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Should the History of Chinese Mathematics be Rewritten?[1]

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Abstract

More than one and a half decades have elapsed since the Zhangjiashan archaeological findings showed excitingly that a new collection of mathematical texts on bamboo strips were excavated, dating earlier than the then earliest known and most influential Chinese mathematical classic *Jiu Zhang Suan Shu*. Now that the contents of the findings have finally been made available to the world outside of the group of archaeologists involved, should the history of Chinese mathematics be rewritten? After comparing the new material with the *Jiu Zhang Suan Shu* I believe that the place of the *Jiu Zhang Suan Shu* remains an irreplaceable cornerstone in the history of Chinese mathematics; however, the *Jiu Zhang Suan Shu* can no longer be seen as the earliest sources for the mathematical knowledge therein.

Keywords: history of Chinese mathematics, *Suan Shu Shu*, *Jiu Zhang Suan Shu*

1 Introduction

It has by now fairly widely known that there is an East Asian equivalent of Euclid's *Elements*, called *Jiu Zhang Suan Shu* (hereafter *JZSS*) 九章算術 (*Nine Chapters on Mathematics*). Quite different in style from the *Elements*, the *JZSS* served as a model for mathematics in China and, more widely, East Asia, for more than a millennium. Its influence is comparable to that of the *Elements* in the

[1] This text is an edited English translation of an invited lecture delivered at Uppsala University on March 17th, 2003. Some technical details are omitted to fit the majority readers of the journal. I thank Professor S. Kaijser, then chairman of the Swedish Mathematical Society for the invitation, and everybody in the audience who contributed with stimulating questions and discussions. The dinner party afterwards was also memorable. I am grateful to Professor K. Carlitz for her valuable help in editing the text.

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Western world. As one of the ten mathematical classics compiled in the Tang Dynasty (Qian 1963), the level of mathematical sophistication exemplified in the *JZSS* is higher than that from other Chinese mathematical classics of later periods.

In the *JZSS*, 246 problems are divided into nine chapters according to the methods applied to solve these problems. After the presentation of one problem or several problems of the same type showing variations, correct answers are offered, in some cases followed by explanations of the solution procedures. The book was used as a mathematical textbook in East Asia for centuries. It was a common practice to learn mathematics by reading this book and writing commentaries on it. Some of the commentaries, predominantly those of Liu Hui have become an integral part of the *JZSS* (263 CE), providing explanation and justification of the rationale and methods used in the book.

Neither the authorship nor the exact date of compilation of the *JZSS* is known. Presumably, generations of scholars contributed to the collection and selection of the problems, classification according to solution procedures, correction of mistakes, and so on. Although it has been generally accepted that the work can be dated from the first century BCE to the first century CE, the exact date has been unsettled (see for example, Guo 1990, Li 1990, Needham 1959, Qian 1964, etc.). I believe that close to the end of the first century CE is a reliable dating for the completion of the work (Ma 1996).

In spite of this uncertainty, the *JZSS* was long regarded as the earliest work specifically devoted to mathematics that has survived from China. However, the situation changed due to archaeological findings from central China where a new collection of mathematical texts on bamboo strips was unearthed.[2]

The excavation took place at Zhangjiashan near Jiangling County in Hubei province from December 1983 to January 1984. Three tombs catalogued as M247, M249, M258 were unearthed. More than a thousand bamboo strips were found from M247, almost two hundred of them relevant to mathematics. On the back of one of these strips are the three characters *Suan Shu Shu* 筭數書 (to be abbreviated as *SSS* below), which have been used by the archaeologists as the title of this collection and translated into English as a *Book on Arithmetic*.

[2] I thank my teacher Professor Du Shiran 杜石然 who spoke to me about the findings when his book (Li and Du 1963) was being translated into English in the 1980s.

The contents of these bamboo strips were kept within the study group consisting of archaeologists for more than one and a half decades. In the beginning of this century these findings were made available to the outside world (Peng 2000, *Working group* 2000, Peng 2001).[3]

The title *Suan Shu Shu* does not seem to have been mentioned or cited anywhere in historical records. Another document from the same tomb was a calendar which ended in the second year of Empress Lü's reign in the Western Han period, viz. 186 BC. If the occupant of the tomb died that year the texts must have been written before 186 BC.

I chose to translate the title *Suan Shu Shu* 筭數書 literally as *En bok om tal och beräkningar* (*A Book on Numbers and Computations*). The last character 書 as a noun can mean book, script etc. In this case I think it better to use the word "book" for such a large collection.[4]

The first character has two variants 筭 and 祘, meaning to count, to calculate, to compute. Both 筭 and 算 have the stroke denoting "bamboo" on the top, indicating the most common material used for counting rods.[5] The lower part in 算 is a noun indicating a device while the lower part in 筭 is a verb meaning to play with.[6] The character 祘 consists of two 示 indicating a divine meaning.

The most literal meaning of the middle character 數 is "number" as a noun (the fourth falling tone) and "to count" as a verb (the third falling-rising tone), as in the expression shu4 yuan2 yu2 shu3 數源於數 (numbers originated from counting).[7] The first character literally corresponds to "beräkning(ar)" in Swedish; and can mean both calculation(s) or computation(s).

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[3] I am grateful to Professor N. Sivin for sending me the material and for encouraging me to explore these new findings.

[4] For the same character as in *Luoshu* 洛書, I chose to translate it as script, although it is actually a diagram (Ma 1996, Ma 2004). Incidentally, the legendary of He Tu and Luo Shu dating Chinese mathematics to several millennia back can still not be verified.

[5] Indeed Chinese culture can be described as a bamboo culture. We use *yi shi zhu xing* 衣食住行, (clothing, food, living, transport) to refer to the basic necessities of life. Hats and rain capes can be made of bamboo, bamboo shoots can be eaten, houses as well as furniture can be built of bamboo and so are sedans.

[6] As explained in the first Chinese dictionary of the Han Dynasty (Xu 2004: 120): 筭： 長六寸。計歷數者。从竹从弄。言常弄乃不誤也. 算：數也。从竹从具。讀若筭。

[7] The character shu 數 as a noun also has other meanings, for example jiushu 九數 refers to the nine areas of mathematics prior to the formation of the nine chapters in the *JZSS*.

Arithmetic as a modern branch of elementary mathematics is *suanshu* 算術 in Chinese.[8] The same two characters in classical Chinese, however, refer to the whole body of mathematics as in 九章算術 *Jiu Zhang Suan Shu*, in which arithmetic as well as algebra, geometry, etc are dealt with. The character *shu* 術 alone can mean art, technique, or algorithm. I used to translate *JZSS* as *Nine Chapters on the Mathematical Art*, until an occasion when I was preparing for a talk entitled "mathematics as art/science/technique".[9] Indeed different aspects of mathematics can be emphasized using different translations but as far as the mathematics in the *JZSS* is concerned, the most suitable translation seems to me simply "mathematics".

That 算術 as 算數之術 is used in ancient China for mathematics actually reflects the nature of traditional Chinese mathematics which centers on 算 and emphasizes 術. Another word used is *sunxue* 算學. From the Song and Yuan periods or the thirteenth century onwards, *shuxue* 數學 has also been used although at that time, the so-called "internal mathematics" (*neishu* 內數) as well as "external mathematics" (*waishu* 外數) were included in 數學. In 1939, the Chinese Mathematical Society decided to adopt *shuxue* 數學 as the standard term for "mathematics".[10]

2 A Closer Look at Some Examples in the *SSS*

The *SSS* covers a variety of topics. On some strips the text begins with a heading of one to four characters. There are a total of 68 headings and one without any explicit heading (*Workig group* 2000). Mathematical problems are presented in question and answer form, as in *JZSS*. But unlike in the *JZSS*, problems are not

[8] I was a young student when a commented version of *JZSS* (Bai 1983) was published. At a bookshop I looked around but could not find it. A shop assistant suggested politely but mistakenly that I should try the primary school section.

[9] As the first translation into a Western language, the characters *suansh*u 算術 in the title was translated as "Arithmetischer Technik" (Vogel 1968).

[10] Interestingly, 算術（さんじゅつ, *sanjutu*) has also been used in Japan in pre-modern times referring to the whole body of mathematics. Nowadays 算數 (さんすう, *sansū*) is in use meaning "arithmetic", the branch of elementary mathematics, while 數學 (すうがく, *sūgaku*) is used to mean the mathematical science as well as the educational subject from junior high school onward. I thank Professor Kobayashi Tatsuhiko 小林龍彦 for interesting communications and Professor Sasaki Chikara 佐々木力 for further information that 數學 (すうがく) rather than 数理学 was adopted by the Tokyo Mathematical Society 東京数学会社 at a meeting of 訳語会 on 7 January 1881.

classified according to the methods used to solve them. Moreover, there is no explicit formulation of those methods as in the *JZSS*. Some problems do remind us of the similarity to those in the *JZSS*, and so do the headings. However, such cases are very few in number. Problems equivalent to those in the chapters 8 and 9 of the *JZSS* are completely missing in the *SSS*; in other words, no problems dealing with right-angled triangles or systems of linear equations are found in the *SSS*.

Concerning additions of two fractions, *He fen shu* 合分術 is given in the *SSS* as below:

> If the two denominators are (of) the same (category), add the numerators.
>
> If the denominators are not (of) the same (category) but
>
> > one denominator can be twice of the other, double the numerator;
> >
> > one denominator can be three times of the other, three times the numerator;
> >
> > one denominator can be four of the other, four times the numerator;
> >
> > one denominator can be five times of the other, five times the numerator;
> >
> > one denominator can be six times of the other, six times the numerator;
>
> When the denominators are the same, add the numerators.
>
> If they are not (of) the same (category), multiply the denominators as divisor, cross-multiply the numerators with the denominators, combine (i.e. add) them to be the dividend.

Here we can see that the description is more detailed than that given in the first chapter of the *JZSS*.

The title of the first chapter in the *JZSS* is *Fang Tian* 方田, which is often translated as *Rectangular Field*. Since that chapter deals with finding areas of various geometric figures in terms of fields, I thought "squaring fields" would be an ideal translation (Ma 1996), using "square" as a verb. Here in *SSS*, interestingly, this problem under the heading *Fang Tian* in the *SSS* is not about finding the square but rather the inverse: to find the side given the area of a square

(Ma 2004). It seems to me, therefore, that is more suitable to translate the heading as “square field” where square is used as an adjective.

This type of problems in *JZSS* belongs to Chapter four and is solved by root extraction. In *SSS*, however, it is solved by the method of excess and deficit, which is systematically dealt with in the seventh chapter of the *JZSS* (Ma 1993):

> [In] a field whose square is 1 *mu*, how many *bu* is a side?
>
> [The answer] says: side 15 and 15/31 *bu*.
>
> The rule says: let the side be 15 *bu*, deficit 15 *bu*;
>
> let the side be 16, a surplus of 16 *bu*.
>
> It says: combine (i.e. add) the gain and the deficit as the divisor,
>
> multiply the deficit numerator with the gain denominator,
>
> multiply the gain numerator with the deficit denominator,
>
> add [the products] as the dividend.

Considering the numerical values, let us look at the very first problem in the *JZSS* Chapter one:

> Now let there be a [rectangular] field of width 15 and length 16.
>
> Question: what is the area of the field?
>
> Answer: 1 *mu*.

It is worth noting that the relation 1 *mu* = 240 (square) *bu* was introduced by Shang Yang in his 383 BC reform. Before that, 1 *mu* was considered to equal 100 (square) *bu*. It seems therefore reasonable to believe that both this problem in *JZSS* and the one in *SSS* were constructed after 383 BC.

If the compiler of the *SSS* is familiar with this problem, it is natural to choose the two assumptions as above, because the length of the side must be between 15 and 16. Or knowing that 240 = 15 × 16 one can also divide the area by the arithmetic mean of 15 and 16 to get the same result: 240/(15+16). Interestingly, the deficit corresponding to 15 is 15 while the excess to 16 is 16.

3 Concluding Remarks

The recently excavated mathematical collection *SSS* has great significance for the history of Chinese mathematics. The compilation of the texts (at least some of the problems) is probably between 383 BCE and 186 BCE, a couple of centuries earlier than the *JZSS*. While *JZSS* was gradually systematized and improved through generations, *SSS* is an original first-hand document written on bamboo strips. The significance of *SSS* can therefore be compared to that of ancient Egyptian papyrus or Babylonian mathematical texts on clay tablets.

The level of sophistication exemplified in the *SSS* is generally lower that that of the *JZSS*. Mathematical problems are presented in question and answer form, as in the *JZSS*, but are not classified according to the methods used to solve them. Certain similarities between the *SSS* and the *JZSS* are undeniable, yet it seems unlikely that the *JZSS* evolved from the *SSS*. It is possible that the compilers of both texts had access to other even earlier sources in common.

Until other earlier mathematical writings in Chinese are to be found, the *JZSS* will still remain the most important and influential Chinese mathematical classic. It is worth investigating any parallels between the *SSS* and *JZSS* as well as between *SSS* and Babylonian or Egyptian mathematics.

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