# CHARACTERIZATION OF CIRCUITS IN GRID OBTAINED BY REGULAR AND SEMI-REGULAR TESSELLATIONS

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Jovиša Žunić, Novi Sad
Faculty of Engineering
Dr Ivan Stojmenović, Institute of Mathematics, Novi Sad

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ABSTRACT: In this paper circuits in grids which are obtained by using plane tessellations are observed. Isomorphism and congruence of circuits in these grids is defined in natural way. Conection between these relations is discussed.

## 1. INTRODUCTION

A tesselation of plane is a covering of the plane by using polygons. It is known that there are exactly eleven ways to cover plane by using regular polygons. Three of these are regular tessellations, where each vertex is surrounded by identical regular polygons (see fig.1). The other eight are semi-regular tessellations, in which each vertex is surrounded by an identical cycle of regular polygons (see fig.2).

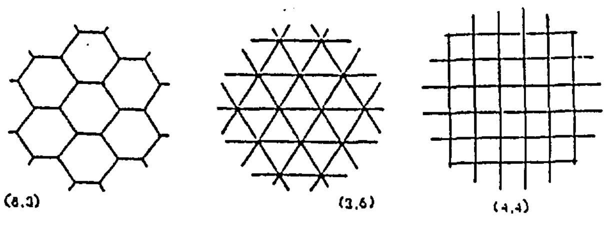



Figure 1. The three regular tesselations

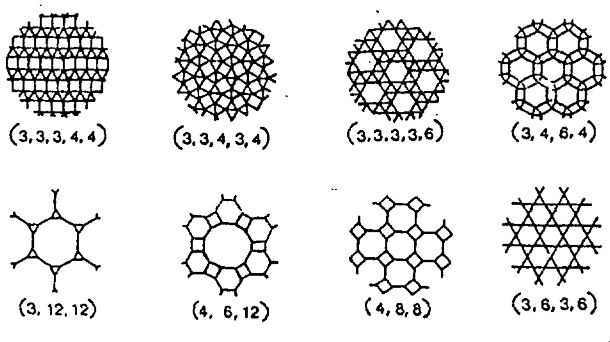



Figure 2. The eight semi-regular tesselations

In this way eleven infinite periodic grids are obtained. (This grids are plane representation of infinite plane graphs.). Let G be one of obtained grids. A circuit of the lenght m in a grid G is a oriented closed path without repeated vertices, containg m edges.

A circuit C in the grid G determines a simple polygon which consists of the edges of C. We will say that a circuit $C_1$ is congruent to a circuit $C_2$ iff polygons determined by circuits $C_1$ and $C_2$ are congruent polygons. Also, in natural way we define an isomorphism of circuits in the grid G which is obtained by regular and semi-regular tessellations. Let $C_1$ and $C_2$ be the circuits in the grid G. Then: the circuits $C_1$ and $C_2$ are isomorphic circuits iff there exists a congruence transformation T such that:

1) T maps the grid G into itself and
2) T maps a polygon determined by the circuit $C_1$ into polygon determined by circuit $C_2$.

Also we say that simple polygons A and B in the grid G are isomorphic polygons iff circuits determined by A and B are isomorphic circuits.

## 2. WORD REPRESENTATION OF CIRCUITS

Let G be one of grids obtained by using tessellations. Grid G is periodic. Let us determine period of grid G. If n is the number of edges in period of G then the number of oriented edges is 2n and we shall denote these oriented edges (vectors) by v(0),v(1),...,v(2n-1). In this way for any oriented edge of the grid G there is corresponding uniquely determined vector from the set v={v(0), v(1),...,v(2n-1)}.

Let A and B be points in the grid G, and P oriented path of lenght t, from A to B. If path P consists of oriented edges $v(i_1),v(i_2),\ldots,v(i_t)$ respectively, then the word $f(P)=i_1 i_2 \ldots i_t$ which corresponds to path P is uniquely determined. Specialy, for i=1 f(v(i))=i. Let $A^k$ be the set of all words of lenght k over the alphabet A={0,1,...,2n-1} and $A' = \cup A^k$ k>0.

Then denote by A" the set of all words which corresponds to oriented pats in the grid G. That means: $a \in A'' \Leftrightarrow$ exists path P such that f(P)=a.

If the word $a=i_1 i_2 \ldots i_t$ is from A", then a determines the path $v(i_1)\ldots v(i_t)$ such that f(P)=a. The circuit C of lenght n determines 2n closed oriented paths, depending on the choice of the initial vertex and the orientation of the circuit. A function f maps these 2n oriented paths into 2n words of the set A". Let us denote the set of these 2n words by Q(C) (for circuit C). Let T be isometry which maps grid G into itself. Let $T(v(i))=v(i')$ i=0,1,2, ...,2n-1 then (0',1',...,(2n-1)') is permutation of (0,1, ...,2n-1). Transformation T maps path $P=v(i_1)v(i_2)\ldots v(i_t)$ into path T(P) such that $T(P)=T(v(i_1)v(i_2)\ldots v(i_t)) = T(v(i_1))T(v(i_2))\ldots T(v(i_t)) = v(i_1')v(i_2')\ldots v(i_t')$ or $f(T(P))=i_1' i_2' \ldots i_t'$.

Let A''' be the set of all words which correspond to circuits in the grid G. Also every word a from A''' ($a=i_1 \ldots i_t$) determines circuit $C=v(i_1)v(i_2)\ldots v(i_t)$ (which determines simple polygon with edges of C).

Let a and b be words from A''' . We say that a and b are in the relation α iff circuits, which are determined by words a and b, are isomorphic circuits.

LEMMA 1: Relation α is equivalence relation.

PROOF: The set τ of all congruence transformations which map grid G into itself is a group.
Specialy: If T=I (identical mapping) then for a,b∈Q(C)⇔ aαb.

In the set of vectors {v(0),v(1),...,v(2n-1)} we define relation ρ by: v(i) v(j) ⇔ exists isometry T which maps the grid G into itself such that

T(v(i)) = T(v(j))

LEMMA 2: Relation ρ is a relation of equivalence.

PROOF: Directly from definition.

Also, we say that: $i\rho j$ iff $v(i)\rho v(j)$.

If P is a path from point A to point B then vector $\vec{AB}$ is equal to the vector sum of oriented edges which the path P contains.

LEMMA 3: Word $a=i_1i_2\ldots i_t$ from A" is from A''' (or $v(i_1)v(i_2)\ldots v(i_t)$ is circuit) iff 1) $v(i_1)+v(i_2)+\ldots+v(i_t)=0$ (vector summ) and 2) $v(i_j)+v(i_{j+1})+\ldots+v(i_{j+k})\neq 0$ for $k<j+k\leq t$.

## 3. ALGORITHM FOR COUNTING NONISOMORPHIC CIRCUITS

Using observations from previous section we can propose one common algorithm for determination numbers of nonisomorphic circuits on each of grids obtained by tessellations (regular and semi-regular).

Let $e_1$ and $e_2$ be the vectors from $\{v(0),v(1),\ldots,v(2n-1)\}$ which are not colinear. Now, we determine coordinates of each vector from $\{v(0),v(1),\ldots,v(2n-1)\}$ with respect to vectors $e_1$ and $e_2$.

Let $v(i)=\alpha_1e_1+\beta_1e_2$ $(i=0,\ldots,2n-1)$ then follows (from Lemma 3):

LEMMA 4: If $a=i_1i_2,\ldots,i_t$ is word from A''' then

$$\sum_{i=0}^{2n-1}\alpha_i l(i)=0 \quad \text{and} \quad \sum_{i=0}^{2n-1}\beta_i l(i)=0$$

where $l(i)$ is the number of occurences of character i in the word $a=i_1i_2\ldots i_t$. CONDITION 3 will be called CONDITION-G.

LEMMA 5: If P is an oriented path in the grid G then: P is a circuit iff:

1) the word f(P) satisfies CONDITION-G and
2) no subword b of a satisfies CONDITION-G.

We use following input data and their notations:

1) Number of vectors in period of G.
2) Number of continuations of each vector denoted by C. (Vector v(i) is a continuation of a vector v(j) if word ji is from A").
3) Continuations of each vector. Corresponding characters for continuations of vector v(i) denoted by c(i,1),c(i,2),...,c(i,C).
4) Number of initial vectors-denoted by I. Note: Initial vector can be any vector. It is clear that if the word $a=i_1i_2\ldots i_t$. Is from A" and $i_1\rho j_1$ then there exists a word $b=j_1j_2\ldots j_t$ such that $a\alpha b$. That means: the number of initial vectors can be equals to the number of equivalence classes with respect to relation $\rho$. In this way can be reduce the computation time.
5) Initial vectors-corresponding characters denoted by Iv(1),Iv(2),...,Iv(I).
6) Number of transformations (of $\tau$).
7) One isometry of first class which maps grid into itself.
8) One isometry of second class which maps grid into itself.
9) For every vector v(i) its opposite vector v(j). $(v(i)=-v(j))$.

Note: for grid $3^4.6$ isometry of first class no exists so 7 is identical mapping.

Let $a=i_1i_2\ldots i_t$ be word from A", and let a(j) denoted the word $i_ji_{j+1}\ldots i_t$ $1\leq j\leq t$. If CONDITION-G is not satisfied for any $j(1\leq j\leq t)$ then we call the word $i_1i_2\ldots i_t$ addable. It is clear that $i_1i_2\ldots i_t$ can be completed to a word $i_1i_2\ldots i_m$ $(m>t)$, representing a circuit iff $i_1i_2\ldots i_t$ is addable. Also it is obvious that $a=i_1i_2\ldots i_t$ denotes a circuit iff a(j) satisfied CONDITION-G only for j=1.

All words that are $\alpha$-equivalent to a word a representing a circuit we can obtain using 6,7,8,9 (see input data). We consider only the equivalent words begining by one of initial vectors (input data-5) and sort them in lexicographic order. We choose the first word a´ as a representative of this class. Hence, if the word a is equal to a´, then word a represents a circuits of lenght t and print it.

Our algorithm can be conveniently explained using two phases: extend and reduce. These phases correspond to the addable and nonaddable cases respectively.

```
READ (t)
FOR k=1 to I DO
 BEGIN
  i1=Iv(k); m:=1
  REPEAT
  IF i1i2...im is addable
   THEN extend
   ELSE IF i1i2...im is representative of
           a nonisomorphic circuits
           THEN print i1i2...im
   reduce
  UNTIL m=1
 END
where
extend ≡ BEGIN m:=m+1; im:=c(im-1,1) END
reduce ≡ WHILE im=c(im-1C) and m≥2
            DO m:=m-1
         IF m≠1
            THEN BEGIN
                   t:=0
                   REPEAT t:=t+1
                   UNTIL im=c(im-1,t)
                   im:=c(im-1,t+1)
                 END
```

Data obtained by proposed algorithm will be given in next section, k(t) denoted the number of nonisomorphic circuits lenght of t.

Grids which is obtained by tessellations $6^3,4^4,3^6$ are not specialy treated, but they have been observed in the papers: |1|,|2|,|3|. The algorithm presented here could be also directly applied to these a rids.

## 4. CONNECTION BETWEEN ISOMORPHISM AND CONGRUENCE OF CIRCUITS

It is clear that if $C_1$ and $C_2$ are isomorphic circuits then $C_1$ and $C_2$ are congruent circuits. In this section we will show that for grids obtained by tessellations $3^3.4^2$, $3^2.4.3.4$, $3.4.6.4$, $3.6.3.6$, $3.12^2$, $4.6.12$, $4.8^2$ (all semi-regular except $3^4.6$) is satisfied: if circuits $C_1$ and $C_2$ are congruent circuits then they are isomorphic circuits. For grids obtained by regular tessellations previous statament follows obviously because of that they are not specialy treated.

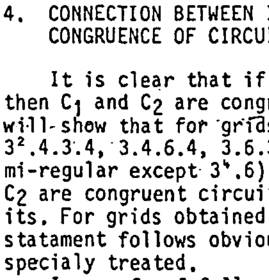
In proofs of following lemmas we will use:

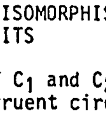
LEMMA 6: Let $M=M_1M_2\ldots M_t$ and $N=N_1N_2\ldots N_t$ be congruent polygons such that $M_{i_1}\equiv N_{i_1},M_{i_2}\equiv N_{i_2},M_{i_3}\equiv N_{i_3}$ (for some integres $i_1,i_2,i_3$ from $(1,2,\ldots,t)$, then: if points $M_{i_1},M_{i_2}$ and $M_{i_3}$ are not colinear then $M_i\equiv N_i$ for all $i\in\{1,2,\ldots,t\}$.

We shall denote by $\sphericalangle(i,j)$ the angle between vectors v(i) and v(j). By r(M) will denoted the word $m_1,_2\ldots$ determined by a simpl polygon $M=M_1M_2\ldots M_t$ such that $m_i=f(M_iM_{i+1})$.

LEMMA 7: Let G be the grid obtained by tessellation $3^3.4^2$, then: if M and N are congruent polygons in grid G then $r(M)\alpha r(N)$.

PROOF: Equivalence classes with respect to relation $\rho$ are: I={0,5} II={1,4,9,6} III={2,3,7,8} (see fig.3)
Let $M=M_1M_2\ldots M_t$ and $N=N_1N_2\ldots N_t$ are congruent polygons in the grid G and $r(M)=m_1m_2\ldots m_t$ and $r(N)=n_1n_2\ldots n_t$.




VECTORS: 5 6 7 8 9 are opposite
for: 0 1 2 3 4

| t: | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|
| k(t): | 1 | 2 | 2 | 4 | 6 | 17 | 32 | 90 | 204 |

f(P) = 0 2 3 1 2 0 8 9 3 5 7 5 9 0 2 0 2 3 5 7 5 7 5 4 4 4 0 8 4

Fig. 3.

1-case: $m_1,n_1\in I$ then $m_1m_2\ldots m_t\alpha 0m_2'\ldots m_t'$ and $n_1n_2\ldots n_t\alpha 0n_2'\ldots n_t'$ (words $0m_2'\ldots m_t'$ and $0n_2'\ldots n_t'$ exist because $m_1$ and $n_1$ are from same equivalence class) if $m_2'=n_2'$ then by Lemma 6 $m_i'=n_i'$ for $i=3,\ldots,t$ if $m_2'\neq n_2'$ then we apply reflection in a line determined by vector v(0) which maps grid G into itself. The image of $0m_2'...m_t'$ is $0m_2''...m_t''$.

$$\sigma_0 \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 0 & 6 & 8 & 7 & 9 & 5 & 1 & 3 & 2 & 4 \end{pmatrix}$$

since $\}(0,m_2'')=\}(0,n_2')$ we have $m_1''=n_1'$, $m_1m_2...m_t\alpha 0n_2'...m_t\alpha 0n_2'm_3''...m_t''$, $n_1n_2...n_t\alpha 0n_2'...n_t'$ that means (by Lemma 6) $n_3'=m_3'$, ..., $n_t'=m_t'$ or $m_1m_2...m_t\alpha n_1n_2...n_t$.

2-case: $m_1n_1 \in ||$ then $m_1m_2...m_t\alpha 1m_2'...m_t'$ and $n_1n_2...n_t\alpha 1n_2'...n_t'$

$\}(1,m_2')=\}(1,n_2') \Rightarrow m_2'=n_2'$

If $m_2'=n_2'=1$ then we continue until $m_i'\neq 1$ but then using Lemma 6 $m_i'=n_i'$ or $m_1m_2...m_t\alpha 1m_2'...m_t'=1n_2'...n_t'\alpha n_1n_2...n_t$.

3-case: $m_1,n_1 \in |||$ then $m_1m_2...m_t\alpha 2m_2'...m_t'$ and $n_1n_2...n_t\alpha 2n_2'...n_t'$

$\}(2,m_2')=\}(2,n_2')$

if $m_2'=n_2'$ then clearly $m_i'=n_i'$ for $i=3,...,t$ and $m_1m_2...m_t\alpha n_1n_2...n_t$ if $m_2'\neq n_2'$ then, let be for example, $m_2'=3$ and $n_2'=4$, now $\}(3,m_3')=\}(4,n_3')$, $\}(2,m_2')=\}(2,n_3')$ $\Rightarrow$ $m_3'=6$ and $n_3'=3$

that means $2m_2'...m_t'=236$ and $2n_2'...n_t'=243$ but $2\,3\,6\,\alpha\,2\,4\,3$

4-case: $m_1 \in |\, n_1 \in |$ then $m_1m_2...m_t\alpha 0m_2'...m_t'$ and $n_1n_2...n_t\alpha 0n_2'...n_t'$

$\}(0,m_2')=\}(1,n_2') \Rightarrow m_2'=1$ and $n_2'=5$ continuing we have $0m_2'...m_t'=0\,1\,5\,4$ and $1n_2'...n_t'=1\,5\,4\,0$, but $0\,1\,5\,4\,\alpha\,1\,5\,4\,0$

5-case: $m_1 \in |$, $n_1 \in |||$ then $m_1m_2...m_t\alpha 0m_2'...m_t'$ and $n_1n_2...n_t\alpha 2n_2'...n_t'$

$\}(0,m_2')=\}(2,n_2') \Rightarrow n_2'=0$ and $m_2' \in \{8,2\}$ if $m_2'=8$ applying $\sigma_0$ we have $m_1m_2...m_t\alpha 0\,2\,m_3''...m_t''$ $n_1n_2...n_t\alpha 2\,0\,n_2'...n_t'$

$\}(0,m_3'')=\}(2,n_3') \Rightarrow m_3''=0$ $\}(2,m_3'')=\}(0,n_3') \Rightarrow n_3'=2$ continuing we get $0\,2\,m_3''...m_t''=0\,2\,0\,2\,...$ and $2\,0\,n_3'...n_t'=2\,0\,2\,0\,...$ but this words are not from A'''.

6-case: $m_1 \in ||$, $n_1 \in |||$ then $m_1m_2...m_t\alpha 1m_2'...m_t'$ and $n_1n_2...n_t\alpha 2n_2'...n_t'$

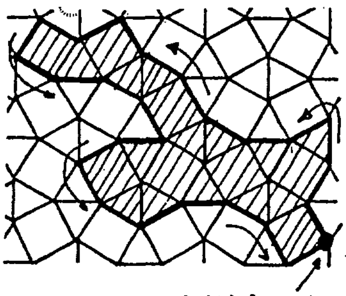
We are interested in the case when $m_1$ and $n_1$ do not satisfy any of previous observed cases. If for some i one of them is satisfied then we observe polygons $M_iM_{i+1}...M_{i-1}$ and $N_iN_{i+1}...N_{i-1}$ where $M_{t+k}=M_k$ and $N_{t+k}=N_k$. Since $\}(1,m_2')=\}(2,n_2')$ and $m_2,n_2$ do not satisfy cases 1,2,3,4,5 then $m_2'=2$ and $n_2'=9$. Next, we have $1m_2'...m_t'=1\,2\,9\,7$ and $2n_2'...n_t'=2\,9\,7\,1$ but $1\,2\,0\,7$ $2\,9\,7\,1$ therefore $m_1m_2...m_t\alpha n_1n_2...n_t$.

LEMMA 8: Let G be the grid obtained by tessellations $3^2.4.3.4$. Then: if A and B are congruent polygons then $r(A)\alpha r(B)$.

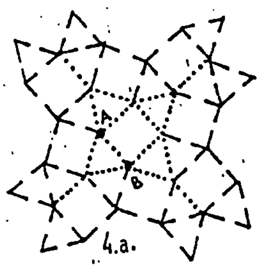
PROOF: Equivalence classes with respect to relation $\rho$ are:

$|=\{0,7,10,17\}$ $||=\{1,3,6,8,12,14,15,19\}$

$|||=\{2,4,5,9,11,13,16,18\}$ (see fig.4)



initial vertex




f(P) = 13 12 7 15 10 9 13 9 13 16 8 14 19 0 18 11 10 16 11 18 6 0 18 11 5


VECTORS: 10 11...18 19 are opposite
for : 0 1... 8 9

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| t: | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| k(t): | 1 | 2 | 1 | 3 | 6 | 17 | 35 | 101 |

Fig. 4.

Let $M=M_1M_2...M_t$ and $N=N_1N_2...N_t$ are congruent polygons and $r(M)=m_1m_2...m_t$ and $r(N)=n_1n_2...n_t$. Case of interest is:

case: $m_1 \in ||$, $n_1 \in ||$ then $m_1m_2...m_t\alpha 1m_2'...m_t'$ and $n_1n_2...n_t\alpha 1 1n_2'...n_t'$. Let us observe polygons $M'=ABM_3'...M_t'$ and $N'=BAN_3'...N_t'$ (see fig.4a):

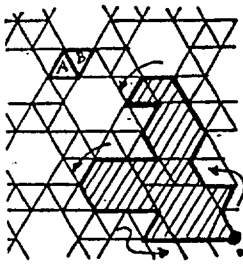
($r(M')=1m_2'...m_t'$ and $r(N')=11n_2'...n_t'$)). Since M' and N' are congruent, there exist isometry S which maps plane into itself such that $S(M')=S(ABM_3'...M_t')=S(A)S(B)S(M_3')..S(M_t')=BAN_3'...N_t'=N'$. But then S is either

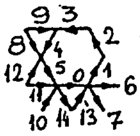
1) reflection in line s which is symmetry axes of segment |AB|.

or 2) half turn with centre in middle of segment |AB|.

If S is reflection then images of edges denoted by broken line (---) do not belong to grid G; therefore edges of polygon $BAN_3'...N'$ can be some of edges denoted with ····. Since polygonis conected, we conclude $n\leq 7$. But for $n\leq 7$ there are thirteen different α-equivalence classes and representatives of this classes are not congruent polygons so statement follows. In the case when S is half turn, proof is analogous.

For grid G obtained by tessellation $3^4.6$ (see fig.5) words which correspond to congruence polygons do not have to be α-equivalent. For example: for congruent triangles A and B (as it is shown in fig. 5) $r(A)\alpha r(B)$ but there is no isometry which 1) maps grid G into itself, 2) maps A into B.




VECTORS:15 16 ... 28 29 are opposite
for: 0 1 ... 13 14

| | | | | | | |
|---|---|---|---|---|---|---|
| t: | 3 | 4 | 5 | 6 | 7 | 8 |
| k(t): | 2 | 2 | 2 | 5 | 5 | 13 |

initial vertex

f(P) = 23 10 26 22 1 2 22 15 25 9 4 5 26 3 4 5 0 6 27 11 0 6

Fig. 5.

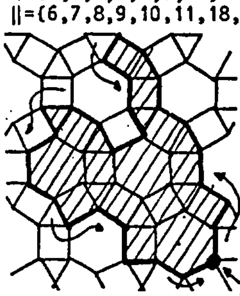
The proofs of following lemmas are omited, since they are analogous to proofs of Theorem 1 and Theorem 2.

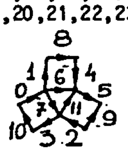
LEMMA 9: Let G be the grid obtained by tesselations 3.4.6.4. Then: if M and N are congruent polygons in grid G then $r(M)\alpha r(N)$.

PROOF: Equivalence classes with respect to relation $\rho$ are:

$|=\{0,1,2,3,4,5,12,13,14,15,16,17\}$

$||=\{6,7,8,9,10,11,18,19,20,21,22,23\}$






VECTORS:12 13 ... 22 23 are opposite
for: 0 1 ... 10 11

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| t: | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| k(t): | 1 | 0 | 0 | 4 | 4 | 3 | 3 | 4 |

initial vertex

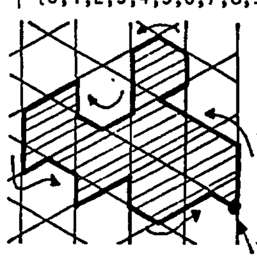
f(P) = 1 9 17 16 10 17 16 10 17 23 14 13 7 3 10 17 18 7 3 4 5 22 15 14 13 6 5 0

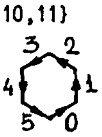
Fig. 6.

LEMMA 10: Let G be the grid obtained by tesselation 3.6.3.6. Then: if M and N are polygons in grid G then $r(M)\alpha r(N)$.

PROOF: There exist only one equivalence classes with respect to relation $\rho$.

$|=\{0,1,2,3,4,5,6,7,8,9,10,11\}$






VECTORS: 6 7 8 9 10 11 are opposite
for: 0 1 2 3 4 5

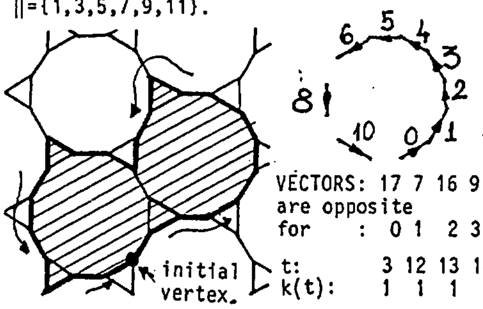
| | | | | | | |
|---|---|---|---|---|---|---|
| t: | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| k(t): | 1 | 1 | 1 | 2 | 1 | 2 | 4 |

initial vertex

f(P) = 10 1 2 11 2 0 1 2 3 4 7 6 11 10 6 3 4
7 9 8 7 9 0 4 5 0 9 8

LEMMA 11: Let G be the grid obtained by tessellations $3.12^2$. Then if M and N are congruent polygons in grid G then r(M)αr(N).

PROOF: Equivalence classes with respect to relation ρ are:
|={0,2,4,5,8,10,12,13,14,15,16,17}
||={1,3,5,7,9,11}.




VECTORS: 17 7 16 9 12 11 14 13 15 are opposite
for : 0 1 2 3 4 5 6 8 10

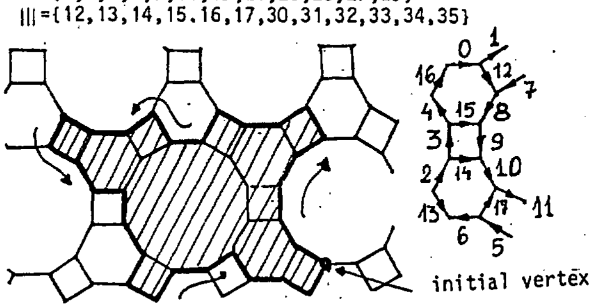
| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| t: | 3 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| k(t): | 1 | 1 | 1 | 3 | 3 | 3 | 1 | 1 |

f(P) = 1 14 11 0 1 2 3 4 5 15 16 7 8 4 5 15
16 7 8 9 16 14 11 0

Fig. 8.

LEMMA 12: Let G be the grid obtained by tessellation 4.6.12 then: if M and N are congruent polygons in grid G then r(M)αr(N).

PROOF: Equivalence classes with respect to rellation ρ are:
|={0,2,4,6,8,10,18,20,22,24,26,28}
||={1,3,5,7,9,11,19,21,23,25,27,29}
|||={12,13,14,15,16,17,30,31,32,33,34,35}




f(P) = 29 28 27 26 25 31 19 18 29 35 6 31 19
18 29 35 23 22 15 9 10 35 23 16 0 1
13 24 23 16

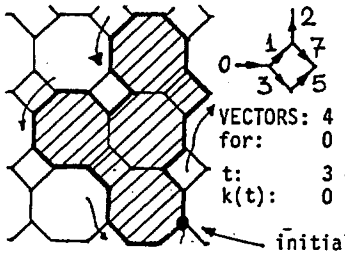
VECTORS: 18 19 ... 34 35 are opposite
for: 0 1 ... 16 17

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t: | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| k(t): | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 3 | 0 | 2 | 0 | 9 |

Fig. 9.

LEMMA 13. Let G be the grid obtained by tessellation $4.8^2$ then: if M, N are congruent polygons in grid G then r(M)αr(N).

PROOF: Equivalence classes with respect relation ρ are:
|={0,2,4,6} ; ||={1,3,5,7,9,10,11}




VECTORS: 4 11 6 8 9 10 are opposite
for: 0 1 2 3 5 7

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| t: | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| k(t): | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 2 | 0 | 4 |

initial vertex

f(P) = 2 8 1 2 5 10 2 8 4 9 6 7 9 8 4 9 6 7
0 3 6 7 0 1

Fig. 10.

Let grid G be obtained by one of semi-regular tessellations $3^3.4^2$, $3^2.4.3.4$, 3.4.6.4, 3.6.3.6, $3.12^2$, 4.6.12, $4.8^2$ then from lemmas 6-13 follows:

THEOREM 1: Circuits $C_1$ and $C_2$ in grid G are isomorphic circuits iff they are congruent circuits.

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