

# Braid representatives minimizing the number of simple walks\*

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Received 15 November 2021, accepted 19 July 2022, published online 21 November 2022

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## Abstract

Given a knot, we develop methods for finding a braid representative that minimizes the number of simple walks. Such braids lead to an efficient method for computing the colored Jones polynomial of the knot, following an approach developed by Armond and implemented by Hajij and Levitt. We use this method to compute the colored Jones polynomial in closed form for the knots  $5_2$ ,  $6_1$ , and  $7_2$ . The set of simple walks can change under reflection, rotation, and cyclic permutation of the braid, and we prove an invariance property which relates the simple walks of a braid to those of its reflection under cyclic permutation. We study the growth rate of the number of simple walks for families of torus knots. Finally, we present a table of braid words that minimize the number of simple walks for knots up to 13 crossings.

*Keywords:* Knots, braids, simple walk, colored Jones polynomial.

*Math. Subj. Class. (2020):* 57K10, 57K14

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\*The authors are especially grateful to Alexander Stoimenow for providing crucial input. They would also like to thank Hodayun Karimi, Robert Osburn, Andrew Nicas, Will Rushworth, and Cornelia Van Cott for valuable feedback.

<sup>†</sup>Corresponding author. The author would like to acknowledge funding from the Natural Sciences and Engineering Research Council of Canada.

<sup>‡</sup>The author acknowledges funding from a USRA award and a Stewart award from McMaster University.

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# Predstavniki pletenic, ki minimizirajo število enostavnih sprehodov\*

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Prejeto 15. novembra 2021, sprejeto 19. julija 2022, objavljeno na spletu 21. novembra 2022

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## Povzetek

V članku razvijemo metode za določitev predstavnika pletenic, ki minimizira število enostavnih sprehodov v danem vozlu. Takšne pletenice vodijo k učinkoviti metodi za izračun barvnega Jonesovega polinoma vozla, s čimer sledimo pristopu, ki ga je razvil Armond, implementirala pa sta ga Hajij in Levitt. To metodo uporabimo za izračun barvnega Jonesovega polinoma v sklenjeni obliki za vozle  $5_2$ ,  $6_1$ , in  $7_2$ . Množica enostavnih vozlov enostavnih sprehodov se lahko spremeni z zrcaljenjem, rotacijo in ciklično permutacijo pletenice; dokažemo invariantno lastnost, ki povezuje enostavne sprehode pletenice s tistimi, ki pripadajo njeni zrcalni obliki pri ciklični permutaciji. Preučujemo stopnjo rasti števila enostavnih sprehodov za družine torusnih vozlov. Nazadnje predstavimo tabelo pleteničnih besed, ki minimizirajo število enostavnih sprehodov za vozle z do 13 križišči.

*Ključne besede:* Vozli, pletenice, enostaven sprehod, barvni Jonesov polinom.

*Math. Subj. Class. (2020):* 57K10, 57K14

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\*Avtorja sta še posebej hvaležna Alexandru Stoimenowu za njegov ključni prispevek. Rada bi se zahvalila tudi Homayunu Karimi, Robertu Osburnu, Andrewu Nicasu, Willu Rushworthu in Cornelijji Van Cott za dragoceno povratno informacijo.

<sup>†</sup>Kontaktni avtor. Avtor je bil financiran s strani Natural Sciences and Engineering Research Council of Canada.

<sup>‡</sup>Avtor je bil financiran z nagrado USRA in s Stewartovo nagrado, ki jo podeljuje McMaster University.

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