Braid representatives minimizing the number of simple walks[*]

Hans U. Boden [†] ⬤, Matthew Shimoda [‡]

Mathematics & Statistics, McMaster University, Hamilton, Ontario, Canada

Abstract

Given a knot, we develop methods for finding a braid representative that minimizes the number of simple walks. Such braids lead to an efficient method for computing the colored Jones polynomial of the knot, following an approach developed by Armond and implemented by Hajij and Levitt. We use this method to compute the colored Jones polynomial in closed form for the knots $5_2, 6_1$, and 7_2. The set of simple walks can change under reflection, rotation, and cyclic permutation of the braid, and we prove an invariance property which relates the simple walks of a braid to those of its reflection under cyclic permutation. We study the growth rate of the number of simple walks for families of torus knots. Finally, we present a table of braid words that minimize the number of simple walks for knots up to 13 crossings.

Keywords: Knots, braids, simple walk, colored Jones polynomial.

Math. Subj. Class. (2020): 57K10, 57K14

Predstavniki pletenic, ki minimizirajo število enostavnih sprehodov[*]

Hans U. Boden [†] [iD],   Matthew Shimoda [‡]

Mathematics & Statistics, McMaster University, Hamilton, Ontario, Canada

Povzetek

V članku razvijemo metode za določitev predstavnika pletenic, ki minimizira število enostavnih sprehodov v danem vozlu. Takšne pletenice vodijo k učinkoviti metodi za izračun barvnega Jonesovega polinoma vozla, s čimer sledimo pristopu, ki ga je razvil Armond, implementirala pa sta ga Hajij in Levitt. To metodo uporabimo za izračun barvnega Jonesovega polinoma v sklenjeni obliki za vozle 5_2, 6_1, in 7_2. Množica enostavnih vozlov enostavnih sprehodov se lahko spremeni z zrcaljenjem, rotacijo in ciklično permutacijo pletenice; dokažemo invariantno lastnost, ki povezuje enostavne sprehode pletenice s tistimi, ki pripadajo njeni zrcalni obliki pri ciklični permutaciji. Preučujemo stopnjo rasti števila enostavnih sprehodov za družine torusnih vozlov. Nazadnje predstavimo tabelo pleteničnih besed, ki minimizirajo število enostavnih sprehodov za vozle z do 13 križišči.

Ključne besede: Vozli, pletenice, enostaven sprehod, barvni Jonesov polinom.

Math. Subj. Class. (2020): 57K10, 57K14