FS_{gs,r}$	2.5
Required safety factor for local sliding, $FS_{ls,r}$	1.5

Table 2. The optimal solution of the continuous footing.

Design variables	Value
Footing width, x_1 (m)	2.839
Footing thickness, x_2 (m)	0.886
Soil embedment depth, x_3 (m)	0.5
Cross-sectional area of reinforcement, x_4 (m ² /m)	17.724×10^{-4}

The proposed optimization presented in Section 2 is applied to determine the optimal solution of this continuous footing. In this analysis, the nonlinear minimization problem is programmed in MATLAB and solved by MIDACO [27, 28, 29, 30] using the MATLAB toolbox. It should be noted that there is no need to try several initial solutions in order to ensure that the obtained optimal solution is the global minimum since MIDACO is the global optimization algorithm. Table 2 summarizes the global optimal solution of this actual case study using MIDACO. This solver handles the objective function and constraints as a black-box function or library. The constraints must be converted into the standard form as $G_i(X) \geq 0$ or $G_i(X) = 0$. The user needs to provide a function call to the optimization problem, i.e., objective function and constraints, which evaluates the objective function $F(X)$ and the constraints $G_i(X)$ for a given design variable X . The MIDACO solver does not require the user to determine the explicit form of the constraints in terms of the design variables $x_1, x_2 \dots x_n$. Several local variables in a function call can be used to store the values of some expressions, which are functions of the design variables. Then, the function calls return the computed value of the objective function and all the constraints back to MIDACO.

The stopping criterion in MIDACO was setup to find the minimum cost of the function for a period of 5 minutes during the optimization. This timing was sufficient to find the global optimal solution of the selected problem consisting of four design variables. Within this timing, the cost of the function converged to the lowest value during the running and was verified by manual checking.

A detailed result of the analysis is examined to verify which constraints are active and control the design. Table 3 lists each condition for all the required geotechnical and structural constraints. The active constraint [$G_i(x) = 0$] means that the expression produces the equality sign, while the inactive constraint [$G_i(x) > 0$] produces the inequality sign. The controlled design conditions or the active constraints are listed as follows:

- 1) Long-term safety factor for the bearing-capacity failure based on the stress calculated using $FS_{bs,l}$ [$G_2(x) = 0$]

Table 3. Results of each design constraint.

Design constraint	Value
Short-term safety factor for bearing-capacity stress, $FS_{bs,s}$ [$G_1(x) > 0$]	3.95
Long-term safety factor for bearing-capacity stress, $FS_{bs,l}$ [$G_2(x) > 0$]	3.00
Minimum applied pressure, σ_{min} (kPa) [$G_3(x) > 0$]	136.4
Short-term safety factor for bearing force, $FS_{bf,s}$ [$G_4(x) > 0$]	4.47
Long-term safety factor for bearing force, $FS_{bf,l}$ [$G_5(x) > 0$]	3.39
Safety factor for overturning, FS_{ov} [$G_6(x) > 0$]	13.54
Short-term safety factor for global sliding, FS_{gfs} [$G_7(x) > 0$]	4.75
Long-term safety factor for global sliding, FS_{gl} [$G_8(x) > 0$]	3.39
Short-term safety factor for local sliding corner 1, $FS_{ls1,s}$ [$G_9(x) > 0$]	4.75
Short-term safety factor for local sliding corner 2, $FS_{ls2,s}$ [$G_{10}(x) > 0$]	4.75
Long-term safety factor for local sliding corner 1, $FS_{ls1,l}$ [$G_{11}(x) > 0$]	3.82
Long-term safety factor for local sliding corner 2, $FS_{ls2,l}$ [$G_{12}(x) > 0$]	2.50
Allowable shear force of concrete, V_c (kN/m) [$G_{13}(x) = 0$]	329
Allowable bending moment of steel, M_s (kNm/m) [$G_{14}(x) = 0$]	207
Allowable bending moment of concrete section M_c (kNm/m) [$G_{15}(x) > 0$]	948
Minimum area of steel reinforcement (cm ² /m) [$G_{16}(x) > 0$]	15.377
Minimum thickness of footing (m) [$G_{17}(x) > 0$]	0.25
Minimum soil embedment depth (m) [$G_{18}(x) = 0$]	0.5

- 2) Allowable shear force of concrete, V_c [$G_{13}(x) = 0$]
- 3) Allowable bending moment of steel, M_s [$G_{14}(x) = 0$]
- 4) Minimum soil embedment depth, [$G_{18}(x) = 0$].

It is clear that there are four active constraints, which are equal to the numbers of design variables. However, the results of these four active constraints do not imply that they are always active for other cases. Similarly, the remaining inactive constraints may not always be irrelevant to the design. In general, the controlled design constraints or the active expressions depend on the relative magnitude of the vertical and horizontal loads, the soil parameters of the short- and long-term conditions, and the required safety factors for each failure mechanism. For example, the constraint of the compressive stress for the minimum applied pressure or the global sliding may become active and control the design for some cases of the input parameters of the loading, the soil parameters, the required safety factor, and the structured parameters.

4 MAJOR LIMITATION OF THE PROPOSED OPTIMAL DESIGN FOR CONTINUOUS FOOTINGS

It should be noted that the proposed optimal design for continuous footings has a major limitation, such that the single steel reinforcements or only the bottom bars are used in the footings. In general, continuous footings with double steel reinforcements may be the most economical scenario, instead of a single reinforcement. To achieve this design consideration, another new design variable, x_5 , the cross-sectional area of the top reinforcements, must be introduced into the proposed optimization problem. In addition, the objective function must be modified to include the cost of the top bar reinforcements. Additional constraints of the reinforced concrete section with a double reinforcement must also be added to the formulated problem, together with the upper and lower limits of this new variable (i.e., $x_5 = 0$ –10% of the total area of the footing base). As a result, this revised optimization problem is the most economical formulation that determines the optimal design consideration, including both single and double reinforcements of the continuous footings. For example, if the optimal solution yields the result such that $x_5 = 0$, this solution corresponds to the single steel reinforcement. On the other hand, if the optimal solution turns out to be $x_5 \neq 0$, this solution corresponds to the double steel reinforcements.

5. CONCLUSIONS

This paper presents a numerical technique for optimizing the dimensions and reinforcement of a continuous footing subjected to vertical and horizontal loads. The mathematical formulations lead to nonlinear programming, whose objective function is to determine the minimum cost of the reinforced concrete footings, including concrete and steel reinforcements. The analysis considers four design variables, i.e., the footing width, the footing thickness, the footing embedment, and the steel reinforcement. The continuous footing subjected to the vertical and horizontal loads generates the complex geotechnical and structural constraints in terms of the design variables, totalling 18 nonlinear constraints. The major advantage of the proposed method is that all the required constraints of the geotechnical and structural designs for the short-term and long-term stability considerations are optimized simultaneously. As a result, the final analysis yields the most optimal dimension of the foundation and the required reinforcements for both types of stability consideration. The proposed method is more efficient and convenient than most conventional footing designs where the short-term and long-term analyses are looked at sepa-

ately. Moreover, a classical or conventional design may have to initially assume some design variables, such as the footing thickness or the footing width. Therefore, it is not always guaranteed that the assumed footing dimensions and reinforcement are the most optimal design.

The formulated optimization problem for the continuous footing is solved by the state-of-the-art ant-colony optimization solver, MIDACO, which is a global optimization algorithm and does not require any derivative of the objective function and the constraints, and does not require changing the initial values of the decision variables. The proposed method is applied to design the continuous foundation of an actual stacker machine used in bulk-material handling applications. The numerical optimization by MIDACO makes it possible to efficiently analyse 18 complex nonlinear constraints from the geotechnical and structural criteria, resulting in the global optimal solution in a single analysis for both the short-term and long-term conditions.

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