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The Void Just Ain't (What It Used To Be): Void, Infinity, and the Indeterminate

“[Q]uae sane unica fuisset causa, ut veritas humanum genus in aeternum lateret;
nisi Mathesis...”

Spinoza, *Ethics*, Book I appendix

I. Taming of the infinite, taming of the void

On the 4th of June 1925, David Hilbert famously announced to the Westphalian Mathematical Congress that “No one shall drive us out of the paradise that Cantor has created for us.”¹ For more than a century now we can comfortably speak of a “modern mathematics”, marked by the twin breakthroughs of Cantor and Dedekind in the last years of the 19th century, through which infinite orders of infinity (the transfinite), along with the field of irrational numbers, have become the mundane operable entities of mathematical and philosophical work. With this “taming of the infinite”, can we also speak of a taming of the void?

In earlier historical contexts, the void-infinite formed a conceptual pair. In some of the earliest conflicts between the partisans of the void and those against, we find Aristotle's famous refutations of atomism. Typical among these rejections was Aristotle's direct association of the void with the indefinite. As he argues in the *Physics*, any locomotive event implies definite speed of such motion and, since the void is itself indefinite, it could not provide a criterion of measure for such motion. In other words, if locomotion exists, then the void does not.”²

The Antique indefinite is of course not the “infinite” understood, across the many transformations of the concepts from the medieval and modern periods, either as the absoluteness of god or the various mathematical distinctions made

¹ David Hilbert, “On the Infinite”, in *Philosophy of Mathematics*, 2nd ed., edited by Paul Benacerraf and Hilary Putnam, Cambridge: Cambridge University Press, 1983, p. 191.

² Aristotle, *Physics*, IV 8, 215a5-12; *The Basic Works of Aristotle*, ed. Richard McKeon, New York: Modern Library, 2001, p. 283.

on the infinite through these periods. At the same time however, it is this same, albeit ambiguous, indefinite-infinite that is irrevocably demystified with the advent of Cantor's transfinite. Along with Cantor, Dedekind can also be given some share of the spotlight here. The very idea of an "irrational" number co-extensive with something as fundamental as the Pythagorean theorem entertains legends of the poor Hippiasus who was apparently murdered (by drowning according to Pappus of Alexandria) by the master Pythagoras himself for making exoteric what was the irrational esoteric secret within the cult. With Dedekind, irrationals come to share the same determinate status as any number whatsoever picked out from the "cut" of the continuous number line. Hence, if the irrational, infinite (indefinite), and other such terms (i.e. imaginary-complex) are no longer the conceptual mirror of the void, we see that the void itself has also, in this, been demystified. The couple void-infinite (indefinite) can only be sustained if the indeterminateness of the former can be imputed on the latter. To make a long historical story very brief (bypassing the long entanglements with the infinite across roughly two millennia), we are no longer in the position to conceive the infinite as indeterminate and thus equally denied the luxury of making this conceptual circuit between the infinite, indeterminate, and the void.

With the taming of the infinite, the void is thus also tamed. From without and within, the void should neither be thought as the abiding negative principle from which a structure of particular things is un-determined nor should it be an undetermined abyss immanent in each determinate being from which transformation spring forth. The conceptual power of the void is neutralized along with that of the infinite.

II. Atomistic Stratagem

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Let us formulate this insight sketched above in the form of a critique of contemporary thought. As our reflections on the void and the infinite (in meta-mathematical terms) above are at a certain remove from contemporary philosophy, we bracket these reflections for the moment.

From thinkers as diverse as Rancière, Agamben, and Žižek, we find a common strategy for thinking with the void. These three represent at least three different traditions of thought, each one represented by some proper subset of Marxism, Heideggerianism, deconstruction, psychoanalysis, and structuralism, and my

criticism against them cannot but remain far from addressing their larger contributions. Yet it is perhaps due to the diversity in this spectrum of thinkers that their commonality on the question of the void might indeed be striking. This common strategy is no doubt familiar to readers of contemporary continental theory. It can be briefly sketched in four steps. This might be named the “atomistic strategy”.

[1] The first step is to outline some quasi-total structure, regime, or framework of representation. This is to be identified as the political state (understood as the neo-liberal police-state, ideology, etc.), universal bio-political regime, global capitalism, or the like.

[2] The second step is to assert the structural difficulties in undermining or resisting such a quasi-total structure or regime in the terms of the structure itself. As such, the dissenting demonstrations of a citizenry or the contestation of labour unions are quickly absorbed by the structural forces and set to reproduce the structures that provoked such acts. The problem is that these resisting forces of the structure are determinate (though antagonistic) features of the structure itself. In other words, there is no non-work time in capitalism, outside the 8 hours of labour, there are 8 hours of rest and 8 hours of leisure (according to the famous slogan but now a tremendous luxury). Both rest *and* leisure are determinate parts of the 24 hour productive day.

[3] The third step is to designate a *singular* abstract subject such as the proletariat, the commons, and the like, often allegorized in figures like Melville's Bartleby, the ignorant school-master (J. Jacotot), the *Tiananmen* protestors, or the Israeli Refusniks. These are singular figures insofar as their actions (or non-actions, not part of the circle labour-rest-leisure) have no valence within the coordinates of the quasi-total structure identified in point [1] and [2] above. These constitute a real challenge or danger to the given regime or structure, unlike those actions in point [2] above, precisely because they not only delegitimise such structures by rendering apparent the latter's arbitrariness or contingency, their actions also constitute an existent yet non-determinate resistance or nascent counter-power to the existent regime.

[4] The fourth and final step is to generalize the status of these singular subjects (or in-actions) in order to make a general point about the relation between a regime or structure and the void. By identifying the singular subject above [3] with the void, we allow ourselves to schematize this structure-regime (power-aesthetic-representation) by identifying a “hole”, “gap”, “lack”, or “blind-spot”. Such figures of the void provide an “immanent-outside”, which, because they are at once part of the regime and unrepresentable in the terms of said regime, allow us to delimit such a regime qua structure outside of the terms (the determinate identities) with which the regime itself operates. This figure of the void not only allows us to determine the contingent or arbitrary limits of any structure, it also allows us to determine the undetermined abyss via the figure of the singular subject that would be capable of undermining such seemingly impenetrable quasi-totalities as the police-state, bio-politics, and global capitalism.

Though far from disagreeing with such a general strategy in the analysis of contemporary politics, it is with the last of these “steps” sketched above that our present critique takes exception. It concerns itself with how this *singular indeterminate* subject noted above [3] should be understood. We see in the above sketch that the problem of the singular is conceptualized within a context of two other terms: a determinate order of representations (regime, state, etc.) and a determinate order of transformations within the order (dissent, contestations, etc.). The problem of the determinate order of transformation is that such a form of difference is not strong enough or radical enough to uproot the ruling order but, through integration or reform, only to recombine the already given group of identities or representations. The third term, that of the void, serves to disrupt such “reformism” precisely insofar as it plays no part in the operation of “normal transformation” within the given structure.

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Here we might look at Ancient Atomism for the understanding of the strategy. We know that the Epicureans formulated a specific solution for a problem that arose in the attempt to account for being in terms of the twin principles of void and atom. The problem, posed in opposition to Aristotle, arose in the context of the genesis of physical reality. If all atoms (the indivisibles) fell in the void with the same speed, what would result in such a “rain” would be tantamount to being no reality at all insofar as it would lack in even minimal differentiability. At

least in Lucretius, a “swerve”, “declination”, or *clinamen* is necessary in “the first beginnings” in order to generate the eventual aggregations and interactions that will generate the physical world of differentiated things and forms.³ As such, the determinate system of atomic interactions requires a principle of indetermination precisely because what is determinate can only be grasped from within the physical system itself. The origin of such a system would by definition be indeterminate, as Lucretius puts it, “at no fixed place and at no fixed time.” It is in this sense that the final step of the four-step strategy described above is nicely illustrated by Epicurean atomism. The contemporary “atomistic strategy” identifies the void (qua indeterminate principle) and the *clinamen*. Such an approach to the indeterminate, found not only in the Epicurean school but also in the contemporary “atomistic strategy”, suffer from a form of ersatz reification of that “something” required by the lacking “ground” for a desideratum of systematic consistency. We see here that the solution to the Atomistic problem can only take the form of a hypostasis, the reification of an indeterminate *clinamen*. If such an origin could be determinate, it would already have to be part of the physical system and hence we would require another, more original indeterminate principle (in a regressive account *ad infinitum*). In the contemporary context, it is the void that plays such a role as the reified repository for the indeterminate.

In philosophical terms, this way of thinking the “indeterminate” qua void is such that it renders it dependent on its relation with the determinate. As a figure of “abyss”, “lack”, or “hole”, the signification of the void is coextensive with that for which it is “other”; dependent on the determinate, the total, the whole. It is here that the meta-mathematical reflection from which we started can provide some insight. In purely figurative terms, it is only within the assumption of a quasi-totalizing regime-structure-state that this conceptual connection between the void qua singular and the indeterminate-indefinite-infinite can constitute a circuit of terms. It is this very circuit between a global or universal state-regime of representations, its normal transformations of particular transformations-recombination, and the radical or singular void, that is the aim of this four-step program outlined above shared by a surprisingly divergent set

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³ Lucretius, *De Rerum Natura*, II 290-293; *On the Nature of Things*, trans. by W.H.D. Rouse and rev. by Martin F. Smith, Cambridge, MA: Loeb Classical Library, 2002, p. 119.

of contemporary philosophical tendencies. By rejecting the identification of the void with the indeterminate, it is also this very circuit that must be rejected.

Although implicit, this circuit of terms sketched above makes use of the identification of the void and the infinite qua indefinite-indeterminate. If, as we have argued, this couple void-infinite is no longer a viable one, how can we reconceive the singular? In other words, how can we conceive of the singular indeterminate outside of its subsumption within the co-extensiveness of the void and totality? What is at stake here is a different possible conception of the singular in the confrontation between philosophy and those singular figures at work in our time out of joint.

III. Zero divides into two

Badiou's long philosophical development, from his early interventions within the Althusserian-Lacanian journal *Cahiers pour l'analyse* in the late 1960s to his recent multi-volumed *Being and Event*, provides an alternative path to the "Atomistic strategy": call it a "formalist" alternative.⁴ This approach, which draws from the actuality of a post-Cantorian meta-mathematical universe, commences from the key insight that the taming of the infinite entails a neutralization of the void. In order to isolate the key difference that this approach makes and to establish a paradigm for understanding this formalist alternative, we shall examine the theoretical conflict between Badiou and Miller, his colleague-interlocutor in his early essays in the *Cahiers*. It is in the context of the *Cahiers* that one can also glimpse a red thread of contention that will prefigure the many polemical episodes that will mark Badiou's work since the 1960s.

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To grasp the context of Badiou's refutation of Miller, we must make a short detour into the problem of structuralism, which provides not only the background for Badiou's philosophical formation but also that of the speculative zero-degree for many participants of the *Cahiers*. From the work of Saussure, an answer is given to the question of how meaning is constituted in language through the internal organization of textual or phonemic signs. Against an essentialist account of how marks or sounds produce meaning, an alternative account is provided whereby sense arises out of the contingent distribution of semantic values

⁴ The *Cahiers pour l'analyse* will be abbreviated as *Cahiers* in what follows.

over a series of mutually distinguished signs. This *functional* account of sense or meaning then relies on the consistency of structure understood as a closed set of mutually distinguished variable-places over which values can be assigned. In turn, these variable-places, before any distribution of values over them, are neutral or without sense.

Such a structuralist account may serve the analysis of any given natural language, but it faces concrete limits in the investigation of the origins of the emergence of the series of signifying marks (textual or phonemic) themselves. In the later adaptation of the structuralist turn in the field of anthropology, Lévi-Strauss was clear to abstain from genetic accounts in his functional *analysis* of morphisms in kinship and mythic structure. On the question of genetic accounts, like Laplace before Napoleon, Levi-Strass notes that, “[I] shall now do no more than repeat that social anthropology has no need of this hypothesis.”⁵ Through long discussions with one of the most important mathematicians of the 20th century, André Weil, Lévi-Strauss borrowed heavily from the algebra structure of groups (group theory) in order to provide the dimension of a “neutral” or “identity” operator responsible for sustaining the internal consistency of anthropological structure.⁶ This algebraic influence allowed Lévi-Strauss to make a further steps toward a functional analysis (whose contingency was always foundational) of the nature of social arrangements and myths precisely by delimiting structural analysis from a non-structural origin or genesis. Far from implying that there is no genesis of structure (linguistic, anthropological), the idea is that there can be no account of such a genesis from the structure itself. Here the tools of *analysis* require a field of determinate terms, and the indeterminate genesis of such a determinate structure is by definition outside of the field of analysis.

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In full view of this problem of genesis in structuralism, both Lacan’s “Science and Truth” and Miller’s “Suture (Elements of the logic of the signifier)”, their respective contributions to the inaugural volume (February 1966) of the *Cahiers*, provide attempts to treat this limit of causality of structure while preserving a

⁵ Claude Lévi-Strauss, *The Elementary Structure of Kinship*, trans. by J.H. Bell, John Richard von Stumer and ed. by Rodney Needham, Boston: Beacon Press, 1969, p. xxix.

⁶ Cf. André Weil, “On the Algebraic Study of Certain Types of Marriage Laws (Murngin system)”, in *The Elementary Structure of Kinship*, p. 221-230.

structuralist background framework.⁷ In these texts, both Miller and Lacan provide a dynamic view of structure by reconceiving the problem of the causality of structure. Since genetic causality cannot be given within structure, this causality must be treated as the indeterminate “other” of the structure (understood in the structuralist framework as a closed system of mutually differentiated variable-places). Both Lacan and Miller provide a sophisticated handling of this problem. Although we have been speaking in a figuratively *chronological* way about genetic causality, both Lacan and Miller understand such a limit to structural analysis in a transcendental way. That is, the problem of causality need not be *chronologically* prior to structure, what is simply indicated is that, insofar as structure is limited by the contingent grounds of its internal consistency, one can retroactively or immanently attend to such an internal indetermination within a given structure.

This starts to look a lot like the “Atomistic strategy” above. Since the origin of physical interaction following the *physical* rules of “weight” and “trajectory” has an origin that cannot be accounted for within physics, an *exception* must be conjectured. This exception, understood in the Atomistic framework as the *clinamen* or “swerve”, is the errant, contingent, and undetermined transformation that causally generates this closed structure of mutual effect at some indeterminate and retroactively posited origin.

Miller’s important contribution to this historical line of conceptual constructions is in his explicit connection of such a structuralist problem and the problem of mathematical consistency. From the outset we see that the development of structuralism has followed in the path of mathematical formalization not the least with Levi-Strauss’ early use of group theory, a tendency he will continue to exploit into the fields of topology in following works. Here, Miller seems to take the tendency to its extreme: the structural reading of Frege’s *Foundations of Arithmetic*. But Miller was not only following an intellectual tendency but, as we shall see, developing the fundamental problem of the contingency in structuralism by the identification of the same problem within the *Grundlagenkrise* in mathematics.

⁷ All references to the *Cahiers pour l'analyse* are taken from the website of the project where original texts and a number of translations are available. <cahiers.kingston.ac.uk>.

Miller's structuralist approach to Frege addresses the problem of the *analytic* nature of the field of arithmetic entities. Much like the problem of structure, the entities of arithmetic provided Miller with the occasional cause to coordinate the problem of the indeterminate in structure with mathematics. This attempt was part of Miller's project of producing a "logic of the signifier" and constituted one of the key intended aims of the *Cahiers* project. Frege's aim was to move the foundations of arithmetic (and mathematics in general) away from the vicious circularity that had shrouded the transcendental character of mathematical foundations in the Kantian tradition which had been widely influential since the 18th century. Kant's definition of the foundations of arithmetic, put forward in the *Critique of Pure Reason* and in the *Prolegomena to Any Future Metaphysics*, attached counting, and, in turn, the successive iteration of numbers, to the intuition of time, the form of inner sense.⁸ Arithmetic was then an *a priori* cognition based on the synthesis of units of time which might be reduced to a feature of consciousness: the intuitive self-representation of a unit of time, a moment. This approach relies on a pre-given notion of unity, a unit of time, pre-given in the cognitive faculty itself. What this implies is the synthetic nature of arithmetic: the combination of these represented units supplied by inner cognition. Yet we seem to be caught in a *petitio principia*. Arithmetic ends up being grounded by something that is already arithmetical. This is no grounding at all.⁹

The question of mathematical *Grundlagenkrise* was how to present a theory of arithmetic without already presupposing numerical concepts. Against the invocation of the "unit" in consciousness, Frege put emphasis on the idea that a certain sort of numericity was already operative in primitive logical relations. When we distinguish things by reference, a certain "number" of them are implicit in such a referentiality. A concept that picks out "featherless bipeds" already implies a certain number of these objects, its *extension*. Yet, in order to bridge this implicit feature of logic with an explicit and unique reference to numbers themselves, Frege has to first establish a univocal arithmetic series. In

⁸ Cf. Immanuel Kant, *Critique of Pure Reason*, trans. Werner Pluhar, Indianapolis: Hackett Publishing, 1996, B 14-17, p. 55-57.

⁹ Of course, there remained great mathematicians who attempted to re-articulate a "Kantian" approach to the foundations of mathematics. The Neo-intuitionists, headed by L.E.J Brouwer, developed an alternative not only to a Fregean-Russelian (logician) approach in the domain of foundations, but also an alternative to set-theoretical analysis of the continuum. Nonetheless, this development was made in view of these historical reconfigurations of the standards and methods made standard in the wake of "modern mathematics".

order to do this, one needs to find a unique reference to the number “zero” as the first number. Instead of “featherless bipeds”, the concept that Frege chose to express this was that which is “not identical with itself”.¹⁰ Frege, by reasoning that there is nothing that is not identical with itself, “ $x \neq x$ ”, identifies a concept with no extension at all, since it does not refer to anything. In turn, with this unique referent in place, numbers would succeed as the re-counting of this empty referent.

Concept	Extension of the concept	Natural (counting) number
Not identical with itself ($x \neq x$)	\emptyset	0
Identical with \emptyset	$\{\emptyset\}$	1
Identical with $\emptyset, \{\emptyset\}$	$\{\emptyset, \{\emptyset\}\}$	2
Identical with $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$	$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$	3
...

Unequivocal reference to the basic natural or counting numbers (0, 1, 2, 3,...) would proceed by reference to this concept. “One” would be the counting of this empty extension, “two” would be the counting of this counted empty extension, and “three” would be the counting of the counting of this empty set. This iterative procedure indeed returns to satisfy the iterative or successive structure of arithmetic progression. Once in place, the expansion of this basic procedure would allow us to map the successive, or iterative structure, generating the variety of other numbers (i.e. the evens, the rationals, the reals).¹¹

¹⁰ Gottlob Frege, *The Foundations of Arithmetic*, trans. by J.L. Austin, New York: Harper Torchbooks, 1960, p. 88. Frege duly notes that, “I could have used for the definition of nought any other concept under which no object falls. But I have made a point of choosing one which can be proved to be such on purely logical grounds; and for this purpose ‘not identical with itself’ is the most convenient...”

¹¹ Of course the structure of these numbers (natural, evens, rationals, primes) qua numbers is not the same. It provides a mapping and the basic iterative structure allows a minimal means to mark differences at the same time as showing a bijective function (isomorphism).

Natural	0	1	2	3	4	5	...
Even	0	2	4	6	8	10	...
Prime	2	3	5	7	11	13	...

Frege’s foundational account for numbers requires a univocal reference to each number. Despite the fact that “featherless bipeds” and “words in this sentence” pick out referents that display a certain numericity, what is necessary for a foundational account is to distinguish a unique referent, such as a zero, a one, a two, and so forth. These references could then allow us to place the extension (“picked out” entities) of these different concepts in correspondence with this numerical ordering. The number of words in this sentence (the extension of “the number of words in this sentence”) could thus refer uniquely to one of the numbers generated by this sequence of counting and recounting of the void set. The number is “38”.

Miller’s interpretation of Frege hinges on the artful minimalism of Frege’s foundational argument. On the one hand, Frege’s recognition of variation and extensive multiplicity in successful reference forced him to provide a unique referential scheme of numbers. On the other hand, this scheme carefully avoids the dangerous circularity of placing unity at its basis. It is through Frege’s minimalism that Miller applies his notion of suture and brings together the content of Frege’s grounding concept and the structural lack in his project for a logic of the signifier. Miller underlines that Frege’s concept, “not identical with itself”, only picks out an empty set when the assumption that all things are actually identical with themselves is in place. This assumes a determination about the world that is not actually accounted for within Frege’s own system. This self-identity need not be ontologically basic. In turn, Miller underlines a certain exclusion of the non-identical as the creation, within a consistent system, of the twin poles of subject and object. He notes that,

the impossible object, which the discourse of logic summons as the not-identical with itself and then rejects as the pure negative, which it summons and rejects in order to constitute itself as that which it is, which it summons and rejects wanting

to know nothing of it, we name this object, in so far as it functions as the excess which operates in the series of numbers, the subject.¹²

In Miller's reading, a lack constitutes structure and guarantees the system of identities that, in being identical with itself, can be minimally differentiated within an immanent series of differences. At the same time, the circulation of this lack, the impossible object ($x \emptyset x$), which is summoned and then rejected, is afforded the place of a mark \emptyset , the null-zero that circulates in the structure. The impossible object, marked as zero, produces the declination of positions in arithmetic succession, it constitutes the object, the zero, and the successive iterations made possible by this object, guaranteed by successful and unique reference. For Miller, this object would also be the subject, the excess of the structure, whose very content, "not identical with itself", calls upon a *suture* that extra-structurally correlates subject and structure (through the impossible object). In philosophical terminology we could understand this suture as the demonstration of the transcendental conditions of the consistency of the system. The inexhaustible variation of subjective apperception is more extensive and varied than what is schematized in a realm of organized and immanently consistent differentiations. This splitting of the transcendently constituted knowing subject and the subject that underlies the transcendental determination itself is what determines the place of the subject. Miller's suture introduces a distinction wherein both sides of this determination can be grasped by a more general logic of the signifier. Suture distinguishes the active creation of the objective qua systematic consistency from inconsistency by means of exclusion, or repression. In this context, analysis is thus precisely what draws out repressed content. In this sense, Miller correlates the structural framework of psychoanalysis with the task of an epistemology of mathematics as the basis for a project for the logic of the signifier.

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There are many reasons to criticize and remain sceptical of Miller's analysis of Frege, but there is no doubt that his use of Frege here is clear in outlining what his concept of suture is meant to do. In the most general terms, a consistent structure is constituted by repressing or excluding some impossible, inconsistent, object, which provides the grounds for a series of differential identities or variable-places as well as animates the repetition and iterative succession of

¹² Jacques-Alain Miller, "Suture (Elements of the logic of the signifier)", in *Cahiers pour l'analyse*, Vol. 1, Feb. 1966, trans. Jacqueline Rose, <cahiers.kingston.ac.uk/pdf/cpa1.3.miller.translation.pdf>.

these mutually differentiated terms. The methodology prescribed by the concept of suture urges us to analyse this disavowed object (i.e. $x \neq x$ inconsistency) and thus gain clarity into the operation of the structure and its necessary limits. In this, a Freudian “repetition” continually covers over the repressed or impossible content of iteration. Over the course of the *Cahiers*, Miller and the members of the *Cercle d'Épistémologie* (the Althusserian students who formed the core editorial group the *Cahiers*) would take this general schema to the analysis of a variety of *repetition* themes from psychoanalysis, philosophy (and its history), to anthropology, literary theory, and political theory.

Badiou's critique of Miller is presented in the 10th and final volume (1969) of the *Cahiers*, entitled “*Mark and lack: on zero*”.¹³ This work provides a resumé of a series of ideas that would eventually constitute Badiou's early project of a “materialist epistemology” more fully developed in the *Concept of Model*, published in the same year.¹⁴ While it is aimed at a critique of the more Millerian tendencies in the *Cahiers*, Badiou in fact sets down key elements from which we can glimpse some characteristic features of his later thought. I shall leave these later developments aside to concentrate on his critique of Miller.

Badiou's central critique strikes Miller's text at its central point. Badiou contests that Frege's use of the non-identical ($x \neq x$) does not in any sense produce a “lack” and, in turn, there is nothing to suture. Badiou argues that the marks that enter into scientific practice such as formal logic or mathematics are generated without any repression of a fundamental lack. Badiou demonstrates this by his alternative account of the problem of structure in mathematics.

Badiou begins by distinguishing three “mechanisms” of the structure and eventually adding a fourth. I present all four together here.¹⁵

¹³ Alain Badiou, “Mark and lack: on zero”, in *Cahiers pour l'analyse*, Vol. 10, Winter 1969, trans. Zachary Luke Fraser with Ray Brassier, <cahiers.kingston.ac.uk/pdf/cpa10.8.badiou.translation.pdf>.

¹⁴ Cf. Alain Badiou, *Le concept de Modèle*, Paris: François Maspero, 1969; *The Concept of Model*, trans. and ed. by Z.L. Fraser and Tzuchien Tho, Melbourne: re.Press, 2007.

¹⁵ Badiou, “Mark and lack: on zero”, pp. 2-5.

[1] Mechanism 1 (M_1), concatenation: Badiou assigns a first level to sign production: an indefinite number of (chains of) mutually differentiable marks.

[2] Mechanism 2 (M_2), syntax: At a second level, he highlights the putting into place of syntactic rules which distinguishes well-formed and nonsensical expressions. In other words, at the M_2 level, arbitrary marks are formed into expressions and the mechanism makes a distinction between well- and ill- formed syntactic expressions according to any number of rules.

[3] Mechanism 3 (M_3), derivation: At the third level, a mechanism of inference-making allows us to distinguish which among the well-formed expressions are derivable and which ones are not. Of course, there are many well-formed expressions that are not derivable. In this M_3 “picks out” a subset from the number of well-formed expressions distinguished by M_2 . The derivable are called “theorems” and the un-derivable are non-theorems of the system.

[4] Mechanism 4 (M_4): With the distinction between theorems and non-theorems, we can distinguish a fourth level, or a systematic level, which operates on the three previous levels to designate the structure, acting on the models constituted from M_1 to M_3 , such as Zermelo-Frankel axiomatic set theory or any such *theory* (an indefinite number of such theories). This M_4 level would also be the place where Frege’s arithmetic would otherwise be found, if it were not for Russell’s famous and ruining critique of Frege’s model of arithmetic, building on the latter’s contradiction in the logic of predication (“a set of all sets”). In a footnote, Badiou underlines that this contradiction in Frege, pointed out by Russell in a 1901 letter, is internal to Frege’s own system and independent of Miller’s notion of repressed inconsistency.¹⁶

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With these four mechanisms in place (M_1, M_2, M_3, M_4), Badiou explains that what Miller clumsily attempt to articulate was a treatment at the level of system (M_4) of something that is obviated at the level of derivation (M_3), that is, the fact that non-identity is non-derivable. Badiou reasons that,

¹⁶ *Ibid.*, p. 9.

for the inscription $\sim I(x,x)$ [ie. $x \neq x$] does not occupy the place of anything else; nor does it mark the place of a nothing. As for the zero [...] [i]t is positively constructed by M_2 . [...] The zero is simply an inscription accepted by M_2 and introduced, along with certain directions for use in M_4 . [...] The zero marks in M_4 (in predicative form) not the lack of a term satisfying a relation but rather a *relation lacking* in M_3 , it is *only insofar as it figures in M_2* .¹⁷

In other words, logical contradiction is an effect of the structure and not its repressed content. Yet, as Badiou carefully points out, non-identity is a well-formed expression ($x \neq x$) and can be found in the previous stage (M_2). Indeed, the process of derivation draws its material precisely from the number of well-formed expressions that were first produced at M_2 , a mechanism that in turn draws its material from the series of mutually differentiated marks produced in M_1 . In turn, Russell's problem with Frege has little to do with the "zero" and, if we temporarily overlook Frege's failure and treat it, as Miller does, as a consistent system, its consistency is not in the least sense guaranteed by a lack or the marking of the lack. Russell's problem concerns the scheme of predication and this issue of "inconsistency" is not that of the self-identity of marks in mathematical syntax.

In pointing out this misdirection in the critique of mathematical structure in Miller's work, Badiou forcefully generalizes his point:

Accordingly, *there is no subject of science*. Infinitely stratified, regulating its passages, science is pure space, without inverse or mark or place of what it excludes. It is a foreclosure, but a foreclosure of nothing, and so can be called a psychosis of no subject, and therefore of all: universal by full right, a shared delirium, it is enough to hold oneself within it to no longer be anyone, anonymously dispersed in the hierarchy of orders. Science is the Outside without a blind spot.¹⁸

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To fully analyse such a bold declaration would take us too far afield. We should simply underline that Badiou's exclusion of the subject in science, one that is exemplified in mathematical logic in the context of this article, entertains no concept of suture. As he argued earlier in the same article, "The logico-math-

¹⁷ *Ibid.*, p. 10.

¹⁸ *Ibid.*, p. 11.

ematical signifier is sutured only to itself.”¹⁹ This is indeed no suture at all. In this, while remaining clear that there is no subject-structure suture in science, Badiou partially agrees with Miller that it is in the context of the analysis of ideology that the analytic tool of suture can do its work. From this, there is no question that, at the level of scientific (mathematical) systems, an ideological treatment of science can no doubt take hold. Badiou will once again address this problem in the *Concept of Model* in the same manner that he addressed Miller, from a bottom-up concept of model that would disavow its ideological appropriation on the basis of model logic qua science.

This bottom-up approach to grasping mathematical or scientific reason through the “materiality of the signifier” places the opposition not between consistency and its necessarily excluded inconsistency but rather between two tendencies in treating the structural role of indetermination. The first tendency submits the scientific production of signs to a pre-given array of oppositions. Here, just as the consistency of a (physical) object is determined by its coordinates within space-time, an algebraic expression or a logical proposition occurs within its own logical space. However, especially in mathematics, scientific production constantly expands beyond its logical space. Rather than the attempt to secure this logical space, the second tendency understands scientific rationality precisely as the constant determination of new spaces of inscription. This is argued in another text of the same period of the 60s, the “Infinitesimal subversion”, published in the ninth issue (1968) of the *Cahiers*. Here we should only highlight that Badiou argues that the supposed “irrationality” of infinite and infinitesimal, attested to in the history of the philosophical critique from Berkeley to Hegel and even Cantor and Fraenkel, of the infinitesimal calculus is made only on behalf of an idealized consistent “whole” of the unity of mathematics.²⁰ The whole drama of the epistemological obstacle of “infinity” only holds in taking too seriously this tendency of treating mathematics as a consistent whole. That is, it is only against the background of a commitment to finitude that the infinite appears subversive.

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¹⁹ *Ibid.*, p. 6.

²⁰ Alain Badiou, “Infinitesimal Subversion”, in *Cahiers pour l'analyse*, Vol. 9, Summer 1968, trans. Robin Mackay with Ray Brassier, < cahiers.kingston.ac.uk/pdf/cpa9.8.badiou.translation.pdf >.

Let us then approach the positive stakes to be drawn from this refutation of Miller. When we reject the strategy of identifying the singular indeterminate as the contingent and excluded-included inconsistency of a given system, what remains of the indeterminate? The alternative that Badiou's early works provide is to see that the emergence of an "excluded", or obstacle, can only appear when the subversive terms have *already* been produced. Against the Millerian path or suture, and by extension, the "atomist path", we must reject the framework in which the break from a given structure is not the result of some repressed lack in the structure. In this, the "subversion" of the infinitesimal lies not in its engendering of an "irrationality" against some contrasted background structure of rationality. Science operates rather in the neutralization of this very opposition through the positive recasting of allegedly "impossible" spaces as new inscriptions. As such, it is in following through with the materiality of formalization and inscription, the literal production of marks, spaces, punctuation, and the like, that the "impossible" is demystified and drawn away from their subservice, as Badiou puts it, "to those constraining illusions whose salvation required an ideal guarantee."²¹

The strongest case that Badiou puts forth for this is Gödel's incompleteness proof. Given the four mechanisms that Badiou argues above, Gödel's proof amounts to a well-formed expression (M_2), say expression g , for which g is not a theorem but the negation of g is not a theorem either. The undecidable expression g is of course not a theorem (M_3), but systematically determines (M_4) what is indetermined within the system, a limit defined by the positive production of the system itself. As such it has nothing to do with either the suture or the place of the void as the point of suture. As Badiou argues, Gödel's incompleteness establishes, immanent within the system, a gap unmoored by the distinctions within the structure. This gap is precisely a singular and indeterminate expression constituted by the system but assignable within the system.

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In philosophical terms, Badiou provides a notion of the indeterminate singular that does not rely on a meta-systematic *suture*. That is, it does not rely on a hypostasized extra-systematic circuit already presumed about the indeterminate and the determinate in order to account for the dynamics of the structure either conceived as iterative or otherwise.

²¹ Alain Badiou, "Infinitesimal Subversion", p. 16.

IV. Singular and indeterminate but not inconsistent

In retrospect it might be easy to see why Miller was so easy to refute. Miller's speculative suture of the problems within a psychoanalytic theory informed by structuralism to the foundation problem of modern mathematics leaves out the fact that there is "no unconscious in science". One might easily suggest that Miller's fault was in bringing these two fields together too hastily. After all, as Badiou himself agrees, suture may well account for the relation between the unconscious and language although it fails for mathematics and scientific discourse in general. Where are the *double entendres* or slips of the tongue in scientific discourse? This may or may not be the case; the point is indeed irrelevant for our purposes here. The stakes in this debate are different (if not higher). They concern how we are to understand indeterminacy, contingency, and rupture within structure. Taking one route, one which we have named the "atomistic strategy" and to which I have associated Miller, the indeterminacy or the contingency of any given structure is to be derived from a certain gap-to-be-sutured between a consistent structure and that inconsistent qua indeterminate term within the structure to which we designate "abyss" or void. Taking another route, that which I have named the "formalist path" and to which I have associated Badiou as one of its sole protagonists, we understand the indeterminate term as a product of the consistent structure itself. Badiou's insight here entails (at least) three philosophical tasks.

The three points, which I will enumerate below with more care, can be summarized by the use of the Pythagorean legend with which we began. The objectionable nature of the irrational (or incommensurable) root (i.e. $\sqrt{2}$) was only taken as such because of the prior assumption that all (geometrical) proportions should be commensurable. This framework allows us to secure the determinate borders of rational proportions in precisely the refusal to think through the terms of incommensurability on their own terms by throwing it together with the inconsistent (hence the name "irrational"). Yet in the positive formalization of the irrational, we do not witness the failure of geometry, but only its illusory representation (governed by the fiat of the master) according to some policing of the normativity of terms (i.e. that all proportions should be rational). The "new" form re-inscribes the space of the prior geometry in a larger logical and technical space and stratifies its determinations against this emergence of the singular indeterminate. This is already Euclid's innovation; banal by now. As such, first-

ly, the identification of indetermination and inconsistency (or irrationality) is *merely* an effect of the refusal to think the indeterminate on its own terms or, in other words, the refusal of its inscription in structure as a new form. Secondly, we see that the Pythagorean story is only one in a series of historical developments where it is clear that new forms are the result of the positive enlargement of the formal and the conceptual rather than arising from the supposedly immanent gaps, lacks, or “inconsistencies” in the system. As such the contingency of structure is hypothetical, that is, independent from the inherent dynamics of the strata of structural determination. Thirdly, the philosophical task of thinking undecidability or incompleteness on their own terms is to render a positive (thought indeterminate) act whose valence can only be grasped on its own grounds rather than the pre-given dynamics of the structure.

Let us look at what is entailed by these three points:

[1] Firstly, the figure of the void is to be separated from the singular indeterminate. If the void continues to be associated with the singular indeterminate, we continue to assert the co-extensiveness of the indeterminate as the necessary “other” of the determinate. This mystification of the void is nothing but the ambiguous stand-in for the gaps of structure. Like the Epicurean clinamen, a reified indeterminacy is posited only to answer for the undetermined origin of the determinate (the nature of atomic interaction). Qua indeterminate, the void remains a catch-all placeholder for an abyss that signifies the origin-genesis, contingency, rupture, for any given system. This reification of the *nihil* ironically does the opposite, insofar that, although by definition indeterminate and contingent, its characterization remains tied to what is determined and necessitated by the gap or gaps of structure. Against this, if the singular indeterminate is to have any role at all outside structure, we must pursue an understanding of the excessive multiplicity of structure rather than its supposed gaps or inconsistency. Like the infinite, the void should be taken as a mundane or neutral element of structure: determinate and equal in status with the standard terms of structure. In other words, the void qua zero in arithmetic is banal insofar as it is the neutral element of arithmetic: the neutral element that guarantees iterative generation of terms. The taming of the void unlinks it from the indeterminate and allows the latter to play a more decisive role in reconceiving any structural rupture.

[2] Secondly, with the void thusly separated from the indeterminate singular, the contingency of structure can be detached from the notion of the “virtual” or immanent possibility. This renders structural contingency anhypothetical. This may seem incorrect at first, but since the singular indeterminate is, within the atomistic strategy, always posited as the ambivalent included-excluded term of a structure, it always remained tied to the gap of that structure; the “other” of structure. As such the singular-indeterminate is always an exclusion that relies on the requirements of the structure itself; it remains an “other” of that structure. When detached from such a framework, the indeterminate singular, understood as the excess of the structure, stands for the non-totalizable nature of the structures. This non-totalizability not only undoes any need for a question of included-excluded inconsistent term but also designates the structural characterization of indeterminacy not as the finitude of structure (to which a lack or void is co-extensive) but rather the infinitude of (any) structure. As such the generalization of the indeterminate across structures does not rely on the terms of any particular structure. Again, undecidability is a feature of structures taken from the ground-up, not a term within an analytics of structure. As such, undecidability qua indeterminate is anhypothetical insofar as it does not rely on the analysis of a hypothetical or conjectural system. This anhypothetic status also means that the singular indeterminate is absolute (or functionally tends towards the absolute) insofar as it is not dependent on the constitution of any particular structural consistency.

[3] Thirdly, and perhaps most importantly, we can grasp the singular indeterminate not as irrational or an inconsistency but rather as the inscription of a new form. The reason why the indeterminate is understood as inconsistency results *only* from the prior assumption that the determinate organization of structure is founded on a closed circulation of terms. This quasi-totalizing representation of structure naturally leads to the privileging of the void as its supposed “indeterminate”. By contrast, the formalist path, in eschewing such systematic totalization, is nothing less than the rejection of such prior assumptions of “completeness”, figuratively speaking. As such, the singular is always the inscription of a new and positive formalization that, though indeterminate insofar as it has no determinate status within the strata of determinations that constitute

a structure, is not the failure of the structure but only the failure of the representation of the structure as total or complete.

V. Concluding remarks

The critique carried out above is one that attempts to extend a philosophical methodology that places thought under condition of the mathematical revolution carried out in the last century. This method is armed with the understanding that the conventional linguistic or logical conception of fundamental relations (i.e. part-whole, local-global, one-multiple) must undergo significant transformation in light of modern mathematics. This methodology can provide important lessons in its critique of contemporary thought. This is not because of any historical inheritance of the *mathesis universalis* or *more geometrico*; the certainty or univocity of mathematics (under the aegis of *judgment*). The formalist methodology labours under the condition of modern mathematics because of the speculative avenues that have been rigorously unfolded for more than a century.

In this examination, I have made use of the early work of Badiou. In tracing the first inklings of the formalist path, Badiou provides, in the late 60s, a powerful alternative to the significations of the void, then emerging from the Althusserian and Lacanian circles. Although Badiou would heavily revise the theoretical armature of this period in his later ontological work of the 80s, what remains continuous is his commitment to rejecting the occultation of the void. As such, in Badiou's later attempts to think the rupture of structure, we find him drawing, not from the power of the void, but rather on the constructive process of articulating an indeterminate yet self-consistent emergence of novelty, under the name of the event. The alternative developed by Badiou, in contrast with contemporary thinkers, would be that of a figure of novelty that would not be the "other" of structure (void or gap), but rather a generic existence which is inscribed through its self-grounding (generic) character.

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We have, in the analysis of the identification of inconsistency and the indeterminate, underlined the stakes of the continuing hypostatization of the void as the repository of the contingent and the singular. The contending point is a rather simple one. With the neutralization of the infinite through the work of Cantor and Dedekind, the void is similarly neutralized. In the same stroke, the conceptualization of the indeterminate singular falls upon the problem of the

excess of structure rather than the indetermination of quantity. The problem is no longer that of infinite, or the indefinite abyss of the void, but that of the undecidable, or the incompleteness of structure. This formalist “subtraction” from the closed circuit of the everything-something-nothing allows us to think the indeterminate singular as a self-grounding multiplicity, a radical cut from the co-extensiveness of structure and its (supposed) gaps.