

RESPONSE SURFACE METHOD-BASED OPTIMIZATION OF OUTER ROTOR PERMANENT MAGNET SYNCHRONOUS MOTOR

NAČIN OPTIMIZACIJE ZUNANJEGA ROTORJA ZA TRAJNI MAGNETNI SINHRONSKI MOTOR

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Abstract

The Finite Element Method (FEM) is a prominent analysis approach. Although it is applicable for simulation and optimization of electrical machines, FEM is a very time-consuming technique. One of the approaches to shorten the optimization runtime is the use of surrogate models instead of FEM. In this paper, the design and optimization of an outer rotor permanent magnet synchronous motor for a hybrid vehicle are investigated. Response surface methodology (RSM) with four input variables is integrated with a sequential quadratic programming algorithm for optimization. Before the optimization, the performance of the surrogate model in the prediction of untried points is validated. Finally, the optimal motor is simulated by FEM to verify the results of RSM-based optimization, and the outputs of both models are compared.

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Povzetek

Metoda končnih elementov (FEM) je eden boljših pristopov analize. Čeprav je FEM uporabna za simulacijo in optimizacijo električnih strojev, je pa zelo zamudna tehnika. Eden od pristopov za skrajšanje trajanja optimizacije je uporaba nadomestnih modelov namesto FEM. V prispevku je raziskano načrtovanje in optimizacija sinhronskega motorja s trajnim magnetom zunanjega rotorja za hibridno vozilo. Metodologija odzivne površine (RSM) s štirimi vhodnimi spremenljivkami je integrirana z algoritmom zaporednega kvadratnega programiranja za optimizacijo. Pred optimizacijo je bila uspešnost nadomestnega modela pri napovedovanju neizkušenih točk potrjena. Končno FEM simulira optimalni motor, da preveri rezultate optimizacije na osnovi RSM in primerja izhode obeh modelov.

1 INTRODUCTION

The most well-known method for the optimization of an electromagnetic device is the finite element method (FEM). Using this method requires a long runtime. It is highly desirable to reduce the computational time of these optimizations. One of the best approaches to shortening the computation runtime is based on surrogate modelling in which the outputs for any new, untried point are calculated using the surrogate model. Different artificial intelligence methods, including Kriging, radial basis function (RBF), and response surface method, have been used thus far. Kriging and RBF are more complicated than the response surface method and have multiple distance matrix calculations for new untried points. In contrast, RSM gives the closed-form equations of the system, which enables the designer to use any optimization method, including sequential quadratic programming (SQP).

Optimization of a linear actuator using RSM has been reported in [2]. In this paper, the objective of the optimization is to minimize the weight with constraints for thrust force and detent force. The design of a slot-less permanent magnet linear synchronous machine (PMLSM) using RSM with three input variables and two responses has been reported in [4]. Rotor pole design in a brushless permanent magnet motor has been proposed in [5] in which cogging torque is expressed in terms of three structural variables of the rotor and air gap length in RSM surrogate model. The design and optimization of a PM synchronous motor using an RSM surrogate model to increase constant power speed range (CPSR) has been studied in [6], in which the RSM has three input variables, and a genetic algorithm has been used to optimize the RSM equations.

In the present paper, an outer rotor permanent magnet motor with particular application for hybrid vehicles is designed. To shorten the computational time, the equations of RSM surrogate model of the system are proposed. Finally, the closed-form equations of the motor are optimized using a sequential quadratic programming (SQP) algorithm to determine the optimum point. The results of the RSM and FEM are compared.

2 INITIAL DESIGN OF MOTOR

The design and optimization of an outer rotor surface-mounted permanent magnet machine with 12 slots and 10 poles are investigated here. The proposed machine delivers 400 Nm torque at 1200 rpm. To maximize the fill factor, prefabricated winding has been used, [7]; thus, the stator has open slots to enable assembling this type of winding. The magnets of the motor are mounted on the surface of the rotor in the air gap.

3 RESPONSE SURFACE SURROGATE MODEL

The best advantage of the RSM surrogate model is its closed-form equation ($y=f(x_1, x_2, \dots, x_n)$). The outputs functions expressed by the inputs for a linear model are as follows [8]:

$$f(x) = \beta_0 + \sum_{i=1}^k \beta_i X_i \quad (3.1)$$

and for quadratic models, it is as follows:

$$f(x) = \beta_0 + \sum_{i=1}^k \beta_i X_i + \sum_{ii=1}^k \beta_{ii} X_{ii}^2 + \sum_{i<j}^k \sum \beta_{ij} X_i X_j \quad (3.2)$$

where β_i , β_{ii} and β_{ij} are the coefficients of the polynomial and X_i and X_j are the input variables of the system.

The coefficients β , are calculated as follows:

$$\beta = [X'X]^{-1} X'y \quad (3.3)$$

where X is the matrix of the input variables and y is the corresponding output values for the sampled points.

4 RSM SURROGATE MODEL OF MOTOR

In this paper, the design space consists of four input variables, and the Box-Behnken RSM surrogate model is used to model this design space. The maximum input current (I_{max}), slot height, the width of the bottom of the slot, and the thickness of the magnets are considered as input variables of this surrogate model. Fig. 1 shows the geometry and three structural input variables.

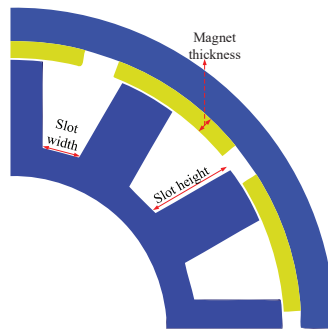


Figure 1: Geometry and structural variables of the motor

The range of these variables in the design space has been shown in Table 1. Making a Box-Behnken set of sample points for the surrogate model for a k-dimensional design space requires $2k(k-1)+C_0$ sample point [9], [10]. C_0 is the centre point, and it determines the curvature of the model. Here, four central points are simulated; 28 sample points are generated by Box-Behnken, and the corresponding outputs are calculated using FEM.

Table 1: Range of input variables

Input variable	Parameter name	Minimum	Maximum
I_{max}	A	130	160
Slot height	B	20	34
Slot width	C	12	18
Magnet thickness	D	7	12

Table 2 presents the list of simulated points selected by Box Behnken with their corresponding responses. The price, torque, and efficiency of the motor are considered as surrogate model responses. Table 3 shows the analysis of variance (ANOVA) of these responses. In each of the responses, the terms with the p-value higher than 0.1 are insignificant and thus are neglected in the RSM equations. All terms of the equations in efficiency and torque are significant, but in price just six terms are significant, and the others are ignored. The quadratic polynomials of the RSM surrogate model of the system are calculated using Eq. (2) by substituting the simulated matrices of Table 2. The closed-form equations of the price, torque, and efficiency are as follows:

$$Price = 211.1693 + 6.0429B + 3.8953C + 46.74D + 1.0097BC + 0.4897C^2 - 0.5266D^2 \tag{4.1}$$

$$Torque = 359.3804 + 36.63A - 12.5602B - 7.6114C + 12.5046D - 2.1576AB - 1.2546AC + 2.4269AD - 4.0435BC - 2.5859BD - 1.6512CD - 0.7935A^2 - 2.9347B^2 - 1.5774C^2 - 4.7137D^2 \tag{4.2}$$

$$Efficiency = 0.9138 - 0.0047A + 0.0129B + 0.0084C + 0.0062D + 0.0012AB + 0.0007AC + 0.0005AD - 0.0022BC - 0.0009BD - 0.0005CD - 0.0003A^2 - 0.0035B^2 - 0.0005C^2 - 0.0014D^2 \tag{4.3}$$

The most expensive part of the motor is the magnet, and in Eq. (4), D has the greatest coefficient (46.74). Torque highly depends on the current, and in Eq. (5) the greatest positive coefficient is the coefficient of the current, which is 36.63. As the current and the conduction loss increase, the efficiency decreases. Thus, the current in Eq. (6) has a negative coefficient.

Table 2: List of sampled points by Box Behnken sampling and corresponding responses

	A	B	C	D	Price (\$)	Torque (Nm)	Efficiency (%)
1	145	34	12	9.5	212.86	354.7	91.63
2	145	27	15	9.5	211.16	359.38	91.38
3	145	20	12	9.5	202.62	371.55	88.58
4	145	34	18	9.5	222.67	330.42	92.91
5	145	20	15	7	157.86	349.38	88.84
6	145	34	15	12	263.35	348.76	92.71
7	130	27	12	9.5	207.76	326	91.01
8	160	20	15	9.5	205.14	406.98	89.09
9	160	27	12	9.5	207.76	402.52	89.94
10	160	34	15	9.5	217.15	377.54	91.9
11	145	27	15	9.5	211.16	359.38	91.38
12	130	20	15	9.5	205.14	329.26	90.32
13	160	27	15	12	257.38	405.42	91.41
14	145	27	12	7	160.49	346.36	0.8967
15	145	34	15	7	169.89	329.25	91.63
16	145	20	15	12	251.37	379.23	90.3
17	145	27	15	9.5	211.16	359.38	91.38
18	145	27	18	7	168.28	334.85	91.45
19	130	34	15	9.5	217.15	308.45	92.64
20	145	27	18	12	261.77	356.33	92.59
21	160	27	15	7	163.89	375	90.04
22	160	27	18	9.5	215.55	385.35	91.74
23	145	20	18	9.5	208.39	363.44	90.77
24	130	27	15	12	257.38	328.21	92.26
25	145	27	15	9.5	211.16	359.38	91.38
26	130	27	15	7	163.89	307.5	91.11
27	145	27	12	12	253.98	374.44	91.03
28	130	27	18	9.5	215.55	313.85	92.50

Table 4 presents the fit statistics of the surrogate model. In this table, C.V% is the coefficient of variance, R^2 is the coefficient of determination. In all responses, the R^2 and adjusted R^2 are close; thus, there is no insignificant variable in the surrogate model, [8]. Finally, the Adeq precision is the measure of the signal-to-noise ratio. In all responses, the Adeq precision is higher than 4, which shows that the model is desirable and can navigate the design space, [8].

Table 3: ANOVA for quadratic form of RSM

Source	p-value		
	Price	Efficiency	Torque
Model	< 0.0001	< 0.0001	< 0.0001
A	1	< 0.0001	< 0.0001
B	< 0.0001	< 0.0001	< 0.0001
C	< 0.0001	< 0.0001	< 0.0001
D	< 0.0001	< 0.0001	< 0.0001
AB	1	< 0.0001	< 0.0001
AC	1	< 0.0001	0.0004
AD	1	0.0006	< 0.0001
BC	< 0.0001	< 0.0001	< 0.0001
BD	0.6392	< 0.0001	< 0.0001
CD	1	0.0004	< 0.0001
A ²	1	0.0075	0.0029
B ²	0.4254	< 0.0001	< 0.0001
C ²	< 0.0001	0.0002	< 0.0001
D ²	< 0.0001	< 0.0001	< 0.0001

Table 4: Fit statistics of the RSM surrogate model

	Price (\$)	Torque (Nm)	Efficiency (%)
Mean	211.15	355.09	91.13
C.V%	0.0272	0.1499	0.0264
R ²	1	0.9998	0.9998
Adjusted R ²	1	0.9996	0.9996
Predicted R ²	1	0.999	0.9988
Adeq Prediction	2510.4112	256.2085	242.4228

Fig. 2 presents the actual values calculated by FEM and the predicted values by RSM. This figure shows that there is a good agreement between the predicted and actual values.

5 OPTIMIZATION OF MOTOR

The motor is optimized using the closed-form equations of the RSM surrogate model. The goal

$$\begin{aligned} &\text{Min: Price} \\ &\text{s.t Torque} > 400 \text{ Nm} \\ &\quad \text{Efficiency} > 91\% \end{aligned} \tag{5.1}$$

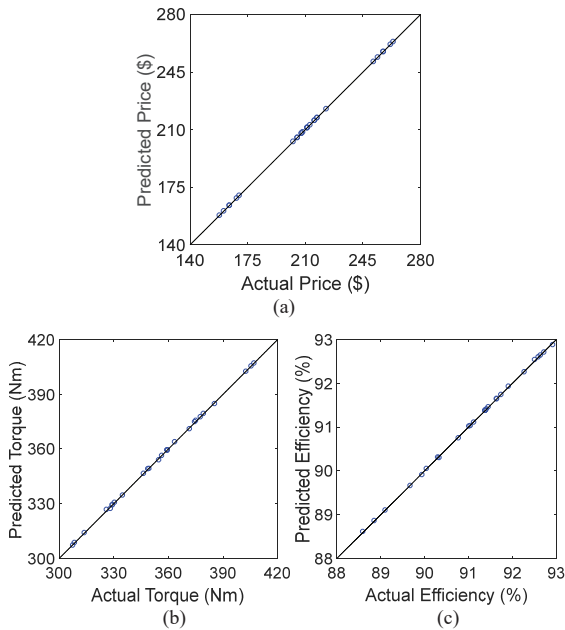


Figure 2: Predicted versus actual values at sampled points for (a) Price, (b) Torque, (c) Efficiency

of the optimization is to minimize the price of the motor, considering the following constraints for torque and efficiency.

Sequential quadratic programming (SQP) is applied to optimize the price. The optimization is performed using the RSM equations. The final values of the RSM input parameters at the

Table 5: Input variables at the optimum point

Input variable	Parameter symbol	Optimum value
I _{max} (A)	A	160
Slot height (mm)	B	24.9084
Slot width (mm)	C	16.2131
Thickness of magnet (mm)	D	10.2329

optimum point is presented in Table 5.

After the optimization, the optimal point is simulated by FEM to verify the results of the RSM based optimization. Fig. 3(a) and (b) show the torque and efficiency of the motor at optimum point using FEA in one complete electrical period of the motor, which indicates that the torque and efficiency constraints are satisfied at the optimal point.

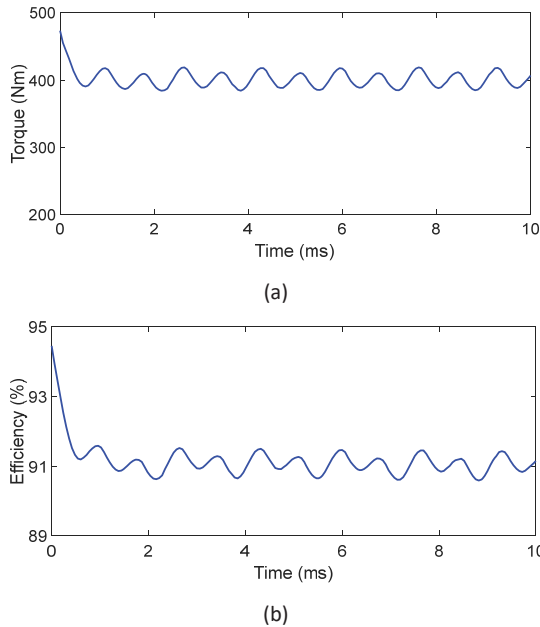


Figure 3: (a) Torque of optimum motor using FEA (b) efficiency of the optimum motor using FEA

A full comparison of the RSM and FEM at the optimum point has been presented in Table 6. According to this table, the error of the RSM surrogate model is acceptable; thus, this method can be used instead of FEM in optimizations. For a precise investigation at the optimal point, the field density of the motor at full load is shown in Fig. 4. This figure indicates that there is no saturation in the motor, and a good performance of the motor is guaranteed.

Table 6: Comparison of RSM and FEA at optimal point

	RSM	FEA	Error
Price (\$)	224.5	224.6	3.87e-04
Torque (Nm)	400	400.4	9.90e-04
Efficiency (%)	91	91.2	2.10e-03

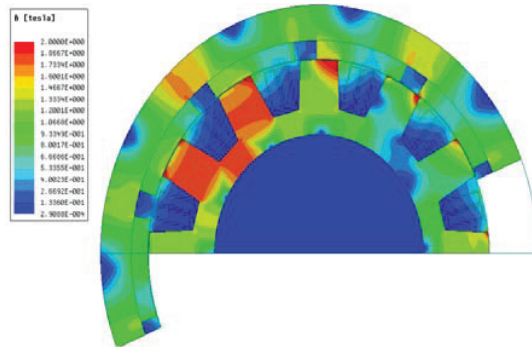


Figure 4: Magnetic field density distribution of optimal motor at full load operation

7 CONCLUSION

Generally speaking, the optimization of electromagnetic devices takes a long computation time using FEM. This paper presented an RSM surrogate-based model optimization. All needed outputs of the model were described by closed-form equations in RSM. Having a closed-form set of equations enables the designer to use the best algorithm of optimization, and SQP was used here. Comparison of RSM and FEM results show that there is a good agreement between the RSM surrogate model and FEM. Simulating the motor by FEM at the optimal point verified the results of the RSM based optimization.

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